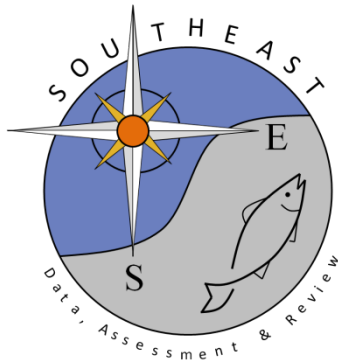# Synthesis of Literature on Von Bertalanffy Growth Parameter Correlations

Nancie Cummings, Skyler Sagarese and Bill Harford

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**Synthesis of Literature on Von Bertalanffy Growth Parameter Correlations**

Nancie Cummings[1], Skyler Sagarese[1] and Bill Harford[2]

[1] NOAA Fisheries, Southeast Fisheries Science Center
Sustainable Fisheries Division
75 Virginia Beach Drive
Miami, FL 33149

[2] Cooperative Institute for Marine and Atmospheric Studies
Rosenstiel School of Marine & Atmospheric Science
University of Miami, Miami, FL 33149

**Purpose and Approach**

In 2016 the Southeast Fisheries Science Center (SEFSC) of NOAA Fisheries conducted an evaluation of six US Caribbean species under the Southeast Data, Assessment and Review (SEDAR 46) process (see http://safmc.net/science-and-statistics/sedar-stock-assessment-program for a description of the SEDAR process). In the SEDAR 46 evaluation, the analytical framework employed was the Data-Limited Methods Toolkit "DLMtool" (Carruthers et al. 2014, 2015). The SEDAR 46 DLMtool results were subsequently reviewed by the Center for Independent Experts (CIE, https://www.st.nmfs.noaa.gov/science-quality-assurance/cie-peer-reviews/index) during February 23 – 25 in Miami FL (www.sefsc.noaa.gov/sedar/46). In April (18-22) 2016, the SEDAR 46 results were presented to the Caribbean Fishery Management Council (CFMC) Scientific and Statistical Committee (SSC). Both the SEDAR 46 Review Panel and the CFMC SSC recommended that additional modeling considerations were needed before results were useful for management. Specific recommendations included the: 1) consideration of biological correlations between growth parameters within the modeling framework; and 2) removal of biologically implausible parameter combinations of the von Bertalanffy growth parameters within the operating models developed for the Management Strategy Evaluation (MSE). This note describes the work conducted since the February 2016 SEDAR Review Workshop and the April 2016 CFMC SSC meeting to address these two specific recommendations.

The process was conducted in two steps. First, a plausible ranges and point estimates of correlations between growth model parameters was identified following a literature review. Second, the source code for conducting MSE using the DLMtool was revised to incorporate the selected range of values. This note details the process and results for item 1. An associated working document (Harford, unpublished) describing step 2 (incorporation of the correlations in the MSE) is currently underway.

*Literature Review*

A search of the available literature was conducted to obtain published estimates of correlations between the three von Bertalanffy growth parameters: asymptotic size (Linf or L∞), growth rate (K) and time at size zero ($t_0$). The search included published and unpublished sources (e.g., referred scientific journals, technical reports, conference proceedings, etc.).

**Results**

Six studies were identified as providing correlations between von Bertalanffy parameters (Table 1). Figure 1 provides histogram summaries of the correlation statistics for the three von Bertalanffy parameter relationships. Table 1 provides summarized statistics on the von Bertalanffy correlations for the three parameter relationships. The summarized values for the estimate of and the range of the correlations for the three von Bertalanffy parameters were used to inform the updated DLMtool application. Table 2 provides summary correlation statistics for the three parameter combinations (Linf, K; Linf, t0; K, t0).

The meta-analysis of the relationship between von Bertalanffy asymptotic size (Linf) and growth rate (K) from the six studies corroborated the belief that the two parameters are negatively correlated (mean = −0. 58, range (mean ± 1SD) = −0.87, −0.29). The negative relationship represents the tradeoff between growth and reproduction such that high rates of growth lead to more rapid onset of maturity and ultimately smaller asymptotic sizes (von Bertalanffy 1937; Beverton 1992). However, it is important to note that very few studies exist that provide direct evidence of this relationship based on analyses of multiple species or populations within a taxon (Helser et al. 2004). The analysis of Helser and Lai (2004) for largemouth bass and Helser et al.'s (2004) meta-analyses of 46 populations of *Sebastes* spp. suggest that although Linf and K are correlated, the level of correlation may be much lower than the values suggested in the literature (−0.8 to −0.9, Quinn and Deriso 1999), and in fact on the order of ~ 50% lower. Helser et al. (2004) even suggests that the Linf and K correlation may be lower, on the order of −0.4, for more diverse taxa. Helser et al. (2004), Helser and Lai (2004), noted the influence of covariates on the relationship (e.g., depth, maturity, latitude, environmental factors). It is important to note that these latter researchers have applied modeling procedures including Bayesian non-linear random effects model (Pilling et al. 2002) and/or Bayesian hierarchical models (Helser and Lai 2004; Helser et al. 2004; He and Bence 2007) which better account for individual variation through the incorporation of random effects. Although computationally more intensive, these improved modeling approaches yield results that are more credible than those resulting from simple regression estimates (i.e., Isaac 1990), which assume fixed effects for parameters and a common variance for the residuals across populations.

To address correlations between growth parameters in DLMtool, we recommend the use of the mean estimate of the von Bertalanffy parameter relationships as a point estimate and the mean estimate ± 1 standard deviation for ascribing a range of plausible values (Table 3). These values will be used to inform the Von Bertalanffy correlation inputs in the DLMtool application for updating the SEDAR46 results.

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Table 1.  Studies identified during the literature review which provided estimated correlations between Von Bertalanffy growth parameters Linfinity (Linf), growth rate (K), and tzero (t0).

| Reference | Study Group | Region | Sample size |
|---|---|---|---|
| Helser et al. 2004 | Meta-analysis of 46 species of *Sebastes* | Eastern Pacific (WA, OR, CA, AK, British Columbia) | 59,421 |
| Helser and Lai 2004 | Meta-analysis of 245 populations of largemouth bass | North American | Simulated length at age data for each of 100 populations similar to the selected 245 populations |
| He and Bence 2007 | Southern  Lake Trout | Northern US (Lake Huron, Michigan) | 15,675 |
| Midway et al. 2015 | Southern Flounder | Eastern US (NC, SC, AL, LA, TX) | 7,343 |
| Pilling et al 2002 | Tropical emperor | Nazareth Bank,  Mascarent Ridge (Indian Ocean) | 72  fish  and 10,000 simulated observations from back calculated lengths at age of the 72 fish |
| Quinn and Deriso 1999 | rougheye rockfish | Southeast Alaska | 545 |

Table 2.  Summary statistics of estimated correlations between Von Bertalanffy parameters Linfinity (Linf), growth rate (K), and tzero (t0).

| | Parameter Combination | | |
|---|---|---|---|
| Statistic | K,t0 | Linf, K | Linf, t0 |
| Min | 0.02 | -0.94 | -0.68 |
| Mean | 0.24 | -0.58 | -0.19 |
| Median | 0.12 | -0.69 | -0.15 |
| Max | 0.89 | -0.09 | 0.09 |
| SD | 0.31 | 0.29 | 0.24 |

Table 3.  Range of correlations between Von Bertalanffy parameters Linfinity (Linf), growth rate (K), and tzero (t0).

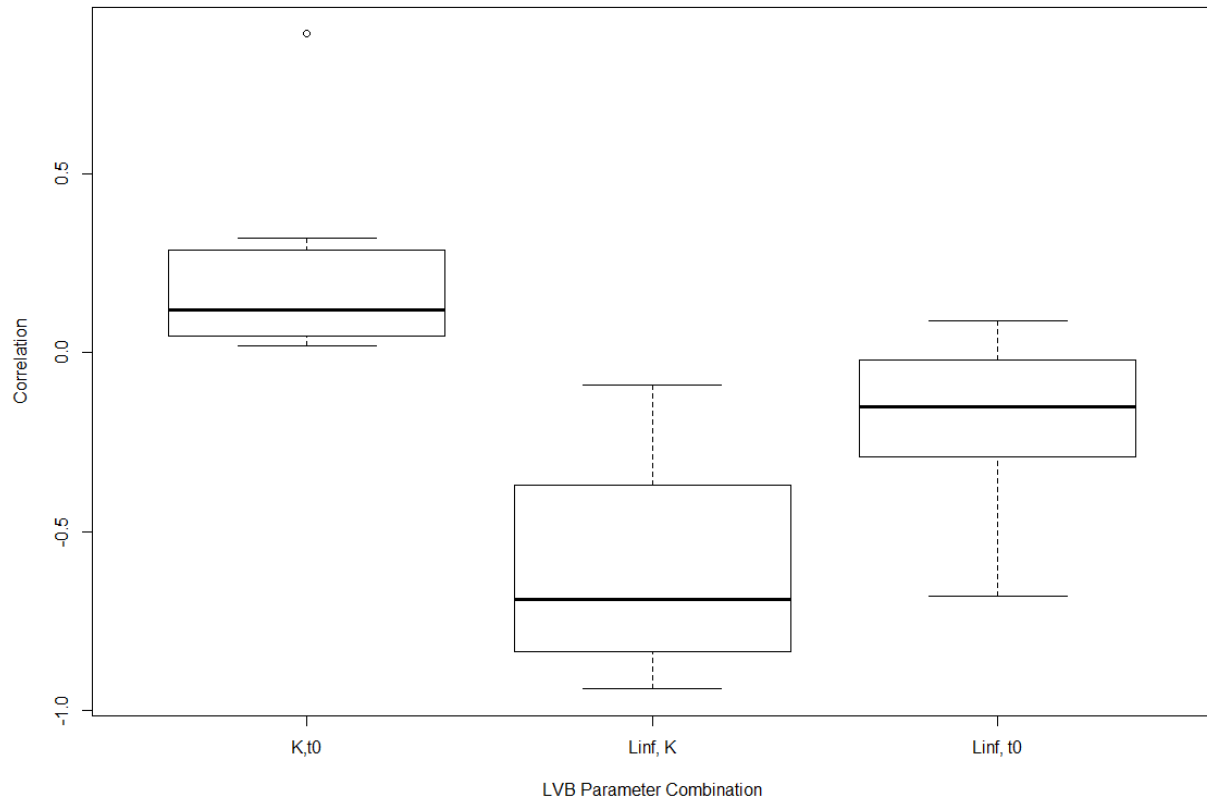| | Statistic | K,t0 | Linf, K | Linf, t0 |
|---|---|---|---|---|
| Value | Mean | 0.24 | -0.58 | -0.19 |
| Range | Mean ± 1 SD | (-0.07,0.55) | (-0.87,-0.29) | (-0.43,0.04) |
| Range | Min, Max | (0.02,0.89) | (-0.94,-0.09) | (-0.68,0.09) |

Figure 1. Boxplot of the von Bertalanffy parameter pair correlations.