# Performance review of simple management procedures: Supplemental Appendix A 

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## Appendix A: Reference methods

## DCAC

In circumstances where the information available is insufficient to derive a catch-limit from stock assessment Depletion Corrected Average Catch has been applied (DCAC, MacCall 2009). DCAC attempts to calculate average catch accounting for the removal of "windfall harvest" of less productive biomass that may have occurred as the stock became depleted. DCAC requires inputs for $M, F_{M S Y} / M$ (or c), $B_{M S Y} / B_{0}$ (stock biomass at MSY relative to unfished, $B_{\text {peak }}$ ) and $B_{\text {cu }} / B_{0}$ (current stock depletion, D). A number of samples are drawn from the following distributions:

App.A.1a) $\quad M_{D C A C} \sim \operatorname{lognormal}(\mu=M, S D=0.5)$

App.A.1b) $\quad c_{D C A C} \sim \operatorname{lognormal}(\mu=c, S D=0.2)$

App.A.1c) $\quad D_{D C A C} \sim \operatorname{lognormal}(\mu=D, S D=0.2)$
where, in keeping with MacCall's (2009) approach, the SDs for $M$ and c are set to 0.5 and 0.2 , respectively.. MacCall (2009) states that "unlike the other parameters, the precision of [depletion $D$ ] is entirely dependent on the data and method used in its estimation, and there is no clear value of precision that can serve as a default". Subsequently, Dick and MacCall (2011) assume a default distribution with a CV of 0.25 . We adopt a beta distribution for depletion:

App.A.2a) $\quad D_{D C A C} \sim \operatorname{beta}\left(\mu=D_{\text {obs }}, C V=0.25\right)$ where $D_{o b s}<0.5$

App.A.2b) $\quad 1-D_{D C A C} \sim \operatorname{beta}\left(\mu=1-D_{\text {obs }}, C V=0.25\right)$ where $D_{\text {obs }}>0.5$

For each sample of these parameters, sustainable yield ( $S Y$ ) is calculated by:

App.A.3)

$$
S Y_{D C A C}=\frac{\sum C_{o b s}}{n+\left(1-D_{D C A C}\right) /\left(\text { Bpeak }_{D C A C} c_{D C A C} M_{D C A C}\right)}=\frac{\sum C_{o b s}}{n+\left(1-D_{D C A C}\right) /\left(0.4 c_{D C A C} M_{D C A C}\right)}
$$

where the $C_{o b s}$ are annual historical catches and $n$ is the number of years of historical catches.

This stochastic approach produces numerous samples of the derived sustainable yield (SY) of which a percentile (typically the median) is used as the TAC.

## $\mathbf{F}_{\text {MSY }} / \mathbf{M}$ ratio 'Fratio'

It has been suggested that ratios of $F_{M S Y} / M(c)$ may be robust to broad life-history types and fisheries exploitation scenarios. Gulland (1971) proposed a simple method of setting maximum sustainable yield $M S Y=0.5 M \cdot B_{0}$, in doing so assuming that $B_{M S Y} / B_{0}=0.5$ and $F_{M S Y} / M=1$. Subsequent publications have revised this $F_{M S Y}$ recommendation downwards. The Fratio MP is simulated by generating imperfect knowledge regarding $M$, current absolute biomass and the ratio of $F_{M S Y} / M$.

## Delay-difference stock assessment (DD)

The performance of a delay-difference model (Deriso, 1980; Schnute, 1985) fitted to catch and effort data is evaluated to provide a reference for the performance of the other MPs. The delay-difference model requires additional auxiliary (independent) information regarding the form of the stock-recruit function, the fraction mature at age, somatic growth, $M$, and the selectivity-at-age curve. The delaydifference stock assessment method provides estimates of $B_{c u r}$ and $F_{M S Y}$ and therefore direct estimates of an appropriate catch limit.

The delay-difference model is fitted to annual total catch and effort data. The model is parameterized according to: maximum sustainable yield, $M S Y_{D D}$ and harvest rate at maximum sustainable yield, $U m s y_{D D}$. The catchability coefficient scaling effort to fishing mortality rate is also estimated. The growth parameters $\alpha$ and $\rho$ of the Ford-Brody growth model $\left(W_{a+l}=\alpha+\rho W_{a}\right)$ are approximated from the known weight at age $W$, for each simulation:

App.A.4)

$$
\alpha=W_{\infty}(1-\rho)
$$

$$
\rho=\frac{W_{V_{o b s}+2}-W_{\infty}}{W_{V_{o b s}+1}-W_{\infty}}
$$

where $W_{\infty}$ is the maximum weight of an individual. Selectivity at age is assumed to follow the maturity schedule and $A M_{\text {obs }}$ is the observed age at $50 \%$ maturity selectivity determined from the ascending limb of the selectivity curve $\omega$ (Eqn. App.A.12). Since bias in the age at $50 \%$ maturity may strongly affect the delay-difference model, $A M_{\text {obs }}$ is simulated subject to imperfect knowledge (Table App.C.1). Survival rate at maximum sustainable yield is given by $S m s y=\exp \left(-M_{o b s}\right)\left(1-U m s y_{D D}\right)$ so that the number of spawners per recruit, $S P R$ is given by:

App.A.5)

$$
S P R=\frac{(\alpha \cdot S m s y) /(1-S m s y)+W_{A M_{o b s}}}{1-\rho \cdot S m s y}
$$

The Beverton-Holt parameter $\alpha_{r e c}$, the maximum recruits per spawner as spawner biomass approaches zero, is calculated:

App.A.6)

$$
\alpha_{r e c}=1 /\left(\left(1-U m s y_{D D}\right)^{2}\left(S P R+U m s y_{D D} \cdot \Delta_{S P R}\right)\right)
$$

The derivative of yield with respect to harvest rate $\Delta_{S P R}$, evaluated at $U m s y_{D D}$ is given by:

App.A.7)

$$
\Delta_{S P R}=-S_{0} \frac{p}{1-\rho \cdot S m s y} \frac{S P R+1}{1-\rho \cdot S m s y} \frac{\alpha}{(1-S m s y)}+\frac{S m s y \cdot \alpha}{(1-S m s y)^{2}}
$$

where $S_{o}$ is unfished survival rate $S_{0}=\exp (-M)$. The Beverton-Holt parameter $\beta_{\text {rec }}$ is calculated as:

App.A.8)

$$
\beta_{r e c}=\frac{U m s y_{D D} \cdot\left(\alpha_{r e c} \cdot S P R-1 /\left(1-U m s y_{D D}\right)\right)}{M S Y_{D D}}
$$

Unfished recruitment $R_{0}$ is allocated to recruitments up to and including the age at recruitment to the fishery $A m_{o b s}$ and is given by:

App.A.9) $\quad R_{0}=\frac{\alpha_{r e c} \cdot S P R_{0}-1}{\beta_{r e c} \cdot S P R_{0}}$
where unfished spawners per recruit $S P R_{0}$ is calculated using Eqn. App.A. 5 when $S_{\text {msy }}$ is replaced by $S_{0}$. It follows that initial biomass $B_{l}$ is given by: $B_{1}=R_{0} \cdot S P R_{0}$ and initial numbers $N_{l}$ is given by $N_{1}=R_{0} /\left(1-S_{0}\right)$. From this initialization, biomass dynamics are calculated by:

App.A.10)

$$
B_{y+1}=S_{y}\left(\alpha \cdot N_{y}+\rho \cdot B_{y}\right)+W_{V} \cdot R_{y+1} ; \quad N_{y+1}=S_{y} \cdot N_{y}+R_{y+1}
$$

where $S_{y}=\exp \left(-E_{y} q_{D D}-\boldsymbol{M}\right)$ is the survival rate in year $y, N$ represents stock numbers, $B$ is the biomass, $W_{k}$ is the weight of an individual at the age at $50 \%$ selectivity $k, M$ is the natural mortality rate (assumed to be known exactly), $q_{D D}$ is the estimated catchability, $E_{\mathrm{y}}$ is the observed fishing effort during year $y$, and $R_{\mathrm{y}}$ represents the number of recruits during year $y$ :

App.A.11)

$$
R_{y+k}=\frac{\alpha_{r e c}\left(B_{y}-C_{y}\right)}{1+\beta_{r e c}\left(B_{y}-C_{y}\right)}
$$

where catches $C$, are given by: $C_{y}=B_{y}\left(1-\exp \left(-q_{D D} E_{y}\right)\right)$.
The model is fitted to observed (simulated) catches by minimizing a global objective $O$ that is calculated by the sum of the negative log likelihood of the catches (excluding constant terms):

App.A.12)

$$
O=\sum_{y}\left[\frac{\log (2 \pi)}{2}+\log \left(\sigma_{c}\right)+\frac{\left(\log \left(C_{y}^{o b s}\right)-\log \left(C_{y}\right)\right)^{2}}{2 \sigma_{c}^{2}}\right]
$$

where $\sigma_{c}$ is the assumed standard deviation (in log space) of the observation error.

## Appendix B. Operating model

## Simulating stock dynamics

A standard age-structured, spatial model identical to that of Carruthers et al. (2014) was used to simulate population and fishery dynamics. Ranges of parameters and variables allowed variation among simulations for a given stock (e.g., natural mortality rate $M$, slope in recent fishing effort, targeting). All parameters that vary as random variables across simulations are denoted with a tilde (e.g. $\tilde{\sigma}$ ). The probability distributions from which these parameters are sampled are detailed in Table App.B.1. Hence, each parameter or variable denoted with a tilde represents a sample from a distribution specific to each stock. This convention alleviates the need for a simulation and stock subscript for every parameter or variable described below. For example, the symbol $\tilde{\sigma}$ represents $\widetilde{\sigma}_{s, i} \sim f\left(\theta_{s}\right)$ which is the sample of the parameter $\tilde{\sigma}_{\tilde{\sigma}}$ corresponding with the $i^{\text {th }}$ simulation for stock $s$, drawn from a distribution function $f()$, which has stock specific parameters $\theta_{s}$.

The numbers of individuals recruited to the first age group $N_{y, a=l, r}$ in each year $y$, and area $r$ is calculated using a Beverton-Holt stock-recruitment relationship with log-normal recruitment deviations:

App. B.1)

$$
N_{y, a=1, r}=\exp \left(P_{y, r}-\frac{\tilde{\sigma}_{p r o c}^{2}}{2}\right) \frac{0.8 R_{0} \tilde{h} S S B_{y, r}}{0.2 S S B_{0}(1-\tilde{h})+(\tilde{h}-0.2) S S B_{y, r}}
$$

where $h$ is the steepness parameter, $R_{0}$ is the mean recruitment given unfished conditions, $S S B_{y, r}$ is spawning stock biomass in the previous year and $S S B_{0}$ is the mean spawning stock biomass under unfished conditions. The process error term $P$, is an autocorrelated random variable:

$$
P_{y, r}=\varsigma \cdot P_{y-1, r}+\phi_{y, r} \cdot \sqrt{1-\varsigma^{2}}, \quad \phi_{y, r} \sim \operatorname{normal}\left(0, \widetilde{\sigma}_{p r o c}\right)
$$

where $\varsigma$ controls the level of autocorrelation in recruitment deviations and $\phi$ is a normally distributed random variable with mean zero and standard deviation $\tilde{\sigma}_{\text {proc }}$.

The spawning stock biomass $S S B$, is given by:

$$
S S B_{y, r}=\sum_{a=1}^{n_{a}} m_{a} W_{a} N_{y, a, r}
$$

where $m_{a}$ is the maturity-at-age $a$, and the maximum age $n_{a}$ is specific to each stock. Maturity-at-age is assumed to follow a logistic relationship with age; the slope of the transition from immature to mature is determined by the precision parameter $\sigma_{A}$, and the inflection point $\widetilde{A}_{m}$ that is the age where $50 \%$ of individuals are mature (sampled form a random uniform distribution):

App.B.4)

$$
m_{a}=\frac{1}{1+\exp \left(\left(\tilde{A}_{m}-a\right) / \sigma_{A}\right)}
$$

Numbers at age are converted to biomass using the von Bertalanffy growth equation:

App.B.5)

$$
L_{a}=\tilde{L}_{\mathrm{inf}}\left(1-e^{-\tilde{\kappa}\left(a-t_{0}\right)}\right)
$$

where $L_{a}$ is the length of an individual of age $a$, the asymptotic length is $L_{i n f}$, and $\kappa$ is the slope at the theoretical age at zero length $t_{0}$. Simulated $L_{i n f}$ and $\kappa$ are sampled independently and assumed to be time-varying with mean percentage slope $\alpha_{\text {Linf }}$ and $\alpha_{\kappa}$ (Table App.B.1.). Parameters $\alpha_{L i n f}$ and $\alpha_{\kappa}$ were sampled independently from uniform distributions between -0.25 and 0.25 percent per year to investigate whether small temporal changes in growth could affect MP performance. Inter-annual variability in $L_{i n f}$ and $\kappa$ were simulated from log-normal distributions with mean 1, and standard deviations $s d_{L i n f}$ and $s d_{K}$.

Weight at age $W_{a}$ is assumed to be related to length by:

App.B.6)

$$
W_{a}=\beta L_{a}{ }^{\alpha}
$$

For ages greater than 1, fishing mortality is assumed to occur before natural mortality and the numbers-at-age are calculated by:

App.B.7)

$$
N_{y, a, r}=\left(N_{y-1, a-1, r}-C_{y-1, a-1, r}\right) \exp (-\widetilde{M})
$$

Similarly to $L_{i n f}$ and $\kappa$, inter-annual variability in natural mortality rate was generated by sampling from a lognormal distribution with mean 1 and standard deviation $s d_{M}$. The underlying trend (per cent per year) in natural mortality was sampled from a random uniform distribution (Table App.B.1.). No "plus group" is modelled; instead the maximum age is set sufficiently high that survival to the maximum age is less than $1 \%$ under unfished conditions.

Movement and spatial targeting dynamics were not the focus of this simulation evaluation. The generic two-area model of the simulation framework was parameterized to mimic a fully diffuse stock that was not subject to spatial targeting.

## Simulating fishery dynamics

The selectivity at age $\omega_{a}$, was calculated using a double normal curve with age at maximum selectivity $m_{\text {sel }}$, an ascending limb standard deviation of $\sigma_{\text {sell }}$ and a descending limb standard deviation $\sigma_{\text {sel2 }}$. These standard deviations were determined for each simulation by numerically solving for two user-specified quantities that are more intuitive: (1) the minimum age at $5 \%$ maximum selectivity $\tilde{a}_{0.05}$, and (2) the selectivity of the oldest age class $\tilde{\omega}_{\text {old }}$. To sample a wide range of selectivity dynamics $\tilde{a}_{0.05}$ was sampled from a uniform distribution between $20 \%$ and $50 \%$ age at maturity. Additionally $\tilde{\omega}_{\text {old }}$ could
range from a $0-100 \%$, representing dome-shaped selectivity curve where older fish are not fished to a 'flatted - topped' selectivity where older fish are fished at the same rate as younger fish.

The ascending limb age selectivity $A_{a}$ (before normalization to a maximum value of 1 ) is given by:

App.B.8)

$$
A_{a}=\frac{1}{\sqrt{2 \pi \tilde{\sigma}_{\text {sel1 }}{ }^{2}}} \exp \left(-\frac{\left(a-m_{\text {sel }}\right)^{2}}{\tilde{\sigma}_{\text {sel } 1}^{2}}\right)
$$

The descending limb selectivity $D_{a}$ is given by:

App.B.9)

$$
D_{a}=\frac{1}{\sqrt{2 \pi \tilde{\sigma}_{\text {sel2 }}{ }^{2}}} \exp \left(-\frac{\left(a-m_{\text {sel }}\right)^{2}}{\tilde{\sigma}_{\text {sel } 2}^{2}}\right)
$$

The selectivity at age is given by:

App.B.10)

$$
\omega_{a}= \begin{cases}A_{a} / \max \left(A_{j}\right) & j \leq m_{\text {sel }} \\ D_{a} / \max \left(D_{j}\right) & j>m_{\text {sel }}\end{cases}
$$

Catch in numbers is calculated by:

$$
C_{y, a, r}=N_{y, a, r}\left(1-\exp \left(-\omega_{a} p_{y, r} F_{y, a}\right)\right)
$$

where $F$ is the instantaneous fishing mortality rate.

Observed catch is calculated by multiplying simulated catch in numbers-at-age by weight-at-age and adding observation error:

App.B.12)

$$
C_{y}^{o b s}=\exp \left(\varepsilon_{y, a, r}-\frac{\tilde{\sigma}_{o b s}^{2}}{2}\right) \sum_{a} \sum_{r} C_{y, a, r} W_{a}, \quad \varepsilon_{y, a, r} \sim \operatorname{dnorm}\left(\mathrm{O}, \tilde{\sigma}_{o b s}\right)
$$

The error term $\varepsilon$, is drawn from a standard normal distribution whose standard deviation $\sigma_{o b s}$ is sampled at random in each simulation:

Fishing mortality rate $F$, was assumed to be proportional to effort according to the constant $\widetilde{\boldsymbol{q}}$ which was determined by numerical optimizing for sampled current depletion $D$ (Table App.B.1. below).

$$
F_{y}=\tilde{q} E_{y}
$$

Total effort is not related to biomass levels, and in historical and future projections can remain high even at very low biomass levels. The maximum instantaneous fishing mortality rate is limited to $90 \%$ to prevent the simulation of large declines in stock biomass in any year due to TAC recommendations that are occasionally very high.

Log-normal variability in effort is added to a general effort trend $V$ :

$$
E_{y}=\exp \left(\varphi_{y}-\frac{\tilde{\sigma}_{e f f}^{2}}{2}\right) V_{y}, \quad \varphi_{y} \sim \operatorname{dnorm}\left(0, \tilde{\sigma}_{e f f}\right)
$$

where the effort variability term $\varphi_{y}$ is randomly sampled from a standard normal distribution that has a standard deviation, $\tilde{\sigma}_{e f f}$ drawn at random for each simulation from a uniform distribution ranging from 0.1 to 0.4 .

A range of effort variability is sampled to assess how the degree of auto-correlation affected the performance of stock status classification methods. The general trend in effort is determined by a linear model of change in effort over time with slope $a_{E}$, and intercept of 0.5 :

$$
\frac{d V_{y}}{d y}=a_{E} y+0.5
$$

This functional form allows effort to increase, decrease or remain unchanged over time. This effort model is constrained by sampling positive values for initial changes in effort (effort is increasing at the start of the time series). The final annual change in effort $\tilde{\Delta}_{E}$, was sampled from a uniform distribution between -1 and 1 to simulate a range of final effort trajectories including strongly decreasing and increasing effort:

$$
\tilde{\Delta}_{E}=\frac{d V_{\text {final }}}{d y}
$$

For any simulated effort time series, the slope $a_{E}$, can then be calculated from the total number of years in the time series $n_{y}$, and the sampled intercept of 0.5 :

App.B.17)

$$
a_{E}=\left(\widetilde{\Delta}_{E}-0.5\right) / n_{y}
$$

Simulated effort time series that included negative values were discarded. All of the stocks experienced the same effort dynamics.

In any given year, spatial fishing effort is assumed to be proportional to the distribution of the vulnerable biomass in the previous year, modified by a targeting parameter $\lambda$, that controls how strongly fishing effort will be distributed in relation to vulnerable biomass. The fraction of fishing effort $P$, allocated to each region $r$, in a given year $y$, is calculated:

$$
p_{y, r}=\left(\sum_{a} \omega_{a} W_{a} N_{y, a, r}\right)^{\lambda} / \sum_{r}\left(\sum_{a} \omega_{a} W_{a} N_{y, a, r}\right)^{\lambda}
$$

The values for $p$ sum to 1 in any year so they can be used to distribute total effort $E_{y}$ across areas in each year such that mean $F$ among areas is the same as total annual $F$. Fishing is distributed evenly regardless of the vulnerable biomass in the previous year when the targeting parameter $\lambda$ is zero. Spatial fishing will be distributed in favour of areas of high vulnerable biomass when $\lambda$ is positive and distributed away from such areas when $\lambda$ is negative. For all stocks a range of the targeting parameter was sampled from a random uniform distribution between -0.5 and 1 to evaluate the impact on MPs of the distribution of fishing relative to the population.

## Parameterization of stock dynamics

Given the availability of full stock assessments with which to characterize their stock dynamics, we chose Pacific herring (DFO, 2012), Atlantic bluefin tuna (ICCAT, 2012), and canary rockfish (Wallace and Cope, 2011) as case-studies that span a range of longevity. The values of input parameters and the sources of these inputs are detailed in Table App.B.1.

Table App.B.1. Summary of the variables/parameters that define each of the stock simulations, including values and/or the range over which they are sampled. The values for simulations were taken from recent stock assessments for Pacific herring (DFO, 2012), eastern Atlantic bluefin tuna (ICCAT, 2012) and canary rockfish (Wallace and Cope, 2011). Where two values are provided, variables are sampled from a uniform distribution with the lower and upper bounds listed.

| Name |  | Pacific herring |  | Eastern Atlantic bluefin tuna |  | Canary rockfish |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Maximum age | $n_{a}$ | 10 |  | 32 |  | 64 |  |
| Steepness | $h$ | 0.4 | 0.6 | 0.6 | 0.9 | 0.35 | 0.7 |
| Mean natural mortality rate | $\mu_{M}$ | 0.28 | 0.38 | 0.12 | 0.16 | 0.04 | 0.08 |
| Interannual variability in natural mortality rate | $s d_{M}$ | 0 | 0.1 | 0 | 0.1 | 0 | 0.05 |
| Gradient in natural mortality rate (per cent $\mathrm{y}^{-1}$ ) | $\alpha_{M}$ | -0.5 | 0.5 | -0.5 | 0.5 | -0.5 | 0.5 |
| Theoretical age at length zero | to | -0.025 |  | -0.97 |  | -0.04 |  |
| Mean maximum length | $\mu_{\text {Linf }}$ | 25 | 29 | 315 | 325 | 62 | 68 |
| Interannual variability in maximum length | $s d_{\text {Linf }}$ | 0 | 0.025 | 0 | 0.025 | 0 | 0.025 |
| Gradient in maximum length (per cent $\mathrm{y}^{-1}$ ) | $\alpha_{\text {Linf }}$ | -0.25 | 0.25 | -0.25 | 0.25 | -0.25 | 0.25 |
| Mean von Bertalanffy growth coefficient | $\mu_{\text {K }}$ | 0.43 | 0.53 | 0.08 | 0.1 | 0.122 | 0.128 |
| Interannual variability in the growth coefficient k | $s d_{k}$ | 0 | 0.025 | 0 | 0.025 | 0 | 0.025 |
| Gradient in the growth coefficient k (per cent $\mathrm{y}^{-1}$ ) | $\alpha_{\text {K }}$ | -0.25 | 0.25 | -0.25 | 0.25 | -0.25 | 0.25 |
| Weight-length parameter $\mathrm{a}\left(\mathrm{W}=\mathrm{aL}^{\text {b }}\right.$ ) | $\alpha_{W L}$ | $4.50 \mathrm{E}-06$ |  | $1.96 \mathrm{E}-05$ |  | $1.55 \mathrm{E}-05$ |  |
| Weight-length parameter $\mathrm{b}\left(\mathrm{W}=\mathrm{aL}^{\text {b }}\right.$ ) | $b_{W L}$ | 3.127 |  | 3.009 |  | 3.03 |  |
| Stock depletion, biomass relative to unfished | D | 0.025 | 0.6 | 0.025 | 0.6 | 0.025 | 0.6 |
| Age at 50\% maturity | $A_{\text {m }}$ | 1.7 | 2.3 | 3.5 | 5 | 6.5 | 9.5 |
| Spatial targetting parameter | $\lambda$ | 0.5 | 1 | 0.5 | 1 | 0.5 | 1 |
| Log-normal recruitment variation | $\sigma_{R}$ | 0.2 | 0.4 | 0.1 | 0.3 | 0.2 | 0.5 |

## Appendix C: Simulating imperfect information

Table App.C.1. Summary of the bias /error parameters and related distributions that control the accuracy and precision of knowledge of the simulated system that is subsequently used by the datalimited methods and harvest control rules. The log-normal distribution described in the table below
where $\sim \operatorname{lognormal}(\mu, \sigma)$ is the exponent of the normal distribution with mean $\mu$ and standard deviation
$\sigma$, parameters: dnorm $\left(-0.5 \log \left(1+\sigma^{2} / \mu^{2}\right), \sqrt{\log \left(1+\sigma^{2} / \mu^{2}\right)}\right)$.

| Variable | Symbol | Related functions |
| :---: | :---: | :---: |
| The standard deviation of the log-normally distributed bias in natural mortality rate $M$ ( $\mu_{M}$ varies among simulations) | $\Upsilon_{M}$ | $\begin{aligned} & M_{o b s}=M \times \mu_{M} \\ & \mu_{M \sim \operatorname{lognormal}}\left(\mu=1, \Upsilon_{M}\right) \end{aligned}$ |
| The standard deviation of the log-normally distributed bias in von Bertalanffy growth rate parameter $K$ ( $\mu_{K}$ varies among simulations) | $\Upsilon_{K}$ | $\begin{aligned} & K_{o b s}=K \times \mu_{K} \\ & \mu_{K} \sim \operatorname{lognormal}\left(\mu=1, \Upsilon_{K}\right) \end{aligned}$ |
| The standard deviation of the log-normally distributed bias in biomass at maximum sustainble yield $B_{M S Y}$ ( $\mu_{B m s y}$ varies among simulations) | $\Upsilon_{\text {Bmsy }}$ | $\begin{aligned} & \text { Bmsy }_{o b s}=\text { Bmsy } \times \mu_{B m s y} \\ & \mu_{B m s y} \sim \operatorname{lognormal}\left(\mu=1, Y_{B m s y}\right) \end{aligned}$ |
| The standard deviation of the log-normally distributed bias in biomass at maximum sustainable yield relative to unfished Bpeak ( $B_{M S Y} / B_{0}$, $\mu_{\text {Bpeak }}$ varies among simulations) | $\Upsilon_{\text {Bpeak }}$ | $\begin{aligned} & \text { Bpeak }_{\text {obs }}=\text { Bpeak } \times \mu_{\text {Bpeak }} \\ & \mu_{\text {Bpeak }} \sim \operatorname{lognormal}\left(\mu=1, \Upsilon_{\text {Bpeak }}\right) \end{aligned}$ |
| The standard deviation of the log-normally distributed bias in the ratio of maximum sustainable fishing mortality rate to natural mortality rate $F M S Y_{-} M\left(\mu_{F M S Y_{-} M}\right.$ varies among simulations) | $\Upsilon_{\text {FMSY_M }}$ | $\begin{aligned} & F M S Y_{-} M_{o b s}=F M S Y_{-} M \times \mu_{F M S Y_{-} M} \\ & \mu_{F M S Y_{-} M} \sim \operatorname{lognormal}\left(\mu=1, \Upsilon_{F M S Y_{-} M}\right) \end{aligned}$ |
| The standard deviation of the log-normally distributed bias in $\operatorname{MSY}\left(\mu_{M S Y}\right.$ varies among simulations) | $\Upsilon_{M S Y}$ | $\begin{aligned} & M S Y_{o b s}=M S Y \times \mu_{M S Y} \\ & \mu_{M S Y} \sim \operatorname{lognormal}\left(\mu=1, \Upsilon_{M S Y}\right) \end{aligned}$ |
| The standard deviation of the log-normally distributed bias in the age at first maturity Am ( $\mu_{A m}$ varies among simulations) | $\Upsilon_{\text {Am }}$ | $\begin{aligned} & A m_{o b s}=A m \times \mu_{A m} \\ & \mu_{A m} \sim \operatorname{lognormal}\left(\mu=1, \Upsilon_{A m}\right) \end{aligned}$ |
| Uniformly distributed observation error in recruitment ( $R_{\text {obs }}$, varies among years and simulations, $\sigma_{\text {Robs }}$ varies among simulations) | $\sigma_{\text {Robs }}$ | $\begin{aligned} & \text { Robs }=\operatorname{lognormal}\left(\mu=R, \sigma_{\text {Robs }}\right) \\ & \sigma_{\text {Robs }} \sim \cup\left(L_{\text {Robs }}, U_{\text {Robs }}\right) \end{aligned}$ |
| The standard deviation of the log-normally distributed bias in the current level of stock depletion $D\left(B / B_{0 ;} D_{o b s}\right.$ and $j_{D}$ vary among projected years and simulations; $\mu_{D}$ and $\sigma_{D}$ vary among simulations) | $r_{D}$ | $\begin{aligned} & D_{o b s}=D \times j_{D} \\ & j_{D^{\sim}} \sim \operatorname{lognormal}\left(\mu_{D}, \sigma_{D}\right) \\ & \mu_{D} \sim \operatorname{lognormal}\left(\mu=1, \Upsilon_{D}\right) \end{aligned}$ |
| Uniformly distributed observation error in current stock depletion $\mu_{D}$ for projected years | $\sigma_{D}$ | $\sigma_{D} \sim U\left(L_{D}, U_{D}\right)$ |
| The standard deviation of the log-normally distributed bias in catches |  | $C_{\text {obs }}=C \times \gamma C$ |
| $C$ ( $C_{\text {obs }}$ and $\gamma_{C}$ vary among projected years and simulations; $\mu_{C}$ and | $r_{C}$ | $r_{C} \sim \operatorname{lognormal}\left(\mu_{C,} \sigma_{C}\right)$ |
| $\sigma_{C}$ vary among simulations) |  | $\mu_{C} \sim \operatorname{lognormal}\left(\mu=1, \Upsilon_{C}\right)$ |
| Uniformly distributed observation error in catches | $\sigma C$ | $\sigma_{C} \sim U\left(L_{C}, U_{C}\right)$ |
| Standard deviation in log-normal error in the relative abundance index for projected years ( $I$ and $\gamma_{I}$ vary among years and simulations, $\sigma_{I}$ varies among simulations) | $\sigma_{I}$ | $\begin{aligned} & I=B^{\beta} \times \gamma_{I} \\ & r_{I \sim \operatorname{lognormal}\left(1, \sigma_{I}\right)} \\ & \sigma_{I} \sim U\left(L_{I}, U_{I}\right) \end{aligned}$ |
| The beta parameter controlling hyperstability / hyperdepletion in the abundance index ( $\beta$ varies among simulations) | $\beta$ | $L N(\beta) \sim U\left(L N\left(\beta_{\text {min }}\right), L N\left(\beta_{\text {max }}\right)\right)$ |
| Loguniform bias in current biomass ( $B_{\text {obs }}$ and $j_{B}$ vary among years and simulations, $\mu_{B}$ and $\sigma_{B}$ vary among simulations) | $\begin{aligned} & \min _{B} \\ & \max _{B} \end{aligned}$ | $\begin{aligned} & B_{o b s}=B \times j_{B} \quad j_{B} \sim \operatorname{lognormal}\left(\mu_{B}, \sigma_{B}\right) \\ & \log \left(\mu_{B}\right) \sim U\left(\min _{B}, \max _{B}\right) \end{aligned}$ |
| The maximum standard deviation for log-normal error in current biomass for projected years | $\sigma_{B}$ | $\sigma_{\text {Bcur }} \sim U\left(L_{B}, U_{B}\right)$ |

