# FINAL REPORT FOR THE ASSESSMENT METHODS WORKING GROUP SUMMARIZING THE DOMESTIC SHARK P* STANDARDIZATION WORKSHOP 

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## EXECUTIVE SUMMARY

A three day workshop was held June 11-13, 2013 in Panama City, Fla. to investigate $P^{*}$ statistical analysis techniques for use in age-structured stock assessments of U.S. domestic shark stocks managed under the consolidated Atlantic Highly Migratory Species (HMS) Fisheries Management Plan (FMP) (Appendix A). The workshop proceedings are summarized in this report. During the workshop, several shortcuts to published $P^{*}$ approaches were discussed that are currently being implemented or evaluated within the framework of the Southeast Fisheries Science Center (SEFSC) Southeast Data Assessment and Review (SEDAR) process. Preliminary analyses at the workshop indicated that results from some of the shortcuts were comparable to those obtained from published $P^{*}$ approaches. However, the application of the published $\mathrm{P}^{*}$ approaches to the existing HMS domestic shark age-structure stock assessment model would require model modifications because the current model structure does not provide estimates of the distribution of fishing mortality $(F)$ at maximum sustainable yield ( $F_{\mathrm{MSY}}$ ).

Alternative probabilistic approaches for reducing the over fishing limit (OFL) to account for scientific uncertainty (i.e. approaches that are not typically implemented within the SEDAR process) were also discussed at the workshop. In particular, it was noted that the use of shortterm probabilistic projections at alternative fixed harvest levels might provide a useful proxy to a typical $P^{*}$ approach for HMS domestic shark stock assessments. The use of short-term probabilistic projections at fixed harvest levels does not require an estimate of the distribution of $F_{\text {MSY }}$, accommodates multiple year lags at fixed harvest levels, and could be used to provide a buffer from OFL based on a pre-specified acceptable probability of overfishing (e.g., analogously to $P^{*}=0.3 ;<0.5$ ). However, the recommendation to use probabilistic projections at fixed harvest levels as a proxy for a $P^{*}$ approach was not evaluated explicitly at the workshop, e.g., with comparative model runs.

Following the workshop, a probabilistic projection approach based on the short-term projection approach discussed at the workshop was implemented for an HMS shark stock assessment in order to provide examples for reducing the OFL to account for scientific uncertainty in the assessment (Appendix B). The probabilistic projection model structure
(Appendix C) was based on a projection model previously developed and implemented for HMS shark stock assessments and was modified to include several recommendations provided during the $P^{*}$ workshop to improve the projection model. The HMS shark stock assessment and the probabilistic projection approach were subsequently reviewed during a Center for Independent Experts (CIE) desk review. Several research recommendations to further improve the HMS shark probabilistic projection approach resulted from the $\mathrm{P}^{*}$ workshop, subsequent projection model development, and the CIE review (Appendix B).

## INTRODUCTION

The Magnuson-Stevens Reauthorization Act (MSRA) of 2006 (MSRA 2006) established new requirements to end and prevent overfishing through the use of annual catch limits (ACLs), and mandated that ACLs must be established by 2011 for all stocks included within a federal fishery management plan (FMP). The National Standard 1 (NS1) guidelines (U.S. Office of the Federal Register 2009) provide guidance on implementing the MSRA and recommend that acceptable biological catch (ABC) be determined by reducing the overfishing limit (OFL) to account for scientific uncertainty and that the ACL must then be set less than or equal to the $\mathrm{ABC}(\mathrm{ACL} \leq \mathrm{ABC} \leq \mathrm{OFL})$. Within this context, $P^{*}$ is the allowable probability that ABC will exceed OFL (Figures 1 and 2; Prager and Shertzer 2010; Shertzer et al. 2010).

Many domestic U.S. shark stocks managed under the consolidated Atlantic Highly Migratory Species (HMS) FMP—e.g., Amendment 5a to the 2006 consolidated HMS FMP (U.S. Office of the Federal Register. 2013)—are assessed with age-structured production models (e.g., NMFS 2012a). For these stocks, OFL is defined relative to the maximum fishing mortality threshold (MFMT), a value of instantaneous fishing mortality rate $(F)$ above which overfishing is deemed to be occurring. For these stocks, the MFMT is set equal to the $F$ at maximum sustainable yield ( $F_{\mathrm{MSY}}$ ) estimated in the stock assessment model (NMFS 2012a), and OFL is calculated directly as the maximum sustainable yield (MSY) obtainable from the current stock biomass by applying $F_{\text {MSY }}$ (NMFS 2012a). However, an explicit procedure for reducing the OFL to account for scientific uncertainty (e.g., a $P^{*}$ approach; Figures 1 and 2; Prager and Shertzer 2010; Shertzer et al. 2010) has not been developed for Atlantic HMS domestic shark stocks that are not under a rebuilding plan (NMFS 2012a, 2012b).

A three day workshop was held June 11-13, 2013 in Panama City, Fla. to investigate $P^{*}$ statistical analysis techniques for use in age-structured stock assessments of domestic U.S. shark stocks managed under the consolidated Atlantic HMS FMP that are not under a rebuilding plan. Atlantic HMS domestic shark stocks managed under the consolidated Atlantic HMS FMP are currently assessed within the framework of the Southeast Data Assessment and Review (SEDAR) process (e.g., NMFS 2012a, 2012b). As a result, our investigation focused on typical
$P^{*}$ approaches currently being implemented or evaluated for management by scientific advisors for regional fisheries management councils within the SEDAR process. Experts in $P^{*}$ statistical analysis techniques currently being implemented or evaluated within the SEDAR process were brought together at a central workshop location (NOAA/NMFS Panama City Laboratory) (Appendix A). A summary of the workshop presentations, discussions, and recommendations are provided below, along with a summary of the work completed following the workshop (Appendices B and C).

## SUMMARY OF WORKSHOP PRESENTATIONS

## $\boldsymbol{P}^{*}$ Projection Approaches Presented at the Workshop

Five $P^{*}$ projection approaches were presented at the workshop for potential application to the existing HMS domestic shark state-space age-structured production model (SSASPM) (e.g., NMFS 2012a). The projection approaches included a published $P^{*}$ approach and four shortcuts to the published $P^{*}$ approach.

Published $P^{*}$ approach.-The published $P^{*}$ approach presented at the workshop was similar to the approach implemented for stocks assessed within the SEDAR process for the South Atlantic Fishery Management Council (SAFMC) (e.g., Caddy and McGarvey 1996; Prager et al. 2003; Shertzer et al. 2008, 2010). In the published $P^{*}$ approach presented at the workshop, the distribution of $F_{\text {limit }}$ was obtained through Monte Carlo bootstrap replicates from fits to the original assessment model. Projections were stochastic with many iterations ( $n=\sim 10,000$ ) and contained two layers of uncertainty. Each iteration represented an extension of a Monte Carlo bootstrap replicate, chosen at random, and was projected forward with lognormal recruitment variability. Each year of the projection, an optimization routine was used to solve for the level of landings (ABC) that provided a predetermined probability $\left(P^{*}\right)$ of overfishing. The value of $P^{*}$ was set by the Scientific and Statistical Committee (SSC) of the SAFMC, according to prespecified criteria.

First shortcut to the published $P^{*}$ approach.-In the first shortcut approach, the distribution of $F_{\text {limit }}$ ( $F_{\text {MSY }}$ for HMS domestic shark stocks) was estimated independently from the projection model as in the published $P^{*}$ approach. However, in contrast to the published $P^{*}$
approach, a value for $F_{\text {proj }}$ was then chosen from the distribution of $F_{\text {limit }}$ (e.g., $F_{\mathrm{MSY}}$ ) that provided a predetermined probability $\left(P^{*}\right)$ of overfishing. The population was then projected forward from a Monte Carlo bootstrap replicate as in the published $P^{*}$ approach, except that the landings ( $\sim \mathrm{ABC}$ ) were computed from the fixed value of $F_{\text {proj }}$ with a constant $F$ projection. In contrast to the published $P^{*}$ approach, the annual $\sim \mathrm{ABC}$ values were no longer a fixed value, but rather belonged to a distribution. As a result, a central estimate of annual $\sim \mathrm{ABC}$ was computed from the distribution of annual $\sim \mathrm{ABC}$ (e.g., median) available for each projection year.

Second shortcut to the published $P^{*}$ approach.-The second shortcut approach was similar to that implemented for stocks assessed within the SEDAR process for the Gulf of Mexico Fishery Management Council (GMFMC). In the second shortcut approach, the distribution of $F_{\text {limit }}\left(\right.$ e.g., $F_{\mathrm{MSY}}$ ) was obtained from bootstrap replicates of the original assessment model. For each bootstrap replicate ( $n$ ), a new value of $F_{\text {MSY }}$ was calculated from the bootstrap replicate parameter values for the assessment model. For each projection year $(t)$, yield was calculated with a constant $F$ projection at $F=F_{\text {MSY }}$ and removed from the population. The resulting distribution of yield at $F_{\text {MSY }}$ approximated the distribution of OFL in each projection year. The level of landings ( ABC ) was chosen that provided a predetermined probability $\left(P^{*}\right)$ of overfishing (defined as the probability of exceeding OFL in year $t$ ). The value of $P^{*}$ was set by the SSC of the GMFMC, according to pre-specified criteria.

Third shortcut to the published $P^{*}$ approach.-The third shortcut approach combined the first and second shortcut approaches. The distribution of $F_{\text {limit }}$ ( $F_{\text {MSY }}$ for HMS domestic sharks) was first estimated independently from the projection model, e.g., either with bootstrap replicates of the original assessment model or with Monte Carlo simulation (parametric bootstrap) from uncertainty in selected influential parameter estimates from the original assessment model. A value of $F_{\text {proj }}$ was then chosen that provided a predetermined probability $\left(P^{*}\right)$ of overfishing from the distribution of $F_{\text {limit }}$ independently from the projection model. A constant $F$ projection ( $F=$ $F_{\text {proj }}$ ) was then used to calculate and remove approximate annual yield at ABC each projection year $(t)\left(\sim \mathrm{ABC}_{\text {proj, }, t}\right)$. A new distribution of $F_{\text {limit }}\left(F_{\mathrm{MSY}}\right.$ for HMS domestic sharks) was then calculated that was dependent upon the projection model. For each bootstrap replicate ( $n$ ), a new value of $F_{\text {MSY }}$ was calculated from the bootstrap replicate parameter values for the assessment model ( $F_{\mathrm{MSY}, n}$ ). For each bootstrap replicate and each projection year, a new value of yield $\left(\sim \mathrm{OFL}_{n, t}\right)$ was calculated with a constant $F$ projection $\left(F=F_{\mathrm{MSY}, n}\right)$. The resulting distribution of
yield each year approximated the distribution of OFL in that projection year $\left(\sim \mathrm{OFL}_{t}\right)$. The yield that provided a pre-specified $P^{*}$ probability of exceeding $\sim \mathrm{OFL}_{t}$ was the approximate ABC in that projection year $\left(\sim \mathrm{ABC}_{t}\right)$.

Fourth shortcut to the published $P^{*}$ approach.-The fourth shortcut approach was based on a probabilistic projection approach previously developed and implemented for HMS domestic sharks stocks under a rebuilding plan (NMFS 2011) and then adapted for HMS domestic sharks stocks that were not under a rebuilding plan (NMFS 2012a, 2012b). The fourth shortcut approach utilized Monte Carlo simulation with parametric bootstrapping of uncertainty in a few key parameters to find the highest fishing mortality rate that achieved a predetermined probability $\left(P^{*}\right)$ of overfishing in the assessment model end year $\operatorname{Pr}\left(F_{\text {trial }}>F_{\text {MSY }}\right) \leq P^{*}$. To find the highest fishing mortality rate that achieved this goal, the age-structured stock assessment model was used to generate profile likelihood approximations to the posterior distribution of $\hat{F}_{\text {end-year }}$, defined as $\hat{P}\left(F_{\text {end-year }}\right)$, and to the posterior distribution of $\hat{F}_{\text {MSY }}$, defined as $\hat{P}\left(F_{\text {MSY }}\right)$. Samples from candidate values of $F_{\text {trial }}=c \hat{F}_{\text {end-year }}$ were then generated from the distribution of $\hat{P}\left(F_{\text {end-year }}\right)$ and multiplied by the fixed constant $c$. In this manner, candidate values of $F_{\text {trial }}$ were drawn from a distribution with the same shape as $\hat{P}\left(F_{\text {end-year }}\right)$ (NMFS 2011). The resulting distribution was defined as $\hat{P}\left(F_{\text {trial }}\right)$. An iterative solution was then found for the highest value of $F_{\text {trial }}$ that resulted in $\operatorname{Pr}\left(F_{\text {trial }}>F_{\text {MSY }}\right) \leq P^{*}$ (NMFS 2011, 2012b). Exploratory target yields in future years were then projected at $F_{\text {target }}=c \times \hat{F}_{\text {end-year }}$ using the fixed constant scalar $c$ from the iterative solution for $F_{\text {trial }}($ NMFS 2011, 2012b).

## Alternative Probabilistic Approaches Presented at the Workshop

Three alternative probabilistic approaches were presented at the workshop for reducing the over fishing limit (OFL) to account for scientific uncertainty in HMS domestic shark stock assessments. The alternative probabilistic approaches, which were not typically implemented within the SEDAR process, included short- and long-term probabilistic projections and an empirical $P^{*}$ approach.

Short-term probabilistic projection approach.-The short-term probabilistic projection approach presented at the workshop utilized 5 to 10 year probabilistic projections at fixed harvest levels similar to a KOBE II Strategy Matrix approach as implemented by ICCAT (e.g., SCRS 2012). In the approach presented at the workshop, short term ( $\sim 5$ to 10 years) probabilistic projections at fixed harvest levels from the final assessment model were utilized to determine the probability that various fixed harvest policies would maintain spawning stock biomass (SSB) above SSB at MSY ( $\mathrm{SSB}_{\mathrm{MSY}}$ ) and maintain fishing mortality, $F$, below $F_{\mathrm{MSY}}$. It was noted that within the context of application to an existing HMS shark dataset and age structured stock assessment model (e.g., NMFS 2012a), short-term probabilistic projections at fixed harvest levels might provide a useful proxy to a typical $P^{*}$ approach. However, application of the shortterm projection approach for HMS domestic sharks would utilize probabilistic projections based on Monte Carlo parametric bootstrap simulation of uncertainty in just a few influential parameters, while the ICCAT approach (e.g., SCRS 2012) utilized probabilistic projections based on bootstrap fits to the original assessment model.

Long-term equilibrium probabilistic projection approach. - In the long-term probabilistic projection approach, projections at fixed harvest levels were used to identify fixed harvest levels that achieved long-term (approximate equilibrium) goals (e.g., MSY). However, details of this approach were not provided.

Empirical $P^{*}$ approach.-The empirical $P^{*}$ approach presented at the workshop was based on the approach developed for use within the Pacific Fishery Management Council (PFMC) (Ralston et al. 2011). Ralston et al. (2011) developed a method for empirically estimating uncertainty in current (e.g., year $t$ ) exploitable biomass $\left(B_{t}\right)$ based upon multiple assessments of stocks assessed within the PFMC. Ralston et al. (2011) noted that calculation of an OFL typically involves three steps: (1) Estimation of current exploitable biomass, $B_{t}$; (2) projection of the population biomass into the future for some number of years; and (3) application of an estimate of $F_{\text {MSY }}$ to the forecasts of future biomass. Consequently, Ralston et al. (2011) noted that the empirical quantification of variation in $B_{t}$ should therefore be considered an estimate of the lower bound on total scientific uncertainty for PFMC stocks.

## SUMMARY OF WORKSHOP DISCUSSIONS

## Pros and Cons of $\boldsymbol{P} *$ Projection Approaches Discussed at the Workshop

Published $P^{*}$ approach.-Preliminary analysis from an example implementation of the published $P^{*}$ approach was conducted at the workshop for South Atlantic vermilion snapper (Table 1). The published $P^{*}$ approach was considered to be the most technically accurate approach presented at the workshop within the context of the current NS1 guidelines (U.S. Office of the Federal Register 2009). However, the published $P^{*}$ approach had some features that made it impractical for immediate application to the existing HMS domestic shark SSASPM model (e.g., NMFS 2012a). In particular, the published $P^{*}$ approach utilized Monte Carlo bootstrap replicates from fits to the original assessment model, which were not available from the existing HMS domestic shark SSASPM model (e.g., NMFS 2012a). The published $P^{*}$ approach can also take a long time to run (typically about one day for each year in the projection with SEFSC Beaufort Laboratory datasets) and can begin to break down (produce undefined results) under output control (i.e., fixed landings) after only a few projection years (typically about three years with SEFSC Beaufort Laboratory datasets).

First shortcut to the published $P^{*}$ approach.-Preliminary analysis from an example implementation of the first shortcut approach at the workshop produced results that were comparable to those obtained from the published $P^{*}$ approach for South Atlantic vermilion snapper (Table 1). However, a potential drawback of the first shortcut approach was that if the distribution of $F_{\mathrm{MSY}}$ was poorly characterized, then a constant $F$ projection $\left(F_{\mathrm{proj}}\right)$ based on the distribution of $F_{\text {limit }}$ might not provide an adequate buffer from $F_{\text {MSY }}$. An extreme example was presented where $F_{\text {limit }}=F_{\text {MSY }}$ and the standard deviation (SD) of $F_{\mathrm{MSY}}=0.0$. In this example, the value of $F_{\text {proj }}$ would be equal to the value of $F_{\text {limit }}$, and the resulting ABC would be equal to OFL (no buffer).

The distribution of $F_{\text {MSY }}$ was poorly characterized in the existing application of the HMS domestic shark SSASPM model (e.g., NMFS 2012a). In particular, the parameter $F_{\text {MSY }}$ was dependent upon an algorithm and was not an estimated parameter (e.g., NMFS 2012b). As a result, the variance of $F_{\text {MSY }}$ was not estimated (e.g., NMFS 2012a). Consequently, the dependence of the first shortcut approach on the distribution of $F_{\text {MSY }}$ made the approach
impractical for immediate application to the existing HMS domestic shark SSASPM model (e.g., NMFS 2012a).

In contrast, uncertainty in the ratio of parameters $F / F_{\text {MSY }}$ was estimated in the HMS domestic shark SSASPM model (e.g., NMFS 2012a). However, the estimate only represented uncertainty in the model ending year fishing mortality, $F$. It may be possible to estimate the variance of $F_{\text {MSY }}$ in the HMS domestic shark SSASPM model (e.g., NMFS 2012a) with the Markov Chain Monte Carlo (MCMC) routine in AD Model Builder (Fournier et al. 2012). However, it was noted that it can be difficult to diagnose convergence with MCMC. An alternative approach for characterizing uncertainty in $F_{\text {MSY }}$ may be to use Monte Carlo bootstrap replicates from residuals of the original assessment (e.g., as in the published $P^{*}$ approach). However, Monte Carlo bootstrap replicates from fits to the original assessment model were not available from the existing HMS domestic shark SSASPM model (e.g., NMFS 2012a).

Second shortcut to the published $P^{*}$ approach.-Preliminary analysis from an example implementation of the second shortcut approach at the workshop produced results that were comparable to those obtained from the published $P^{*}$ approach for South Atlantic vermilion snapper (Table 1). However, a potential drawback of the second shortcut approach was that it might underestimate the true population size over time because it removed yield at a constant fishing mortality rate, $F$, associated with OFL, $F_{\mathrm{MSY}}$, rather than at the $F$ associated with ABC, $F_{\text {proj. }}$. The second shortcut approach also had some features that made it impractical for immediate application to the existing HMS domestic shark SSASPM model (e.g., NMFS 2012a). In particular, the second shortcut approach utilized Monte Carlo bootstrap replicates from fits to the original assessment model, which were not available from the existing HMS domestic shark SSASPM model (e.g., NMFS 2012a).

Third shortcut to the published $P^{*}$ approach.-Preliminary analysis from an example implementation of the third shortcut approach at the workshop produced results that were comparable to those obtained from the published $P^{*}$ approach for South Atlantic vermilion snapper (Table 1). A potential benefit of the third shortcut approach was that it might be less likely to underestimate yield than the second shortcut approach. In particular, the third shortcut approach removed yield with a constant $F$ projection that approximated $\mathrm{ABC}\left(F=F_{\text {proj }}\right)$ rather than OFL ( $F=F_{\text {MSY }}$ ). Another potential improvement of the third shortcut approach was that it might be more likely to provide an adequate buffer from OFL, even if the preliminary estimate
of the uncertainty in $F_{\text {MSY }}$ was poorly characterized. In particular, the buffer from OFL in the third shortcut approach was obtained from bootstrap replicates of the value of $F_{\text {MSY }}$ calculated in the original assessment model and the pre-specified $P^{*}$ probability of exceeding OFL. However, the third shortcut approach also had some features that made it impractical for immediate application to the HMS domestic shark SSASPM model (e.g., NMFS 2012a). In particular, the approach utilized Monte Carlo bootstrap replicates from fits to the original assessment model, which were not available from the existing HMS domestic shark SSASPM model (e.g., NMFS 2012a).

Fourth shortcut to the published $P^{*}$ approach.-A potential drawback was identified at the workshop for application of the fourth shortcut approach to the HMS domestic shark SSASPM model (e.g., NMFS 2012a). In particular, the distribution of $F_{\text {MSY }}$ was required in the approach, but, as discussed above, the distribution of $F_{\text {MSY }}$ was not available from the existing application of the HMS domestic shark SSASPM model (e.g., NMFS 2012a). As mentioned above, it may be possible to estimate the variance of $F_{\text {MSY }}$ in the HMS domestic shark SSASPM model with the MCMC routine available in AD Model Builder (Fournier et al. 2012). However, use of the AD Model Builder MCMC routine for this purpose was not evaluated at the workshop.

## Pros and Cons of Alternative Probabilistic Approaches Discussed at the Workshop

Short-term probabilistic projection approach.-It was noted at the workshop that, within the context of application to an existing HMS domestic shark SSASPM model, short-term ( $\sim 5$ to 10 year) probabilistic projections at fixed harvest levels might provide a useful proxy to a typical $P^{*}$ approach. In contrast to the $P^{*}$ projection approaches presented at the workshop, the use of probabilistic projections at fixed harvest levels would not require an estimate of the distribution of $F_{\text {MSY }}$. The use of probabilistic projections at fixed harvest levels could also accommodate multiple year lags at fixed harvest levels (e.g., multiple year lags between HMS domestic shark assessment cycles) and could be used to provide a buffer based on a pre-specified acceptable probability of overfishing, analogously to a $P^{*}$ approach, by determining the probability that various fixed harvest policies would maintain SSB above $\mathrm{SSB}_{\text {MSY }}$ and maintain $F$ below $F_{\text {MSY }}$. However, recommendations regarding the use of probabilistic projections at fixed harvest levels as a proxy for a $P^{*}$ approach should be interpreted cautiously because they were not evaluated at the workshop, e.g., with comparative model runs.

Long-term equilibrium probabilistic projection approach. -It was also noted at the workshop, that projections at a fixed harvest level that achieved long-term equilibrium might be used to avoid a situation where short-term high projected catches associated with a biomass windfall are followed by a long-term projected decline in catches associated with biomass approaching long-term equilibrium. This approach, however, was not discussed in detail.

Empirical $P^{*}$ approach.-The empirical $P^{*}$ approach discussed at the workshop (Ralston et al. 2011) also had some features that made it impractical for immediate application to existing HMS domestic shark assessments (e.g., NMFS 2012a). In particular, Ralston et al. (2011) only included assessments conducted within the PFMC in their estimate of among-assessment variability. As a result, the method might not provide an accurate estimate of among-assessment variability for Atlantic HMS domestic shark assessments conducted within the SEDAR process.

## SUMMARY OF WORKSHOP RECOMMENDATIONS

## Shortcuts to Published $\boldsymbol{P}^{*}$ Projection Approaches

Several shortcuts to published $P^{*}$ projection approaches were discussed at the workshop that are currently being implemented or evaluated within the framework of the SEDAR process. Preliminary results from some of the shortcut approaches were comparable to those obtained from the published probabilistic $P^{*}$ approach (Table 1). However, when the technical merits of each $P^{*}$ shortcut approach were discussed within the context of application to an existing HMS domestic shark SSASPM model (e.g., NMFS 2012a, 2012b), it became apparent that the shortcuts to the typical $P^{*}$ approaches discussed at the workshop had features that made them impractical for immediate application to HMS domestic sharks. In particular, many of the $P^{*}$ shortcut approaches required the distribution of $F_{\text {MSY }}$ ( $F_{\text {limit }}$ for HMS domestic shark stocks), which was poorly characterized in the existing application of the HMS domestic shark SSASPM model. In addition, many of the $P^{*}$ shortcut approaches utilized bootstrap simulations of the assessment model residuals that were not available from the existing application of the HMS domestic SSASPM model (e.g., NMFS 2012a).

## Alternative Probabilistic Approaches

In contrast, several alternative probabilistic approaches were also discussed at the workshop, including short-term ( $\sim 5$ to 10 year) projections at fixed harvest levels, long-term equilibrium projections, and an empirical $\mathrm{P}^{*}$ approach. The short-term projection approach provided the best alternative because this approach did not require estimates of uncertainty in $F_{\text {MSY }}$ and could also accommodate multiple year lags between assessment cycles at fixed harvest levels. The short-term projection approach could also be used to provide a buffer based on a prespecified acceptable probability of overfishing, analogously to a $P^{*}$ approach. As a result, the short-term projection approach at fixed harvest levels might provide a useful proxy to a typical $P^{*}$ approach for application to the existing HMS domestic shark SSASPM model. However, conclusions regarding the use of probabilistic projections at fixed harvest levels as a proxy for a $P^{*}$ approach should be interpreted cautiously because results from the approach were not evaluated at the workshop relative to other $P^{*}$ approaches, e.g., with comparative model runs.

## Recommended Improvements to the HMS Domestic Shark Projection Methodology

It was noted at the workshop that the existing HMS domestic shark projection methodology (NMFS 2012b) might not adequately characterize recruitment variability. The existing application of the HMS domestic shark projections (NMFS 2012b) were implemented using Monte Carlo bootstrap simulation with parametric uncertainty in a few key parameters. Parameter uncertainty was included in initial numbers ( $N_{\text {end-year }}$ ), fishing mortality ( $F_{\text {end-year }}$ ), and age-0 pup survival ( $e^{-M_{0}}$ ) sampled from a multivariate normal distribution with expectations equivalent to posterior modes from SSASPM. The multivariate normal approximation was used to reduce the probability of selecting values of the different parameters that were unlikely to have generated the data (for instance, high fishing mortality and low pup survival).

An examination of projection output plots (NMFS 2012b) during the workshop, however, indicated that parameter estimates of $F_{\text {end-year }}$, and $N_{\text {end-year }}$ were largely uninformative for some model runs. For these model runs, projection uncertainty was informed only by the distribution of age-0 pup survival, $e^{-M_{0}}$ (NMFS 2012b). An examination of projection output plots (NMFS 2012b) during the workshop also indicated that the 30th and 70th percentiles of projected recruitment appeared to narrow over time, which was an implausible result. Together, these
results suggested that recruitment variability might not be adequately characterized in the projections.

The following changes were recommended at the workshop in order to improve the characterization of recruitment variability in the projections: (1) Remove age-0 pup survival, $e^{-M_{0}}$, from the existing multivariate normal distribution with $F_{\text {end-year }}$ and $N_{\text {end-year; }}$; (2) model $F_{\text {end- }}$ year and $N_{\text {end-year }}$ together with a bivariate normal distribution; (3) modify SSASPM to include the estimated uncertainty in equilibrium recruitment $\left(R_{0}\right)$ with the model output; and (4) add uncertainty in equilibrium recruitment, $R_{0}$, to the projections by modeling uncertainty in $R_{0}$, and age-0 pup survival, $e^{-M_{0}}$, together with a second bivariate normal distribution.

## SUMMARY OF WORK COMPLETED FOLLOWING THE WORKSHOP

Following the workshop, a probabilistic projection approach based on the short-term projection approach discussed at the workshop was implemented for an HMS shark stock assessment in order to provide examples for reducing the OFL to account for scientific uncertainty in the assessment (NMFS 2013a, 2013b; Appendix B). The probabilistic projection model structure (Appendix C) was based on a projection model previously developed in R statistical software (R 2013) and implemented for HMS shark stock assessments and was modified to include several recommendations provided during the $P^{*}$ workshop to improve the projection model (Appendix C).

Application of the probabilistic projection approach resulted in a range of possible reduction values from OFL to account for scientific uncertainty (NMFS 2013a, 2013b, 2013c; Appendix B). For example, application of the probabilistic projection approach for the SEDAR 34 Atlantic sharpnose baseline SSASPM configuration resulted in a $10 \%$ reduction from the OFL to account for scientific uncertainty (NMFS 2013a; Appendix B). In comparison, application of the probabilistic projection approach for a range of sensitivity configurations evaluated for the SEDAR 34 Atlantic sharpnose assessment resulted in a median $23 \%$ reduction from the OFL to account for scientific uncertainty (NMFS 2013a; Appendix B).

The SEDAR 34 HMS shark stock assessments, including the probabilistic projection approach, were subsequently reviewed during a Center for Independent Experts (CIE) desk
review (NMFS 2013c). In general, the CIE reviewer comments relevant to the probabilistic projection approach supported the view that projections offered an insight into possible stock trajectories. However, the CIE reviewers were skeptical about the probabilities associated with the projections and about the stock trajectories obtained from long term projections (greater than a decade), which the reviewers suggested should only be regarded as illustrative of what might happen (NMFS 2013c). Several research recommendations to further improve the HMS shark probabilistic projection approach resulted from the $P^{*}$ workshop, subsequent projection model development, and the CIE review. The research recommendations are summarized in Appendix B.

## CONCLUSIONS

## Benefits

The workshop was a cost effective and efficient method to develop an explicit procedure for reducing the overfishing limit (OFL) to account for scientific uncertainty when setting ACLs for Atlantic HMS domestic shark stocks assessed with age-structured models that are not under a rebuilding plan. The limited scope of the workshop facilitated collaboration among participants required to adapt alternative $P^{*}$ approaches to Atlantic HMS domestic shark stocks.

## Deliverables

Following the workshop, NOAA stock assessment scientists (SEFSC Panama City Laboratory) modified and documented a probabilistic projection approach in R statistical software for reducing the OFL to account for scientific uncertainty for Atlantic HMS domestic shark stocks assessed with age-structured stock assessment models that are not under a rebuilding plan (Appendix C). The documentation (this report) will be submitted to the Assessment Methods Working Group as a final report and to the SEDAR process as an informational document. The projection code in R statistical software is available from the authors.

## ACKNOWLEDGMENTS

We thank participants in the workshop (Appendix A) for their contributions. Funding for the workshop was provided by the NOAA Stock Assessment Methods Working Group.

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TABLE 1. Preliminary analysis from several shortcuts to published $P^{*}$ approaches discussed at the workshop $\left(P^{*}=0.275\right)$ for South
Atlantic vermilion snapper (K. Shertzer, NMFS, personal communication).



FIGURE 1. Graphical example of $P^{*}$ within the context of the National Standard 1 (NS1) Guidelines (U.S. Office of the Federal Register 2009) for Magnuson-Stevens Reauthorization Act of 2006 (MSRA 2006), adapted from the U.S. Office of the Federal Register (2009 Figure 2) and from Shertzer et al. (2010 Figure 1); Acceptable biological catch (ABC) is determined by reducing the overfishing limit (OFL) to account for scientific uncertainty, and the annual catch limit (ACL) is then set less than or equal to the $\mathrm{ABC}(\mathrm{ACL} \leq \mathrm{ABC} \leq \mathrm{OFL})$; Within this context, $P^{*}$ is the allowable probability that ABC will exceed OFL (e.g., Prager and Shertzer 2010; Shertzer et al. 2010); Given an ACL, an annual catch target (ACT) could optionally be set at some lower level, where the ACT would serve as the management goal, with a buffer from ACL to account for management uncertainty.


FIGURE 2. Graphical example of a $P^{*}$ procedure adapted from Prager and Shertzer (2010 Figure 1) for setting acceptable biological catch ( ABC ) from the statistical distribution of the overfishing limit (OFL); Given the distribution of OFL, ABC is adjusted so that the probability of ABC exceeding OFL is equal to the predetermined $P^{*}$ value.

## APPENDIX A. Workshop Participant List

P* Workshop Participants June 11-13, 2013, NOAA/NMFS Panama City Laboratory

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# APPENDIX B. Example Implementation of a Probabilistic Projection Approach for Reducing the Overfishing Limit (OFL) to Account for Scientific Uncertainty for Atlantic HMS Domestic Shark Stocks Assessed With an Age-structured Stock Assessment Model 

## Introduction

Following the $P^{*}$ workshop, a probabilistic projection approach was implemented for the SEDAR 34 HMS Atlantic sharpnose and bonnethead shark state-space age-structured production model (SSASPM) to provide examples for reducing the overfishing limit (OFL) to account for scientific uncertainty in the stock assessment (NMFS 2013a, 2013b). The probabilistic projection approach was used to provide examples of fixed removals associated with $\leq 30 \%$ probability of overfishing occurring, analogous to a $P^{*}=0.3$ approach, and resulted in a range of possible reduction values in the OFL to account for scientific uncertainty (NMFS 2013a, 2013b). Examples of the probabilistic projection approach results from the SEDAR 34 Atlantic sharpnose assessment (NMFS 2013a) are summarized below.

The SEDAR 34 HMS Atlantic sharpnose and bonnethead shark stock assessments, including the projections, were reviewed by the SEDAR 34 assessment panel and, subsequently, by a Center for Independent Experts (CIE) desk review (NMFS 2013c). Research recommendations provided during the $\mathrm{P}^{*}$ workshop, the SEDAR 34 assessment panel review, and the subsequent CIE review that have not been completed are summarized below.

## Probabilistic Projection Approach

All projections used 10,000 Monte Carlo bootstrap simulations with initial values drawn from two bivariate normal distributions with expectations equivalent to posterior modes from SSASPM (Appendix C). Projections were implemented for a range trial values of fixed total annual removals due to fishing (in numbers), analogously to the short-term projection approach discussed at the $\mathrm{P}^{*}$ workshop (e.g., Table B.1) (NMFS 2013a, 2013b, 2013c). The trial values evaluated for fixed annual removals (Table B.1) represented commercial catches with longlines,
gillnets, and lines, as well as recreational catches and shrimp trawl fishery discards (NMFS 2013a, 2013b).

The projection model structure was based on a previously implemented probabilistic projection approach developed in R statistical software (e.g., R 2013) for an age-structured catch-free model (ASCFM) during the SEDAR 21 Atlantic HMS shark stock assessment (NMFS 2011) and, subsequently, modified for SSASPM during the SEDAR 29 Atlantic HMS shark stock assessment (NMFS 2012a, 2012b). Following the P* workshop, the projection model structure was further modified as described below.

First, based on recommendations from the $\mathrm{P}^{*}$ workshop to improve the characterization of recruitment variability, the projection model structure was modified to draw initial values for the Monte Carlo bootstrap simulations from two bivariate normal distributions. The first bivariate normal distribution was developed from the estimated terminal assessment year, $t=$ h.endyr, total population numbers ( $\hat{N}_{t=\text { h.endyr }}^{\text {SSASM }}$ ) and the estimated terminal assessment year annual fishing mortality rate ( $\hat{F}_{t=\text { h.endyr }}^{\text {SSASP }}$ ) obtained of the SSASPM assessment model output (NMFS 2013a, 2013b). The second bivariate normal distribution was developed from the estimated survival rate of age-0 pre-recruits ( $e^{-\hat{M}_{0} \text { SSASPM }}$ ) and the estimated unexploited equilibrium recruitment ( $\hat{R}_{0}^{\text {SSASPM }}$ ) obtained from the SSASPM assessment model output (NMFS 2013a, 2013b).

Second, the projection model structure was modified to include a historic (retrospective) period. The projection model population dynamics were implemented at an annual time step with an annual fishing mortality rate calculated from all catch combined in the terminal year of the SSASPM assessment model. In contrast, the SSASPM assessment model population dynamics were implemented at a monthly time step with catch removed sequentially each month by multiple gear types (NMFS 2013a, 2013b). Consequently, a retrospective period was included in the projections as a projection model diagnostic in order to compare retrospective time series projected at an annual time step from all catch combined (without uncertainty) to those obtained directly from the SSASPM assessment model output.

Third, the duration of projections was modified to implement a 30 -year projection interval rather than the 5- to 10-year projection interval, as discussed for the short-term projection approach during the $P^{*}$ workshop. The longer term projection interval (30 years) was
implemented because it resulted in relatively more stable population trajectories towards the end of the projection interval for moderate levels of annual fixed removals (NMFS 2013a, 2013b). However, the choice of the final projection interval (30 years) was ad-hoc, and was based on the projection interval from a previous SEDAR shark stock assessment (NMFS 2012a, 2012b).

The first projection year was implemented in 2012, and projections were run until the year 2041 (30 years). Projections were implemented during the first three years (2012, 2013, 2014) with the 2011 fishing mortality rate obtained from the SSASPM assessment model output, and for the remaining years (2015-2041) with the fishing mortality rate obtained from the trial value of fixed removals evaluated for the projection scenario (Table B.1), as requested in the terms of reference for the stock assessments (NMFS 2013a, 2013b).

For the projection period, selectivity at age was obtained from the annual fishing mortality rate calculated from all catch combined in the terminal year of the SSASPM assessment model (Appendix C). Consequently, the relative proportion of the catch at age (in numbers) among fleets was assumed to be constant for all projection years.

## Reducing the OFL to Account for Scientific Uncertainty

For HMS domestic shark stocks assessed with age-structured models, the OFL is calculated directly as the maximum sustainable yield (MSY) obtainable from the current stock biomass by applying the fishing mortality rate at MSY ( $F_{\mathrm{MSY}}$ ) (e.g., NMFS 2012a). For the purposes of this probabilistic projection approach, both $F_{\text {MSY }}$ and the mature spawning stock fecundity at MSY ( $\mathrm{SSF}_{\mathrm{MSY}}$ ) were used as proxies for the OFL.

The projection approach used 10,000 Monte Carlo bootstrap replicate projections (boot), to identify a trial value of fixed removals during the years $(t)$ 2015-2041 (Table B.1) that resulted in both the $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right) \geq 70 \%$ and the $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right) \leq 30 \%$ in the year 2041 (NMFS 2013a, 2013b). Probabilities were calculated from the 30th percentile of the ratio $\mathrm{SSF}_{t, \text { boot }} / \mathrm{SSF}_{\mathrm{MSY}}$ and the 70 th percentile of the ratio $F_{t, \text { boot }} / F_{\mathrm{MSY}}$. For comparison, the 10,000 Monte Carlo bootstrap replicate projections were also tabulated during the last 10 years of the projection interval ( $t=2032-2041$ ) as the proportion of times (cumulative relative frequency) that $\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}$ and $F_{t, \text { boot }}>F_{\mathrm{MSY}}$. For example, the proportion of times that $\mathrm{SSF}_{t, \text { boot }}>$ $\mathrm{SSF}_{\text {MSY }}$ was defined as $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right)=1-\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }} \leq \mathrm{SSF}_{\mathrm{MSY}}\right)$ and was tabulated in R statistical software ( R 2013 ) as ( 1 - cumulative relative frequency $\left(\mathrm{SSF}_{t, \text { boot }} \leq \mathrm{SSF}_{\mathrm{MSY}}\right.$ ), where
the cumulative relative frequency was calculated as the ratio of (cumulative frequency)/(sample size).

## Example Results for Atlantic Sharpnose Sharks

The 30th percentile of the ratio $\mathrm{SSF}_{t, \text { boot }} / \mathrm{SSF}_{\text {MSY }}$ and the 70th percentile of the ratio $F_{t, \text { boot }} / F_{\text {MSY }}$ were summarized graphically from the 10,000 Monte Carlo bootstrap projections for each projection year (2012-2041) and for each trial value of fixed removals (Table B.1) (Figures B. 1 and B.2). The 30th percentile of the ratio $\mathrm{SSF}_{t, \mathrm{boot}} / \mathrm{SSF}_{\text {MSY }}$ represented the $70 \%$ probability of maintaining $\mathrm{SSF}_{t}$ above $\mathrm{SSF}_{\mathrm{MSY}}$ for a given trial value of fixed removals in a given year. The 70th percentile of the ratio $F_{t, \text { boot }} / F_{\text {MSY }}$ represented the $30 \%$ probability of $F_{t}$ exceeding $F_{\text {MSY }}$ for a given trial value of fixed removals in a given year.

The $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right)$ and the $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right)$ were summarized from the 10,000 Monte Carlo bootstrap projections for the last 10 years of the projection interval (2032-2041) and for each trial value of fixed removals (Tables B.1-B.3). Trial values of fixed removals that resulted in $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right) \geq 70 \%$ represented at most a $30 \%$ probability of exceeding $\mathrm{SSF}_{\text {MSY }}$ and were highlighted in green. Trial values that resulted in $50 \% \leq \operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\text {MSY }}\right)$ $<70 \%$ represented more than a $30 \%$ probability of exceeding SSF $_{\text {MSY }}$ but at most a $50 \%$ probability of exceeding SSF $_{\text {MSY }}$ and were highlighted in yellow. Trial values that resulted in $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right)<50 \%$ represented more than a $50 \%$ probability of exceeding $\mathrm{SSF}_{\text {MSY }}$ and were highlighted in red.

Similarly, trial values that resulted in $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right) \leq 30 \%$ represented at most a $30 \%$ probability of exceeding $F_{\text {MSY }}$ and were highlighted in green. Trial values that resulted in $30 \%<$ $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right) \leq 50 \%$ represented more than a $30 \%$ probability of exceeding $F_{\mathrm{MSY}}$ but less than or equal to a $50 \%$ probability of exceeding $F_{\text {MSY }}$ and were highlighted in yellow. Trial values that resulted in $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right)>50 \%$ represented more than a $50 \%$ probability of exceeding $F_{\text {MSY }}$ and were highlighted in red.

For the SEDAR 34 Atlantic sharpnose baseline SSASPM configuration (NMFS 2013a), examples of trial values of fixed removals during the years (2015-2041) (Table B.1) that resulted in both $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right) \geq 70 \%$ and $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right) \leq 30 \%$ in the year 2041 would result in a $10 \%$ reduction from the OFL to account for scientific uncertainty (Table B.4). In comparison, over the range of SSASPM sensitivity configurations evaluated with projections
during the SEDAR 34 Atlantic sharpnose assessment (NMFS 2013a), examples of trial values of fixed removals during the years (2015-2041) (Table B.1) that resulted in both $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\right.$ $\left.\mathrm{SSF}_{\mathrm{MSY}}\right) \geq 70 \%$ and $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right) \leq 30 \%$ in the year 2041 would result in a median $23 \%$ reduction from the OFL to account for scientific uncertainty (Table B.4). The range of SSASPM sensitivity configurations evaluated with projections in the SEDAR 34 Atlantic sharpnose stock assessment was intended to be representative of the full range of uncertainty in data inputs and model configurations evaluated in the assessment (NMFS 2013a).

Following the CIE desk review, the assessment authors conducted further analyses to investigate how successful using a bivariate normal distribution was in reducing the risk of selecting values of the variables that had not generated the data (NMFS 2013c). First, the assessment authors produced plots of the frequency distributions from the original 10,000 Monte Carlo bootstrap replicates drawn from the bivariate normal distribution for total population numbers in the terminal assessment year, $N_{t=\text { h.endyr }}^{\text {boot }}$, and the annual fishing mortality rate in the terminal assessment year, $F_{t=h \text { hendyr }}^{\text {boot }}$, and from the bivariate normal distribution for pre-recruit pup survival, $e^{-M_{0} \text { boot }}$, and equilibrium recruitment, $R_{0}^{\text {boot }}$ (NMFS 2013c). For example, frequency distributions from 10,000 Monte Carlo bootstrap replicate simulations for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment evaluated at a trial value of fixed annual removals due to fishing equal to $2750(1,000 s)$ (Adapted from NMFS 2013c) were informative (i.e. not uniform) and consistent with the parameter values estimated in SSASPM (Figure B.3).

Second, the assessment authors produced scatter plots from the original 10,000 Monte Carlo bootstrap replicates drawn from $N_{t=\text { h.endyr }}^{\text {boot }}$ and $F_{t=\text { h.endyr }}^{\text {boot }}$ and from $e^{-M_{0} \text {,bot }}$ and $R_{0}^{\text {boot }}$ (NMFS 2013c). For example, scatter plots from the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment evaluated at a trial value of fixed annual removals due to fishing equal to $2750(1,000 \mathrm{~s})$ (Adapted from NMFS 2013c) did not indicate a strong correlation between replicate values drawn for $F_{t=\text { h.endyr }}^{\text {boot }}$ and $N_{t=\text { h.endyr }}^{\text {boot }}$ (Figure B.4, Panel A), which was consistent with the relatively small estimated correlation coefficient $(r)$ for $\hat{F}_{t=\text { h.endyr }}^{\text {SSASP }}$ and $\hat{N}_{t=\text { h.endyr }}^{\text {SSASM }}$ obtained from SSASPM model output ( $r=-0.1238$; NMFS 2013c). In contrast, scatter plots of the replicate values drawn for $e^{-M_{0} \text {,boot }}$ and $R_{0}^{\text {boot }}$ (Figure B.4, Panel B) indicated
a relatively stronger correlation, which was consistent with the relatively larger (more negative) estimated correlation coefficient for $e^{-\hat{M}_{0} \text { SSASPM }}$ and $\hat{R}_{0}^{\text {SSASPM }}$ obtained from SSASPM model output ( $r=-0.4772$; NMFS 2013c).

The assessment authors also provided plots of the retrospective projection period (i.e., 1950-2011) (NMFS 2013c) (Appendix C). For example, results of the retrospective projection period (conducted without uncertainty) together with the projection period (obtained from 10,000 Monte Carlo bootstrap replicate simulations) were provided for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment evaluated at a trial value of fixed annual removals due to fishing equal to 2750 (1,000s) (Adapted from NMFS 2013c) (Figure B.5).

## Research Recommendations

Projection model research recommendations provided during the $\mathrm{P}^{*}$ workshop, the SEDAR 34 assessment panel review, and the subsequent CIE review that have not been completed, are summarized below.

Research recommendations provided during the $P^{*}$ workshop.-One research recommendation provided during the $P^{*}$ workshop that has not been completed was to compare projection results obtained from the HMS domestic shark probabilistic projection approach with results obtained from other approaches for the same stock assessment model output.

Research recommendations provided during the SEDAR 34 Assessment.-Several research recommendations were provided by the SEDAR 34 assessment panel that have not been completed: (1) Add a projection scenario that includes trends in shrimp effort; (2) explore alternative probability distributions for parameter uncertainty in fishing mortality and pup survival; (3) explore a more nuanced approach to modeling selectivity at age and fishing mortality separately by fleet in the projections; and (4) explore the effect of changing effort over time. For example, the SEDAR 34 assessment panel noted that some sensitivity scenarios resulted in very different stock sizes, which would likely affect the resulting age composition of the projected population and the resulting distribution of catch among fleets based on each fleet's selectivity, as well as changing effort over time. However, these considerations were not captured in the projections.

Research recommendations provided during the SEDAR 34 CIE desk review.-Several research recommendations were provided during the CIE desk review (NMFS 2013c) that have not been completed: (1) Limit the projection period to a decade because the probabilities and stock trajectories associated with long term projections, for example longer than a decade, are probably unreliable; (2) consider projection models where future effort is used as the control variable; (3) review probabilities associated with reference points over long time horizons which may be too heavily dependent on the model estimates of fishing mortality; (4) provide criteria and diagnostics for equilibrium conditions in the projections; and (5) consider starting the projections in 1950 instead of 2011 and using the catch data already available for the period 1950-2011 as an alternative to including a bivariate normal distribution for $F$ and $N$.

Research recommendations identified by the SEDAR 34 assessment authors.-Two research recommendations were identified by the stock assessment authors during the preparation of this report that have not been completed: (1) Investigate the incorporation of parameter uncertainty in fishing mortality in the projections; and (2) investigate the relatively poor agreement between retrospectively projected time series of annual yield obtained from the projection model compared with the time series of annual yield obtained directly from the SSASPM model output (-7.1\% average annual percent difference) (Appendix C).

TABLE B.1. Trial values evaluated for fixed levels of total annual removals due to fishing (1,000s) during the years (2015-2041) from the SEDAR 34 Atlantic sharpnose stock assessment (Adapted from NMFS 2013a).

| Description | Value |
| :--- | :--- |
| First projection year | year 2012 |
| Projection duration | 30 (years) |
| Interim projection interval | 3 (years) |
| End projection year | year 2041 |
| Projection criteria | Fixed removals due to fishing |
| Alternative | Trial value of fixed removals $(1,000 \mathrm{~s})$ |
| 1 | 0 |
| 2 | 250 |
| 3 | 500 |
| 4 | 750 |
| 5 | 1000 |
| 6 | 1250 |
| 7 | 1500 |
| 8 | 1750 |
| 9 | 2000 |
| 10 | 2250 |
| 11 | 2500 |
| 12 | 2750 |
| 13 | 3000 |
| 14 | 3250 |
| 15 | 3500 |
| 16 | 3750 |
| 17 | 4000 |
| 21 | 4250 |
|  | 4500 |
|  | 4750 |
| 18 | 5000 |

TABLE B.2. Results of 10,000 Monte Carlo bootstrap replicate projections (boot) for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment (Adapted from NMFS 2013a), summarized as $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right)$ during the years 20322041; bootstrap replicates were tabulated as the proportion of times that $\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}$ for each trial value of fixed removals (Table B.1) and categorized as $\operatorname{Pr} \geq 70 \%$ (green), $50 \% \leq \operatorname{Pr}<$ $70 \%$ (yellow), and $\operatorname{Pr}<50 \%$ (red).

| Trial <br> value of <br> fixed <br> removals <br> $(1,000$ s) | 2032 | 2033 | 2034 | 2035 | 2036 | 2037 | 2038 | 2039 | 2040 | 2041 |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 250 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 500 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 750 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 1000 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 |
| 1250 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.98 | 0.98 | 0.98 |
| 1500 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 |
| 1750 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 |
| 2000 | 0.94 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.92 | 0.92 | 0.92 | 0.92 |
| 2250 | 0.89 | 0.89 | 0.88 | 0.88 | 0.88 | 0.87 | 0.87 | 0.86 | 0.86 | 0.86 |
| 2500 | 0.84 | 0.83 | 0.82 | 0.82 | 0.81 | 0.80 | 0.80 | 0.79 | 0.79 | 0.78 |
| 2750 | 0.77 | 0.76 | 0.75 | 0.74 | 0.73 | 0.73 | 0.72 | 0.71 | 0.70 | 0.70 |
| 3000 | 0.67 | 0.66 | 0.65 | 0.64 | 0.63 | 0.62 | 0.61 | 0.60 | 0.60 | 0.59 |
| 3250 | 0.58 | 0.56 | 0.55 | 0.54 | 0.52 | 0.51 | 0.50 | 0.49 | 0.48 | 0.47 |
| 3500 | 0.47 | 0.45 | 0.44 | 0.43 | 0.41 | 0.40 | 0.39 | 0.38 | 0.37 | 0.36 |
| 3750 | 0.38 | 0.36 | 0.34 | 0.33 | 0.32 | 0.31 | 0.30 | 0.29 | 0.28 | 0.28 |
| 4000 | 0.28 | 0.26 | 0.25 | 0.24 | 0.22 | 0.21 | 0.21 | 0.20 | 0.19 | 0.18 |
| 4250 | 0.20 | 0.19 | 0.17 | 0.16 | 0.16 | 0.15 | 0.14 | 0.13 | 0.13 | 0.12 |
| 4500 | 0.14 | 0.13 | 0.12 | 0.11 | 0.10 | 0.10 | 0.09 | 0.09 | 0.08 | 0.08 |
| 4750 | 0.10 | 0.09 | 0.09 | 0.08 | 0.07 | 0.07 | 0.06 | 0.06 | 0.06 | 0.05 |
| 5000 | 0.06 | 0.05 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 |

TABLE B.3. Results of 10,000 Monte Carlo bootstrap replicate projections (boot) for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment (Adapted from NMFS 2013a), summarized as $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right)$ during the years 2032-2041; bootstrap replicates were tabulated as the proportion of times that $F_{t, \text { boot }}>F_{\text {MSY }}$ for each trial value of fixed removals (Table B.1) and categorized as $\operatorname{Pr} \leq 30 \%$ (green), $30 \%<\operatorname{Pr} \leq 50 \%$ (yellow), and $\operatorname{Pr}>50 \%$ (red).

|  | Projection year |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2032 | 2033 | 2034 | 2035 | 2036 | 2037 | 2038 | 2039 | 2040 | 2041 |
| 0 | < $=0.01$ | < 0.0 .01 | < $=0.01$ | < $=0.01$ | < $=0.01$ | < $=0.01$ | < $=0.01$ | < $=0.01$ | < $=0.01$ | < 0.01 |
| 250 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 500 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 750 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1250 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 1500 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 1750 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| 2000 | 0.03 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 |
| 2250 | 0.07 | 0.07 | 0.08 | 0.08 | 0.08 | 0.09 | 0.09 | 0.10 | 0.10 | 0.10 |
| 2500 | 0.14 | 0.14 | 0.15 | 0.15 | 0.16 | 0.16 | 0.17 | 0.17 | 0.18 | 0.18 |
| 2750 | 0.23 | 0.23 | 0.24 | 0.25 | 0.26 | 0.26 | 0.27 | 0.27 | 0.28 | 0.28 |
| 3000 | 0.36 | 0.37 | 0.37 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.42 | 0.42 |
| 3250 | 0.50 | 0.51 | 0.52 | 0.52 | 0.53 | 0.54 | 0.55 | 0.55 | 0.56 | 0.56 |
| 3500 | 0.64 | 0.65 | 0.66 | 0.67 | 0.67 | 0.68 | 0.69 | 0.69 | 0.70 | 0.70 |
| 3750 | 0.75 | 0.76 | 0.77 | 0.77 | 0.78 | 0.78 | 0.79 | 0.79 | 0.80 | 0.80 |
| 4000 | 0.86 | 0.86 | 0.87 | 0.87 | 0.87 | 0.88 | 0.88 | 0.88 | 0.88 | 0.89 |
| 4250 | 0.92 | 0.92 | 0.92 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.94 | 0.94 |
| 4500 | 0.96 | 0.96 | 0.96 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 |
| 4750 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 |
| 5000 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 |

TABLE B.4. Results of 10,000 Monte Carlo bootstrap replicate projections (boot) for all SSASPM model configuration evaluated with projections during the SEDAR 34 Atlantic sharpnose stock assessment (Adapted from NMFS 2013a); projection results were summarized from trial values evaluated for fixed removals during the years (2015-2041) (Table B.1) that resulted in both $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right) \geq 70 \%$ and $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right) \leq 30 \%$ in the year 2041; the overfishing limit (OFL) for each SSASPM model configuration was obtained directly from the SSASPM model output (Adapted from NMFS 2013a).
$\left.\begin{array}{llccc}\hline & & & \begin{array}{c}\text { Examples of } \\ \text { reductions in } \\ \text { the OFL to }\end{array} \\ \text { account for }\end{array}\right]$
*Some model parameters were fixed within the SSASPM sensitivity configuration for projection scenario-6, which resulted in an unreasonable buffer.

30th Percentiles of (SSFt,boot)/(SSFmsy)


FIGURE B.1. Results of 10,000 Monte Carlo bootstrap replicate projections (boot) for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment (Adapted from NMFS 2013a), summarized as $\operatorname{Pr}\left(\mathrm{SSF}_{t, \text { boot }}>\mathrm{SSF}_{\mathrm{MSY}}\right) \geq 70 \%$ during the years 2012-2041; moments of the distributions were summarized from the 30th percentile of the ratio $\mathrm{SSF}_{t, \text { boot }} / \mathrm{SSF}_{\text {MSY }}$ for each trial value evaluated for fixed removals (Table B.1); the 30th percentile of the ratio $\mathrm{SSF}_{t, \text { boot }} / \mathrm{SSF}_{\text {MSY }}$ represented the $70 \%$ probability of maintaining $\mathrm{SSF}_{t}$ above $\mathrm{SSF}_{\text {MSY }}$ for a given level of fixed removals in a given year.


FIGURE B.2. Results of 10,000 Monte Carlo bootstrap replicate projections (boot) for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment (Adapted from NMFS 2013a), summarized as $\operatorname{Pr}\left(F_{t, \text { boot }}>F_{\mathrm{MSY}}\right) \leq 30 \%$ during the years 20122041; moments of the distribution were summarized from the 70th percentile of the ratio $F_{t, \text { boot }} / F_{\mathrm{MSY}}$ for each trial value evaluated for fixed removals (Table B.1); the 70th percentile of the ratio $F_{t, \text { boot }} / F_{\text {MSY }}$ represented the $30 \%$ probability of $F_{t}$ exceeding $F_{\text {MSY }}$ for a given level of fixed removals in a given year.


FIGURE B.3. Results of 10,000 Monte Carlo bootstrap replicate simulations (boot) for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment evaluated at a trial value of fixed annual removals due to fishing equal to $2750(1,000 \mathrm{~s})$ (Adapted from NMFS 2013c), summarized as frequency distributions of bootstrap replicates from the bivariate normal distribution of total population numbers in the terminal assessment year, $N_{t=\text { h.endyr }}^{\text {boot }},(\mathrm{N}-$ boot, Panel A) and the annual fishing mortality rate in the terminal assessment year, $F_{t=\text { h.endyr }}^{\text {boot }},($ F-boot Panel B), and from the bivariate normal distribution of pre-recruit pup survival, $e^{-M_{0}, \text { boot }},\left(\mathrm{S}-\right.$ boot, Panel C) and equilibrium recruitment, $R_{0}^{\text {boot }}$, (R0-boot, Panel D); frequency distributions were plotted relative to the corresponding parameter estimates obtained from SSASPM model output (dashed lines) and the medians of the 10,000 Monte Carlo bootstrap distributions obtained for each parameter (solid lines).


FIGURE B.4. Results of 10,000 Monte Carlo bootstrap replicate simulations (boot) for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment evaluated at a trial value of fixed annual removals due to fishing equal to $2750(1,000 \mathrm{~s})$ (Adapted from NMFS 2013c), summarized as scatter plots of bootstrap replicates from the bivariate normal distribution of total population numbers in the terminal assessment year, $N_{t=\text { h.endyr }}^{\text {boot }}$, (Nboot) versus the annual fishing mortality rate in the terminal assessment year, $F_{t=\text { h.endyr }}^{\text {boot }},(\mathrm{F}-\mathrm{boot})$ (Panel A), and from the bivariate normal distribution of pre-recruit pup survival, $e^{-M_{0}, \text { boot }}$, (Sboot) versus equilibrium recruitment, $R_{0}^{\text {boot }},(\mathrm{R} 0$-boot)( Panel B), as described above.


FIGURE B.5. Projection results for the ratios of spawning stock fecundity (SSF) and the annual fishing mortality rate (F) to their values at maximum sustainable yield (MSY) from the retrospective projection period ( $t=1950-2011$, conducted without uncertainty) together with the projection period ( $t=2012-2014$, obtained from 10,000 Monte Carlo bootstrap replicate simulations) for the baseline SSASPM model configuration of the SEDAR 34 Atlantic sharpnose stock assessment evaluated at a trial value of fixed annual removals due to fishing equal to 2750 (1,000s) (Adapted from NMFS 2013c); The 30th percentile of the ratio SSF/SSF ${ }_{\text {MSY }}$ (Panel A, lower thin line) was used to represent the $70 \%$ probability of maintaining $\operatorname{SSF}_{t, \text { boot }}$ above $\mathrm{SSF}_{\mathrm{MSY}}$, and the 70th percentile of the ratio $F / F_{\mathrm{MSY}}$ (Panel B, upper thin line) was used to represent the $30 \%$ probability of $\mathrm{F}_{t, \text { boot }}$ exceeding $F_{\mathrm{MSY}}$.

## APPENDIX C. Projection Model

## Introduction

All projections were implemented in R statistical software (R 2013). Projection model data were obtained directly from the output of a state-space age-structured production model (SSASPM) (e.g., NMFS 2013a, 2013b, 2013c) implemented in ADMB (Fournier et al. 2012). The recruitment age for projections was fixed at one ( $a_{r}=1$ ), and the maximum age for all projections ( $a_{\max }$ ) was set equal to the maximum age from the SSASPM assessment model (e.g., NMFS 2013a, 2013b, 2013c). A historic (retrospective) period was included in the projections as a projection model diagnostic in order to compare retrospective time series projected at an annual time step from all catch combined (without uncertainty) to those obtained directly from the SSASPM assessment model output, which were implemented at a monthly time step.

## Projection Period

The projection period was initialized in the terminal year of the SSASPM assessment $\operatorname{model}(t=\mathrm{p} . \operatorname{styr}=\mathrm{h} . e n d y r)$. The ending year of the projection period $(t=\mathrm{p}$. endyr $)$ was the projection period plus the initialization year (e.g., for a 30 year projection period, $\mathrm{p} . \mathrm{endyr}=$ p.styr +30 years).

Terminal assessment year fishing mortality rate at age.-In the projection model, fishing mortality was calculated from all catch combined at an annual time step. For the projection period, the estimated fishing mortality rate at age $a$ in the terminal assessment year ( $t=$ h.endyr) was obtained directly from the SSASPM assessment model $\left(\hat{F}_{t=\text { h.endyr }, a}^{\text {SSASP }}\right)$.

Terminal assessment year selectivity at age.-In the projection model, the terminal assessment year selectivity at age $a\left(\operatorname{sel}_{t=\text { h.endyr }, a}\right)$ was then calculated from $\hat{F}_{t=\text { h.endyr }, a}^{\text {SSAPM }}$ as follows:

$$
\operatorname{sel}_{t=\text { h.endyr }, a}=\hat{F}_{t=\mathrm{h} . \mathrm{endyr}, a}^{\mathrm{SSSPP}} / \max \left(\hat{F}_{t=\mathrm{h} . \mathrm{hendy}, a}^{\mathrm{SSSPM}}\right),
$$

where: (1) the parameter estimate $\hat{F}_{t=\text { h.endyr }, a}^{\text {SSAPM }}$ was assumed to be separable into the terminal assessment year selectivity at age, $\operatorname{sel}_{t=\mathrm{h} . \mathrm{endyr}, a}$, and the terminal assessment year annual fishing mortality rate $\left(F_{t=\mathrm{h} . \text { endyr }}\right)$ as follows: $\hat{F}_{t=\mathrm{h} . \text {.endy }, a}^{\mathrm{SSASM}}=\operatorname{sel}_{t=\mathrm{h} . \text {.endyr }, a} F_{t=\mathrm{h} . \text { endyr }} ;$ (2) the maximum selectivity at age was assumed to be equal to one as follows: $\max \left(\operatorname{sel}_{t=\mathrm{h} . \mathrm{endyr}, a}\right)=1$; and (3), Consequently, $F_{t=\text { h.endyr }}$ was calculated from $\hat{F}_{t=\text { h.endyr }, a}^{\mathrm{SSAPM}}$ as: $F_{t=\mathrm{h} . \mathrm{endyr}}=\max \left(\hat{F}_{t=\mathrm{h} . \text {.endyr }, a}^{\mathrm{SSAPM}}\right)$.

Projection period selectivity at age.-For the projection period, selectivity at age $a$ was assumed to be constant over time and was obtained from the terminal assessment year selectivity at age as: $s e l_{a}^{p}=s e l_{t=h . e n d y r, a}$. By scaling the maximum selectivity at age equal to one, selectivity at age was interpreted as the probability of capturing an animal of a given age relative to the probability of capturing an animal at the age at that the probability of capture is highest (i.e., equal to one). Within this context, the projection model calculation of selectivity at age included both the concepts of gear selectivity (probability of capture once contact with the gear was made) as well as availability to the gear (Punt et al. 2014).

Bootstrap replicates.-All projections used 10,000 Monte Carlo bootstrap simulations with initial values drawn from two bivariate normal distributions with expectations equivalent to posterior modes obtained from the SSASPM assessment model output (e.g., NMFS 2013a, 2013b, 2013c).

Bivariate normal distributions were developed from the AD Model Builder (ADMB) output (Fournier et al. 2012), which included the parameter estimates, their standard deviations, and their correlation coefficients $(r)$. The bivariate normal distributions were implemented using the R statistical software ( R 2013) function mvrnorm obtained from the R library MASS (Crawley 2007 p. 237). For example, for the parameters $x$ and $y$, the function mvrnorm was implemented from the matrix of standard deviations $\left(\mathrm{SD}_{x}\right.$ and $\left.\mathrm{SD}_{y}\right)$ and covariance, $\left[\begin{array}{cc}\mathrm{SD}_{x}^{2} & \operatorname{cov}(x, y) \\ \operatorname{cov}(y, x) & \mathrm{SD}_{y}^{2}\end{array}\right]$, where the covariance of $x$ and $y$ was calculated from correlation coefficient and standard deviations of $x$ and $y$ as follows: $\operatorname{cov}(x, y)=\operatorname{cov}(y, x)=r \sqrt{\mathrm{SD}_{x}^{2} \mathrm{SD}_{y}^{2}}$.

The first bivariate normal distribution was developed from the estimated terminal assessment year, $t=$ h.endyr, total population numbers ( $\left.\hat{N}_{t=\text { h.endyr }}^{\text {SSAPM }}\right)$ and the estimated terminal
assessment year annual fishing mortality rate ( $\hat{F}_{t=\text { h.endyr }}^{\text {SASM }}$ ) obtained of the SSASPM assessment model output. The second bivariate normal distribution was developed from the estimated survival rate of age-0 pre-recruits ( $e^{-\hat{M}_{0} \text { SSASPM }}$ ) and the estimated unexploited equilibrium recruitment ( $\hat{R}_{0}^{\text {SSASPM }}$ ) obtained from the SSASPM assessment model output. The entire bootstrap replicate draw ( $N_{t=\text { h.endyr }}^{\text {boot }}, F_{t=\text { h.endyr }}^{\text {bot }}, e^{-M_{0} \text { boot }}$, and $R_{0}^{\text {boot }}$ ) was rejected if any value was less than zero or if $e^{-M_{0} \text { boot }}$ was greater than one.

Projection period numbers at age. - In the projection model, numbers at age were calculated from recruitment at the beginning of each projection year and from the numbers at age at the beginning of the previous year multiplied by the annual survival rate during the previous year. For the projection period, population numbers at age $a$ at the beginning of each projection year, $t$, were calculated as follows:

$$
\begin{aligned}
& N_{t, a}^{p}=\left\{\begin{array} { l } 
{ \{ N _ { t - \mathrm { p } . \operatorname { s t y } , a } ^ { \mathrm { boot } } }
\end{array} \left\{\begin{array}{l}
R_{t}^{p} \\
N_{t-1, a-1}^{p} e^{-Z_{t-1, a-1}^{p}} \\
N_{t-1, a-1}^{p} e^{-Z_{t-1, a-1}^{p}}+N_{t-1, a}^{p} e^{-Z_{t-1, a}^{p}}
\end{array}\right.\right. \\
& \left.\begin{array}{c}
\{\text { if } t=\mathrm{p} . \text { styr } \\
\text { if } a=1 \\
\text { if } 1<a<a_{\max } \\
\text { if } a=a_{\max }
\end{array}\right\} \text { if p.styr }<t \leq \text { p.endyr }
\end{aligned}
$$

where, for the first projection period ( $t=\mathrm{p} . \mathrm{styr}$ ), population numbers at the beginning of the year were calculated from the SSASPM assessment model terminal year, $t=$ h.endyr, estimates of population numbers at age (for both sexes combined) and from the bootstrap replicate draw for total numbers $\left(N_{t=\text { h.endyr }}^{\text {boot }}\right)$ as follows:

$$
N_{t=\mathrm{p} . \mathrm{styr}, a}^{\mathrm{boot}}=\hat{N}_{t=\mathrm{h} . \mathrm{hendy}, a}^{\mathrm{SSASPM}} * \frac{N_{t=\text { h.endyr }}^{\mathrm{boot}}}{\hat{N}_{t=\mathrm{h} . \mathrm{hPndyr}}^{\text {SSM }}}
$$

Projection period annual fishing mortality rate.-For the projection period, the annual fishing mortality rate in year $t$ was calculated as follows:

$$
F_{t}^{p}=\left\{\begin{array}{ll}
\hat{F}_{t \text { hhandsur }}^{\text {SAPM }} & \text { if } \mathrm{p} . \mathrm{styr} \leq t \leq \mathrm{p} . \mathrm{styr}+3 \\
\text { obtained numerically } & \text { if } \mathrm{p} . \mathrm{styr}+3<t \leq \text { p.endyr }
\end{array} .\right.
$$

For the projection years $\mathrm{p} . \mathrm{styr} \leq t \leq \mathrm{p} . \operatorname{styr}+3$, the parameter $F_{t}{ }^{p}$ was set equal to the terminal assessment year estimate of annual fishing mortality rate obtained of the SSASPM assessment model output, $\hat{F}_{t=h . \text { endyr }}^{\text {SSAPM }}$. For the projection period years $\mathrm{p} . s t y r+3<t \leq \mathrm{p}$.endyr, the parameter $F_{t}^{p}$ was obtained numerically as the annual fishing mortality rate that minimized the squared difference $\left(C_{\text {trial }}^{p}-\sum_{a=1}^{a_{\max }}\left(C_{t, a}^{p}\right)\right)^{2}$, where $C_{\text {trial }}^{p}$ was the trial value of total annual removals (in numbers) evaluated for the projection scenario (e.g., Table B.1), and $C_{t, a}^{p}$ was the catch at age corresponding to $F_{t}{ }^{p}$ obtained from the Baranov catch equation, as described below.

Projection period total mortality rate at age. - In the projection model, the total mortality rate was calculated from the fishing mortality rate plus the natural mortality rate at an annual time step. For the projection period, the total mortality rate at age $a$ in year $t\left(Z_{t, a}^{p}\right)$ was calculated from $F_{t}^{p}$ and sel ${ }_{a}^{p}$, obtained as described above, and from the natural mortality rate at age $a$ obtained from the SSASPM assessment model output ( $M_{a}^{\text {SSASPM }}$ ) as follows:

$$
Z_{t, a}^{p}=\operatorname{sel}_{a}^{p} F_{t}^{p}+M_{a}^{\mathrm{SSASPM}} .
$$

Projection period annual spawning stock size.-In the projection model, the annual spawning stock size was calculated from the annual numbers at age (for females and males combined) and from the net fecundity at age. For the projection period, the annual spawning stock size in year $t\left(S_{t}^{p}\right)$, defined in the SSASPM assessment model as spawning stock fecundity (SSF), was calculated from the annual numbers at age, $N_{t, a}^{p}$ (for both sexes combined), the annual total mortality rate at age, $Z_{t, a}^{p}$, and the net (per capita) fecundity at age $a$ obtained from the SSASPM assessment model ( $\left.f_{a}^{\text {SSASPM }}\right)$ as follows:

$$
S_{t}^{p}=\sum_{a=1}^{a_{\max }} f_{a}^{\text {SSASPM }} N_{t, a}^{p} e^{-\tau_{M} Z_{t, a}^{p}},
$$

where $\tau_{M}$ was the fraction of year from the beginning of the calendar year (January 1 ) to the beginning of the pupping season input into the SSASPM assessment model, and $e^{-\tau_{M} Z_{t, a}^{h}}$ was the expected survival rate at age $a$ from the beginning of calendar year $t$ to the beginning of the pupping season.

For the sex-combined age-structured projection model, the net fecundity at age, $f_{a}$, was assumed to represent the product of maturity, the proportion of females, and the number of eggs per mature female (or a proportional quantity related to number of eggs per mature female) (L. Brooks, National Marine Fisheries Service, personal communication; e.g., Quinn and Deriso 1999 section 8.2.5 and chapter 4) as follows:

$$
f_{a}=m_{a} \rho E_{a},
$$

where $m_{a}$ was the proportion of females mature at age a, $\rho$ was the sex ratio (e.g., proportion female, generally $\rho=0.5$ ), and $E_{a}$ was the number of eggs produced per mature female per year (or a proportional quantity related to the number of eggs per mature female). For example, $E_{a}=(v / \gamma)$, where: (1) $v=$ the average number of pups produced by a mature female shark (during each pregnancy); and (2) $\gamma=$ the average reproductive periodicity of a mature female. Fixing the parameter $\gamma=1$ would imply that females reproduce every year, and $\gamma=2$ would imply that females reproduce every other year. Given this formulation, the net fecundity at age was interpreted in the projection model as the average number of pups produced annually per mature individual (i.e., per capita).

Projection period annual recruitment.-In the projection model, the annual recruitment was calculated from the annual spawning stock size using a Beverton-Holt spawner-recruit curve (e.g., Quinn and Deriso 1999 equation 3.6) re-parameterized following NMFS (2013a equations 2 and 3), as described below. For the projection period, the annual recruitment in year $t\left(R_{t}^{p}\right)$
was calculated from annual spawning stock size in year $t-1\left(S_{t-1}^{p}\right)$ with the re-parameterized Beverton Holt stock recruitment relationship (NMFS 2013a equations 2 and 3) as follows:

$$
R_{t}^{p}=\frac{e^{-M_{0} \mathrm{boot}} \varphi_{0}^{\mathrm{SSASPM}} R_{0}^{\mathrm{boot}} S_{t-1}^{p}}{S_{0}^{\text {SSASPM }}+\left(e^{-M_{0} \mathrm{boot}} \varphi_{0}^{\text {SSASPM }}-1\right) S_{t-1}^{p}}
$$

The parameters $e^{-M_{0} \text { boot }}$ and $R_{0}^{\text {boot }}$ were bootstrap replicate parameter values drawn from the bivariate normal distribution of age-0 pup survival, $e^{-M_{0} \text { boot }}$, and unexploited equilibrium recruitment, $R_{0}^{\text {boot }}$, as described above. The parameter $\varphi_{0}^{\text {SSASPM }}$ was the equilibrium spawning stock size per recruit obtained from the SSASPM assessment model, and the parameter $S_{0}^{\text {SSASPM }}$ was the unexploited equilibrium spawning stock size obtained from the SSASPM assessment model.

Projection period annual catch at age in numbers.-For the projection period, the annual catch at age $a$ in year $t, C_{t, a}^{p}$ in numbers, was calculated from $N_{t, a}^{p}, F_{t, a}^{p}$, and $Z_{t, a}^{p}$ using the Baranov catch equation (e.g., Quinn and Deriso 1999 equation 1.22) as follows:

$$
C_{t, a}^{p}=\frac{F_{t, a}^{p}}{Z_{t, a}^{p}}\left(1-e^{-Z_{t, a}^{p}}\right) N_{t, a}^{p},
$$

where, $F_{t, a}^{p}$ was calculated as: $F_{t, a}^{p}=\operatorname{sel}_{a}^{p} F_{t}^{p}$, and $\operatorname{sel}_{a}^{p}, F_{t}^{p}, N_{t, a}^{p}$, and $Z_{t, a}^{p}$ were obtained as described above.

## Retrospective Projection Period

The retrospective projection period was used as a projection model diagnostic to compare retrospective time series to those obtained directly from the SSASPM assessment model output. Retrospective time series were calculated for the annual fishing mortality rate, total population numbers, spawning stock size, recruitment, catch, and yield. Comparisons to SSASPM output were made for data obtained from the base model configuration of the Atlantic sharpnose projections (NMFS 2013a). Time series were compared using the average of the annual percent
differences over the years in the retrospective period, calculated as follows: ([predictedobserved]/observed)*100. The predicted and observed values were the retrospective projection time series and the time series obtained from the SSASPM assessment model output, respectively. Retrospectively projected time series were considered to have had relatively good agreement with SSASPM output if the absolute value of the average annual percent difference was less than or equal to $5 \%$.

The duration of the historic (retrospective) projection period was defined by the number of years included in the SSASPM assessment model (e.g., NMFS 2013a, 2013b, 2013c). The beginning year $(t)$ of the retrospective period ( $t=\mathrm{h} . \mathrm{styr}$ ) was the first year modeled in the SSASPM assessment model. The ending year of the retrospective period ( $t=\mathrm{h}$.endyr) was the terminal year modeled in the SSASPM assessment model.

Retrospective period fishing mortality rate at age.-For the retrospective projection period, the estimate of annual fishing mortality rate at age $a$ in year $t$ obtained from the SSASPM assessment model output ( $\hat{t}_{t, a}^{\text {SSASPM }}$ ) was used directly in the projections. The total mortality rate at age $a$ in year $t\left(Z_{t, a}^{h}\right)$ was then calculated from the fishing mortality at age $a$ in year $t$ plus the natural mortality rate at age $a$ obtained from the SSASPM assessment model output ( $M_{a}^{\text {SSASPM }}$ ) as follows:

$$
Z_{t, a}^{h}=\hat{F}_{t, a}^{\mathrm{SSASPM}}+M_{a}^{\mathrm{SSASPM}} .
$$

For comparison with the retrospective projections, the annual fishing mortality rate at age $a$ in year $t\left(F_{t, a}^{h}\right)$ was assumed to follow the relationship:

$$
F_{t, a}^{h}=C_{t, a}^{\mathrm{SSASPM}} / \bar{N}_{t, a}^{\mathrm{SSASPM}},
$$

where $C_{t, a}^{\text {SSASPM }}$ was the annual catch at age (in numbers) obtained from SSASPM, and $\bar{N}_{t, a}^{\text {SSASPM }}$ was the average annual year-class size (in numbers) obtained from SSASPM. Catch at age in numbers obtained from SSASPM, $C_{t, a}^{\text {SSASPM }}$, was assumed to be proportional to $\bar{N}_{t, a}^{\text {SSASPM }}$ as:
$C_{t, a}^{\text {SSASPM }}=F_{t, a}^{h} \bar{N}_{t, a}^{\text {SSASPM }}$ (e.g., Quinn and Deriso 1999 equation 1.27). The annual fishing mortality rate was then obtained as $F_{t}^{h}=\max \left(F_{t, a}^{h}\right)$.

For the base model configuration of the Atlantic sharpnose projections (NMFS 2013a), the time series of annual fishing mortality rate, $F_{t}^{h}$, obtained as described above, agreed with the time series of annual fishing mortality rate obtained from SSASPM output as $F_{t}^{\text {SSASPM }}=\max ($ $\left.\hat{F}_{t, a}^{\text {SSASPM }}\right)(0.0 \%$ average annual percent difference). I.e., on average, there was no difference between the time series of assumed annual fishing mortality rate and that obtained from the SSASPM output.

Retrospective period population numbers at age.-For the retrospective projection period, the population numbers at age $a$ at the beginning of year $t$ were calculated (for both sexes combined) as follows:

$$
N_{t, a}^{h}=\left\{\begin{array}{ll}
\hat{N}_{t=\mathrm{h} . \mathrm{styr}, a}^{\mathrm{SSASM}} & \\
R_{t}^{h} & \text { if } a=1 \\
N_{t-1, a-1}^{h} e^{-Z_{t-1, a-1}^{h}} & \text { if } 1<a<a_{\max } \\
N_{t-1, a-1}^{h} e^{-Z_{t-1, a-1}^{h}}+N_{t-1, a}^{h} e^{-Z_{t-1, a}^{h}} & \text { if } a=a_{\max }
\end{array}\right\} \text { if h.styr } t=t \leq \mathrm{h} . \mathrm{styr} . \mathrm{h} . \mathrm{endyr},
$$

where $R_{t}^{h}$ was the annual retrospective recruitment in year $t$, obtained as described below, and $\hat{N}_{t=\mathrm{h} . \mathrm{syy}, a}^{\mathrm{SSAPM}}$ was the annual population numbers at age $a$ in the first retrospective year, $t=\mathrm{h} . \mathrm{styr}$, obtained from the SSASPM assessment model output.

The retrospectively projected time series of annual population numbers, $N_{t}^{h}$, obtained as described above, had relatively good agreement with the time series of annual population numbers obtained directly from SSASPM model output, $N_{t}^{\text {SSASPM }}$, for the base model configuration of the Atlantic sharpnose projections (NMFS 2013a) ( $-2.1 \%$ average annual percent difference). I.e., on average, the retrospectively projected annual population numbers were about $2 \%$ smaller than those obtained from the SSASPM model output.

Retrospective spawning stock size.-For the retrospective projection period, the spawning stock size in year $t\left(S_{t}^{h}\right)$, was calculated from $N_{t, a}^{h}$ (for both sexes combined), $Z_{t, a}^{h}$, and the net (per capita) fecundity at age $a$ obtained from the SSASPM assessment model $\left(f_{a}^{\text {SSASPM }}\right)$ as follows:

$$
S_{t}^{h}=\sum_{a=1}^{a_{\max }} f_{a}^{\mathrm{SSASPM}} N_{t, a}^{h} e^{-\tau_{M} Z_{t, a}^{h}},
$$

where $\tau_{M}$ was defined as described above.
The retrospectively projected time series of annual spawning stock size, $S_{t}^{h}$, obtained as described above, had relatively good agreement with the time series of annual spawning stock size obtained directly from SSASPM model output, $S_{t}^{\text {SSASPM }}$, for the base model configuration of the Atlantic sharpnose projections (NMFS 2013a) ( $4.3 \%$ average annual percent difference). I.e., on average, the retrospectively projected annual spawning stock size was about $4 \%$ smaller than that obtained from the SSASPM assessment model output.

Retrospective period annual recruitment.-For the retrospective projection period, the annual recruitment in year $t, R_{t}^{h}$, was calculated from retrospective spawning stock size in year $t$ $-1, S_{t-1}^{h}$, using the Beverton-Holt spawner-recruit curve, and assuming that recruitment occurred at age one (analogous to the SSASPM assessment model; e.g., NMFS 2013a equations 2 and 3) as follows:

$$
R_{t}^{h}=\frac{e^{-\hat{M}_{0} \text { SSAPSM }} \varphi_{0}^{\text {SSASPM }} \hat{R}_{0}^{\text {SSASPM }} S_{t-1}^{h}}{S_{0}^{\text {SSASPM }}+\left(e^{-\hat{M}_{0} \text { SSAPSM }} \varphi_{0}^{\text {SSASPM }}-1\right) S_{t-1}^{h}},
$$

where $e^{-\hat{M}_{0} \text { SSAPSM }}$ was the estimated age-0 survival rate of pre-recruit pups (age-0 pup survival) obtained from the SSASPM assessment model, $\hat{R}_{0}^{\text {SSASPM }}$ was the estimated unexploited equilibrium recruitment obtained from the SSASPM assessment model, $\varphi_{0}^{\text {SSASPM }}$ was the equilibrium spawning stock size per recruit obtained from the SSASPM assessment model, and
$S_{0}^{\text {SSASPM }}$ was the unexploited equilibrium spawning stock size obtained from the SSASPM assessment model.

The retrospectively projected time series of annual recruitment, $R_{t}^{h}$, obtained as described above, had relatively good agreement with the time series of annual recruitment obtained directly from the SSASPM model output, $R_{t}^{\text {SSASPM }}$, for the base model configuration of the Atlantic sharpnose projections (NMFS 2013a) ( $-1.3 \%$ average annual percent difference). I.e., on average, the retrospectively projected annual recruitment was about $1 \%$ smaller than that obtained from the SSASPM assessment model output.

Retrospective period catch at age in numbers.-For the retrospective projection period, catch at age $a$ in year $t$, in numbers, was calculated from $N_{t, a}^{h}, F_{t, a}^{h}$, and $Z_{t, a}^{h}$ as follows:

$$
C_{t, a}^{h}=\frac{F_{t, a}^{h}}{Z_{t, a}^{h}}\left(1-e^{-Z_{t, a}^{h}}\right) N_{t, a}^{h} .
$$

The retrospectively projected time series of annual catch, $C_{t}^{h}$, obtained as described above, had relatively good agreement with the time series of annual catch obtained directly from the SSASPM model output, $C_{t}^{\text {SSASPM }}$, for the base model configuration of the Atlantic sharpnose projections (NMFS 2013a) ( $-0.7 \%$ average annual percent difference). I.e., on average, the retrospectively projected annual catch was about $1 \%$ smaller than that obtained from the SSASPM assessment model output.

Retrospective period yield at age.-For the retrospective projection period, yield at age a in year $t$, in lb dressed weight, was calculated from $C_{t, a}^{h}$ as follows:

$$
Y_{t, a}^{h}=C_{t, a}^{h} w_{a}(2.2046)(0.5),
$$

where $w_{a}$ was the empirical weight at age $a(\mathrm{~kg})$ input into the SSASPM assessment model, the value 2.2046 was the ratio of $\mathrm{lb}: \mathrm{kg}$, and the value 0.5 is the assumed ratio of ( lb dressed weight): (lb round weight).

Compared to the other times series evaluated in the retrospective projections, the retrospectively projected time series of annual yield, $Y_{t}^{h}$, obtained as described above, had relatively poorer agreement with the time series of annual yield obtained directly from the SSASPM model output ( $Y_{t}^{\text {SSASPM }}$ ) for the base model configuration of the Atlantic sharpnose projections (NMFS 2013a), (-7.1\% average annual percent difference). I.e., on average, the retrospectively projected annual yield was about $7 \%$ smaller than that obtained from the SSASPM assessment model output. One reason for the relatively poorer agreement in yield may have been the use of an annual time step within the projection model compared to the use of a monthly time step within the SSASPM model.

