

Length-Based Risk Analysis (LBRA)

Jerald S. Ault

L. Jay Grove, J. Blondeau, K. McCarthy

SEDAR 103 Methods Workshop

Exploited stock's ability to maintain sufficient reproductive capacity to produce desired "yields" into the indefinite future requires assessment of risks from fishing and environment.

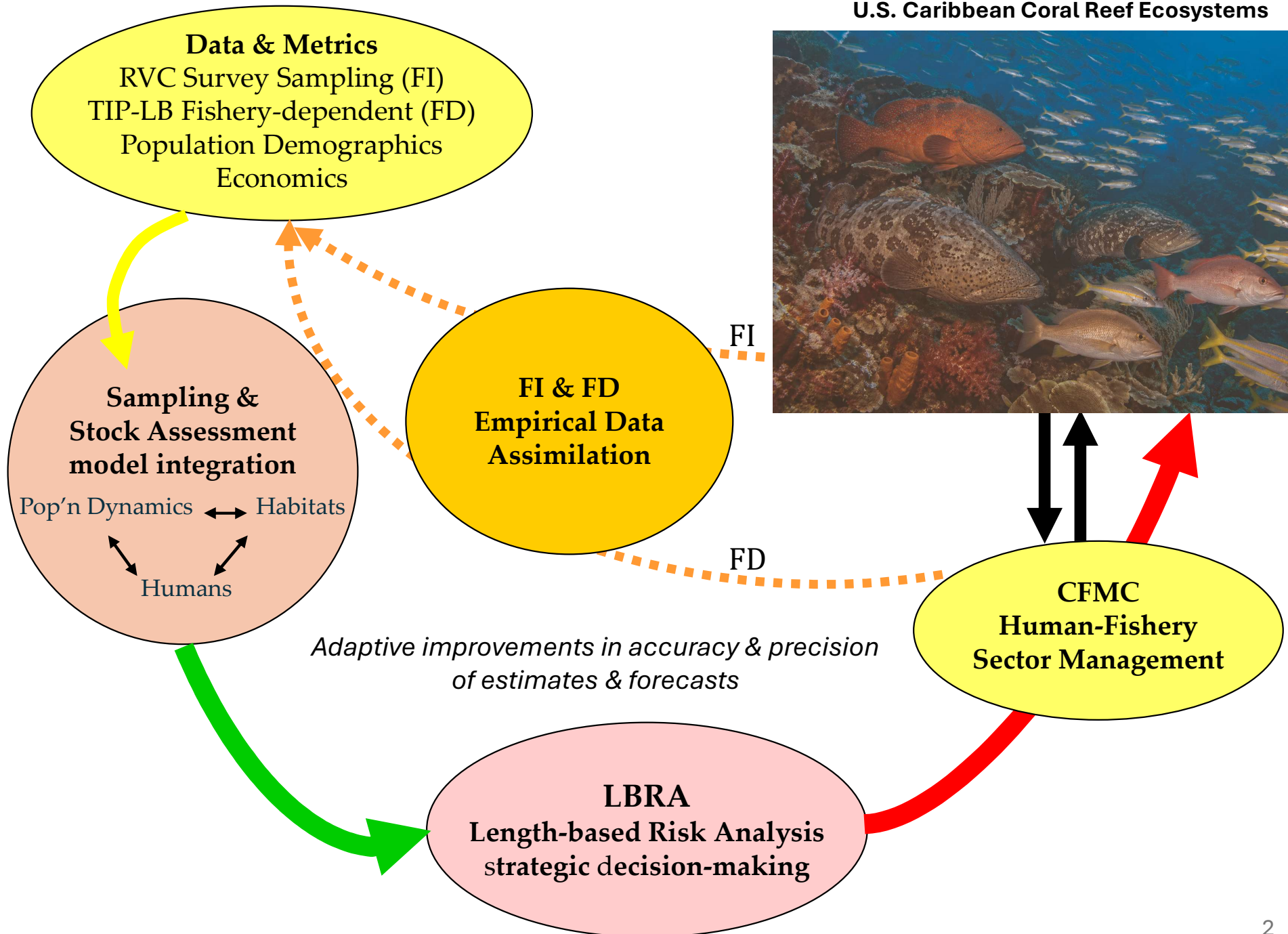


UNIVERSITY OF MIAMI

ROSENSTIEL SCHOOL of
MARINE, ATMOSPHERIC
& EARTH SCIENCE

Fishery Systems Science

U.S. Caribbean Coral Reef Ecosystems



Length-based Risk Analysis (LBRA)

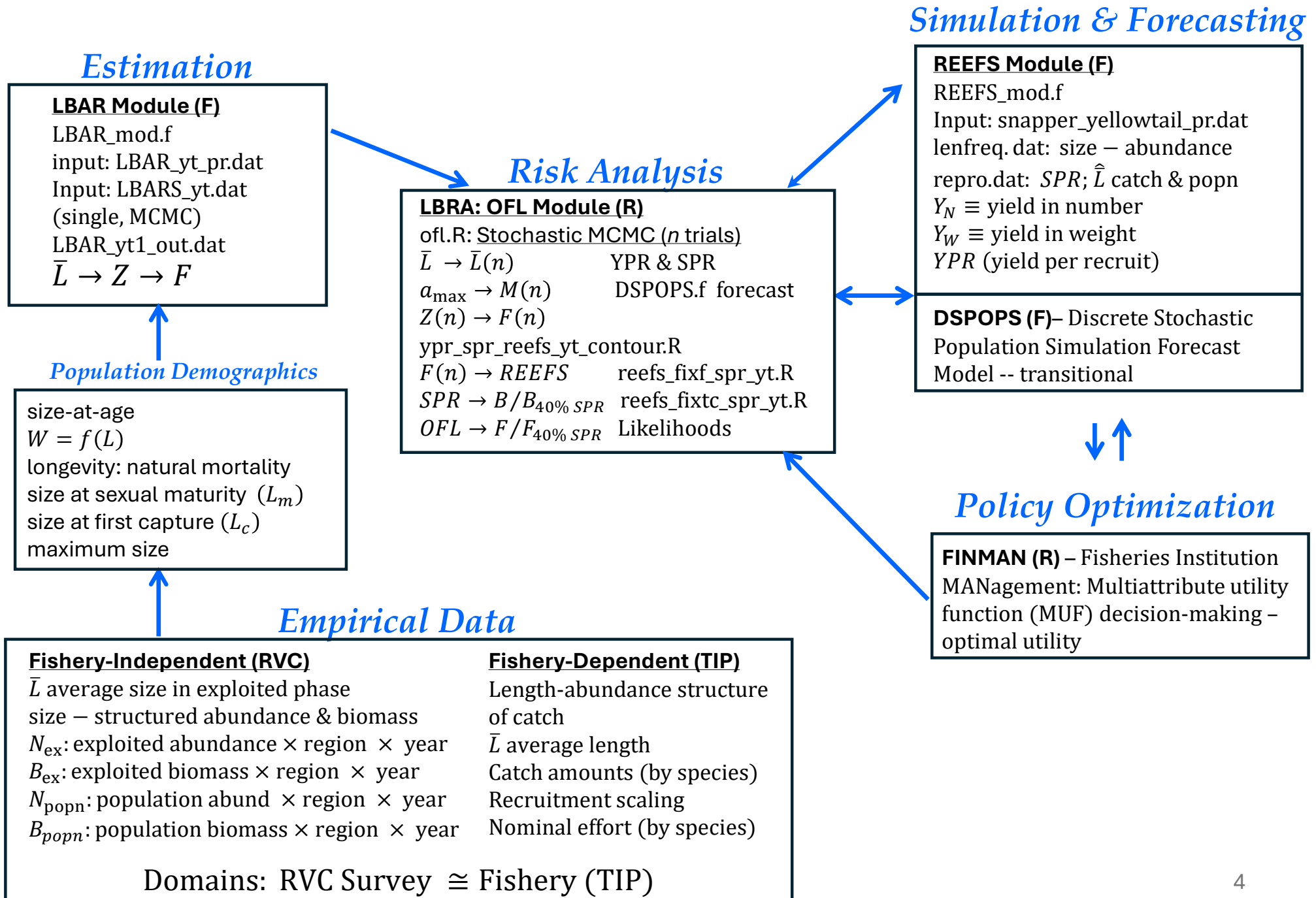
LBRA is a **probabilistic estimation and simulation framework** that integrates size-structured abundance data from fishery-independent and fishery-dependent surveys to quantify population and community sustainability risks, thereby supporting informed fisheries decision-making and policy optimization.

Ault, J.S. 1988. Nonlinear numerical simulation for assessing tropical fisheries with continuously breeding multicohort populations. Ph.D. Dissertation, University of Miami

- Goodman, Scripps - Methot
- Transition North Sea models to the tropics
- Artisanal, small-scale fisheries
- Hard to age, continuous recruitment

1996. Stock Assessment: Quantitative Methods & Applications for small-scale fisheries.

Length-based Risk Analysis (LBRA) Toolbox



LBRA: Estimation, Simulation & Forecasting

Bull Mar Sci. 100(2):283–312. 2024
<https://doi.org/10.5343/bms.2023.0025>

coral reef paper



Integrating the US Caribbean Reef Fish Visual Census into fishery stock assessments (2024)



Contents lists available at ScienceDirect

Fisheries Research

journal homepage: www.elsevier.com/locate/fishres



Length-based risk analysis of management options for the southern Florida USA multispecies coral reef fish fishery (2022)

Towards fishery-independent biomass estimation for Hawaiian Islands deepwater snappers (2018)



Assessing coral reef fish population and community changes in response to marine reserves in the Dry Tortugas, Florida, USA (2013)



Contents lists available at ScienceDirect

Ecological Indicators (2014)

journal homepage: www.elsevier.com/locate/ecolind



Indicators for assessing the ecological dynamics and sustainability of southern Florida's coral reef and coastal fisheries



ARTICLE

Canadian Journal Fisheries and Aquatic Sciences 73:1-11

A stepwise stochastic simulation approach to estimate life history parameters for data-poor fisheries (2016)

Marc O. Nadon and Jerald S. Ault

TOWARDS SUSTAINABLE MULTISPECIES FISHERIES IN THE FLORIDA, USA, CORAL REEF ECOSYSTEM (2005)

Life history demographic parameter synthesis for exploited Florida and Caribbean coral reef fishes (2020)

Molly H. Stevens | Steven G. Smith | Jerald S. Ault

ICES Journal of Marine Science



ICES Journal of Marine Science (2018), doi:10.1093/icesjms/fsy123

Length-based risk analysis for assessing sustainability of data-limited tropical reef fisheries (2019)

Jerald S. Ault^{1*}, Steven G. Smith¹, James A. Bohnsack², Jiangang Luo¹, Molly H. Stevens¹, Gerard T. DiNardo³, Matthew W. Johnson², and David R. Brvan¹

Length-Based Assessment of Coral Reef Fish Populations in the Main and Northwestern Hawaiian Islands (2015)

Marc O. Nadon^{1,2,3*}, Jerald S. Ault¹, Ivor D. Williams³, Steven G. Smith¹, Gerard T. DiNardo³

ICES Journal of Marine Science, 62: 417–423 (2005)
doi:10.1016/j.icesjms.2004.12.001

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Evaluation of average length as an estimator of exploitation status for the Florida coral-reef fish community (2005)

Jerald S. Ault, Steven G. Smith, and James A. Bohnsack

A retrospective (1979–1996) multispecies assessment of coral reef fish stocks in the Florida Keys (1998)

Transactions of the American Fisheries Society 121:115–122, 1992
© Copyright by the American Fisheries Society 1992

Analysis of Two Length-Based Mortality Models Applied to Bounded Catch Length Frequencies (1992)

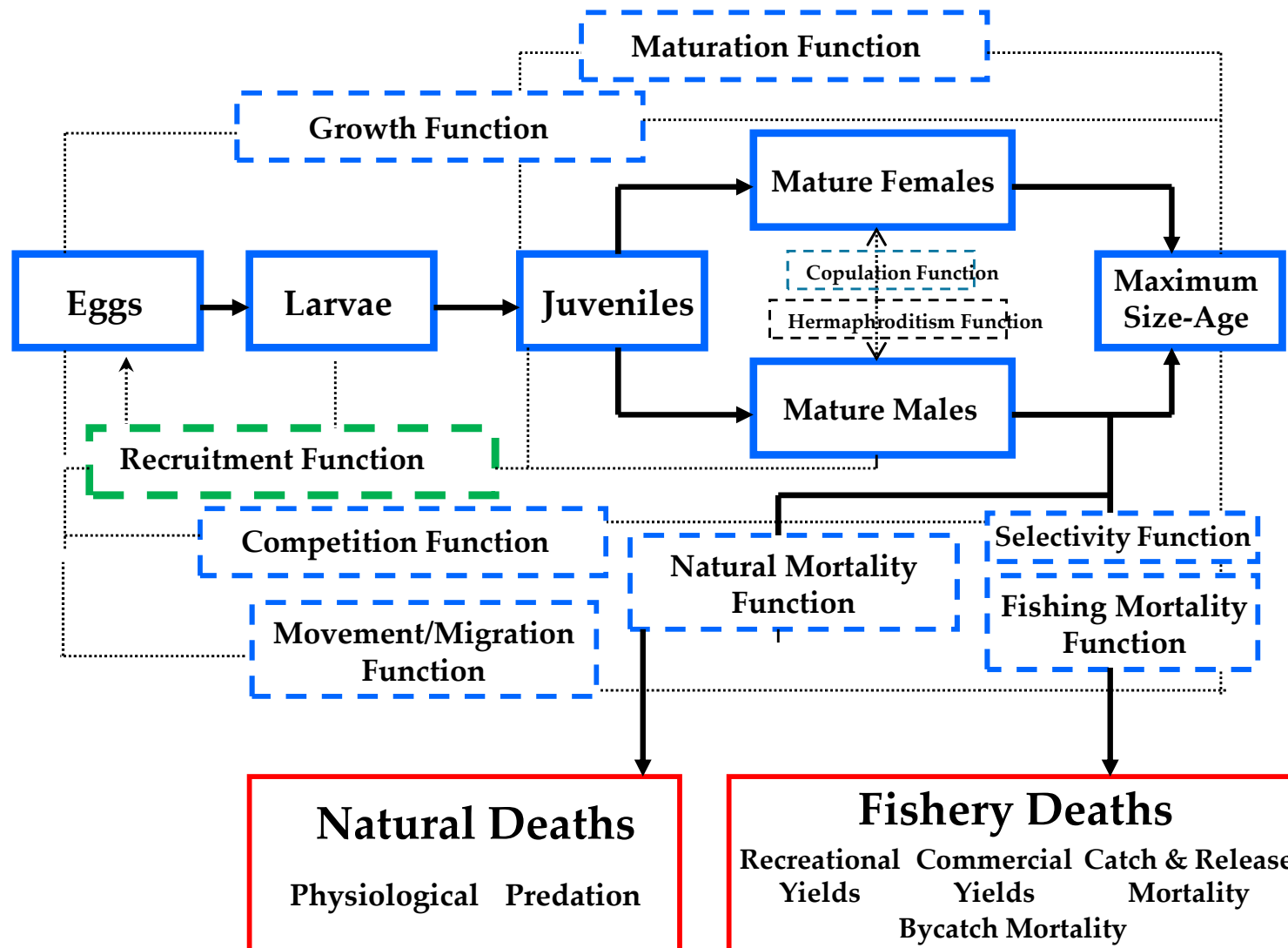
Proceedings of the 11th International Coral Reef Symposium, Ft. Lauderdale, Florida, 7-11 July 2008
Session Number 22

Are the Coral Reef Finfish Fisheries of South Florida Sustainable? (2009)

J.S. Ault, S.G. Smith, and J.T. Tilmant*

FINMAN: Simulated Decision Analysis with Multiple Objectives (1989) ⁵
JERALD S. AULT AND WILLIAM W. FOX, JR.

REEFS: Simulation & Forecasting



$$\frac{dN(a, t)}{dt} = \frac{\partial N(a, t)}{\partial a} + \frac{\partial N(a, t)}{\partial t} = -Z(a, t)N(a, t)$$

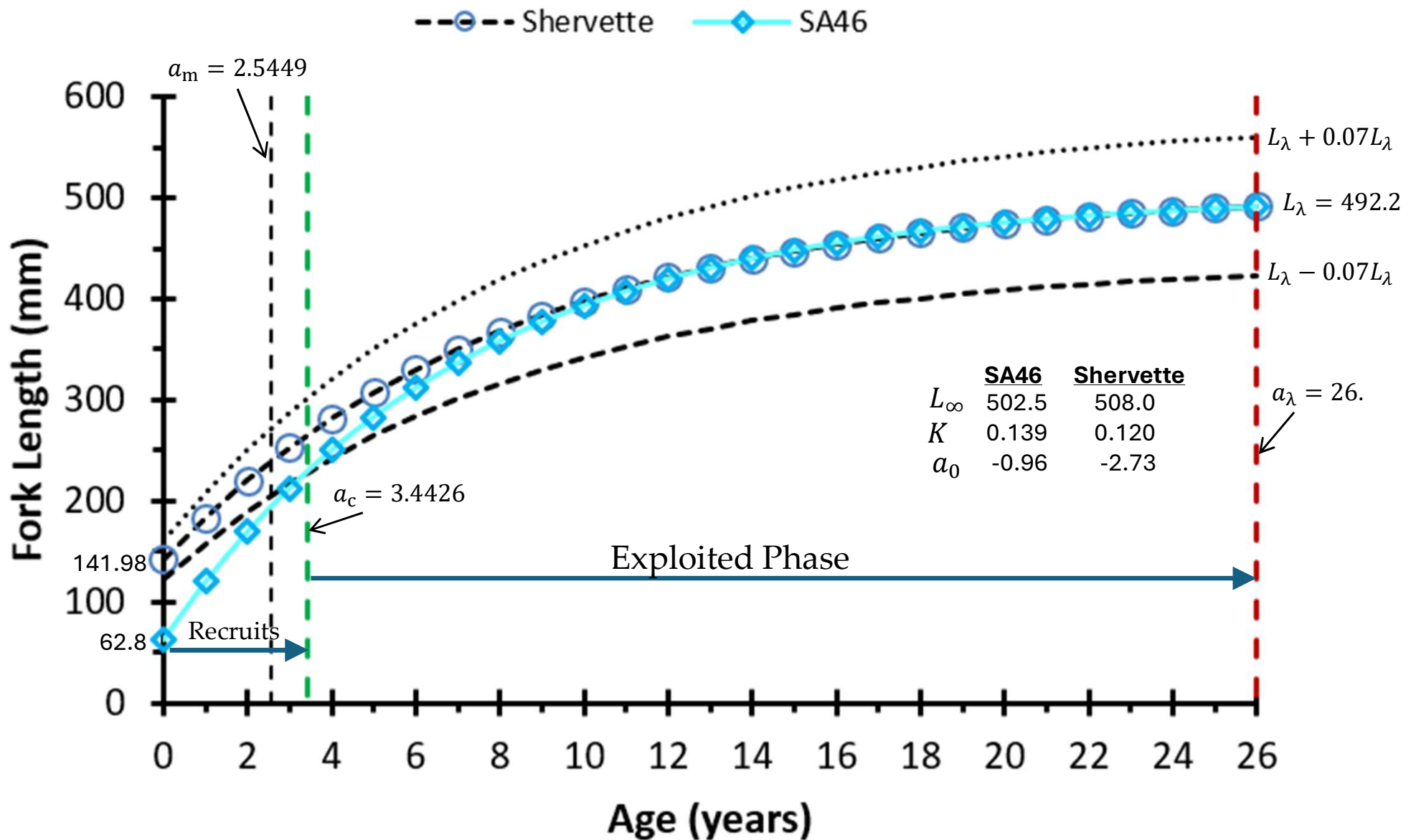
Data Inputs

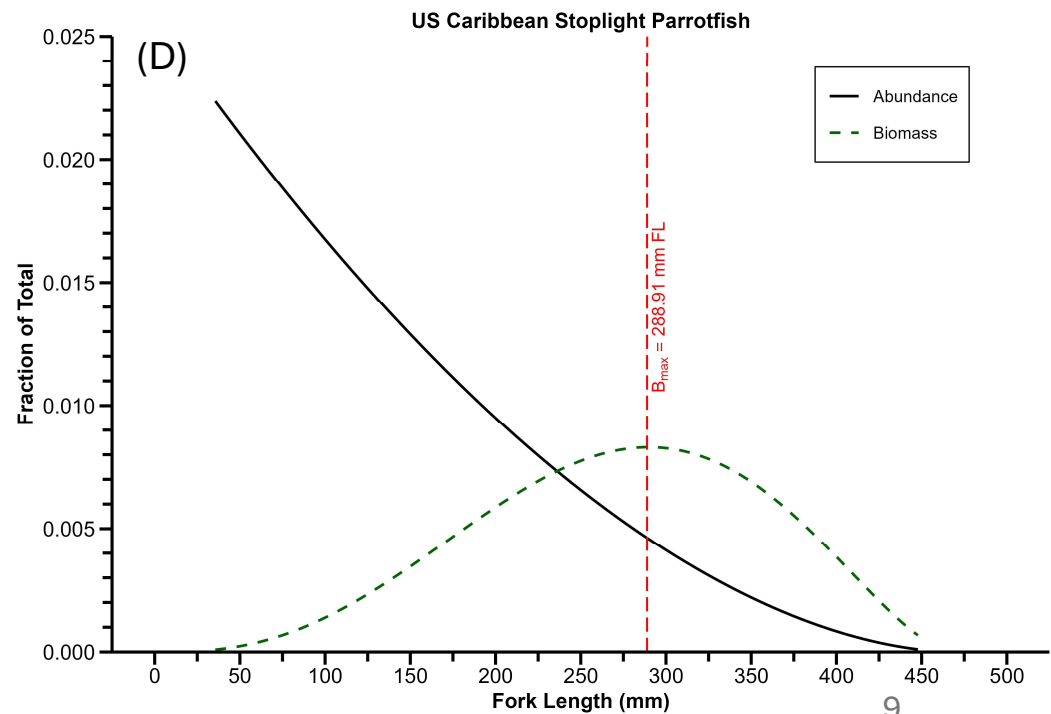
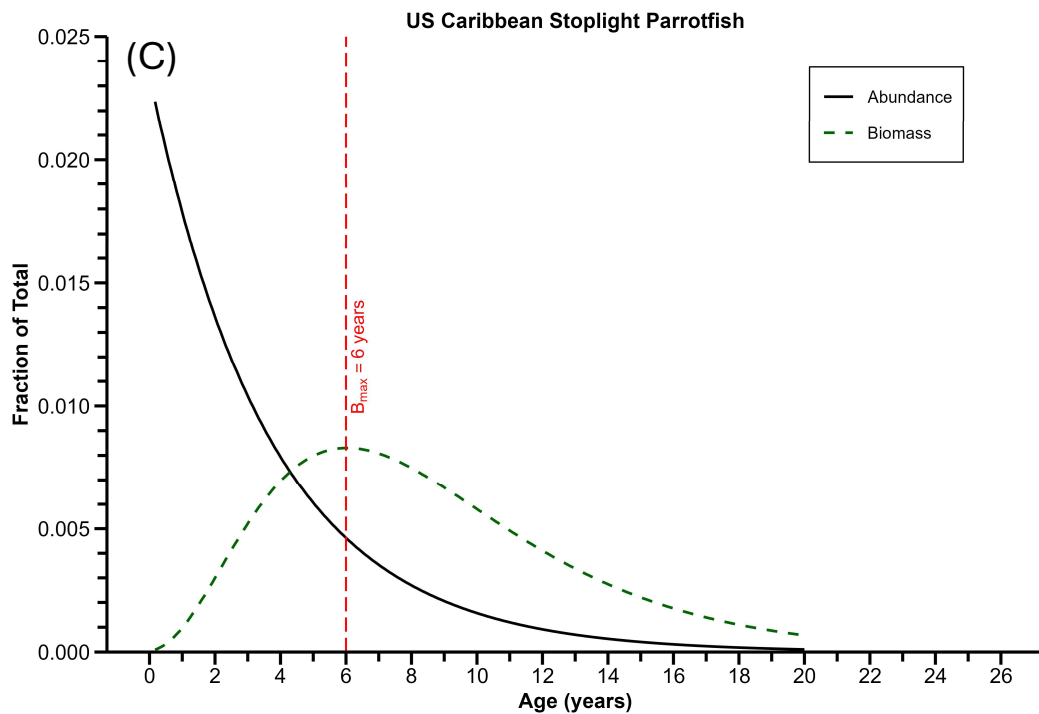
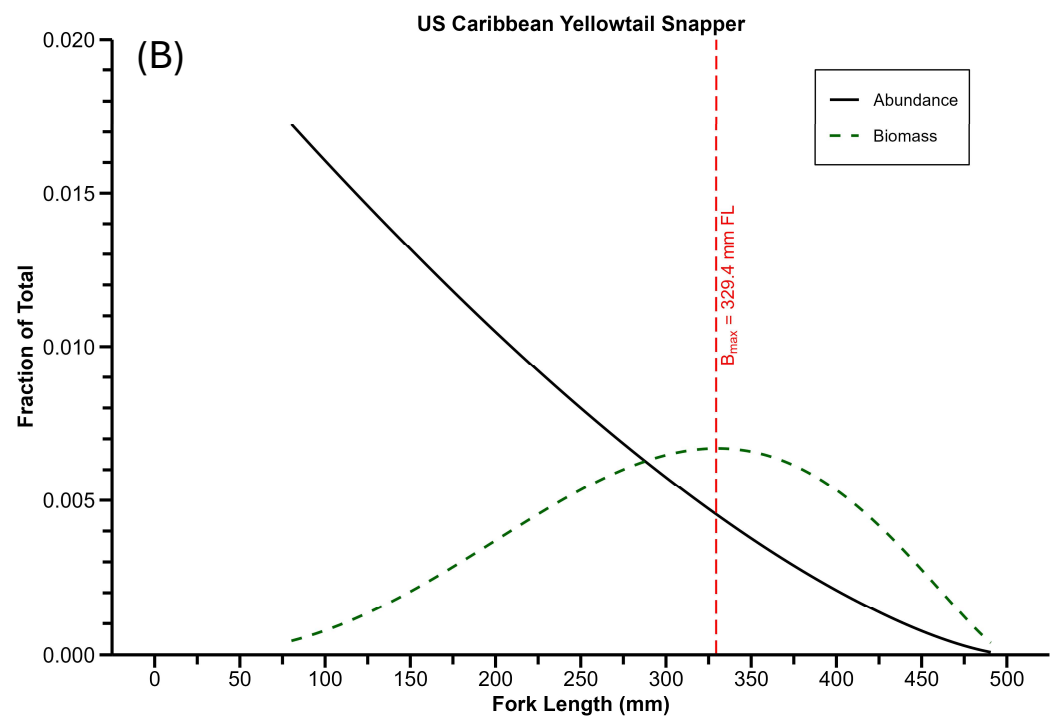
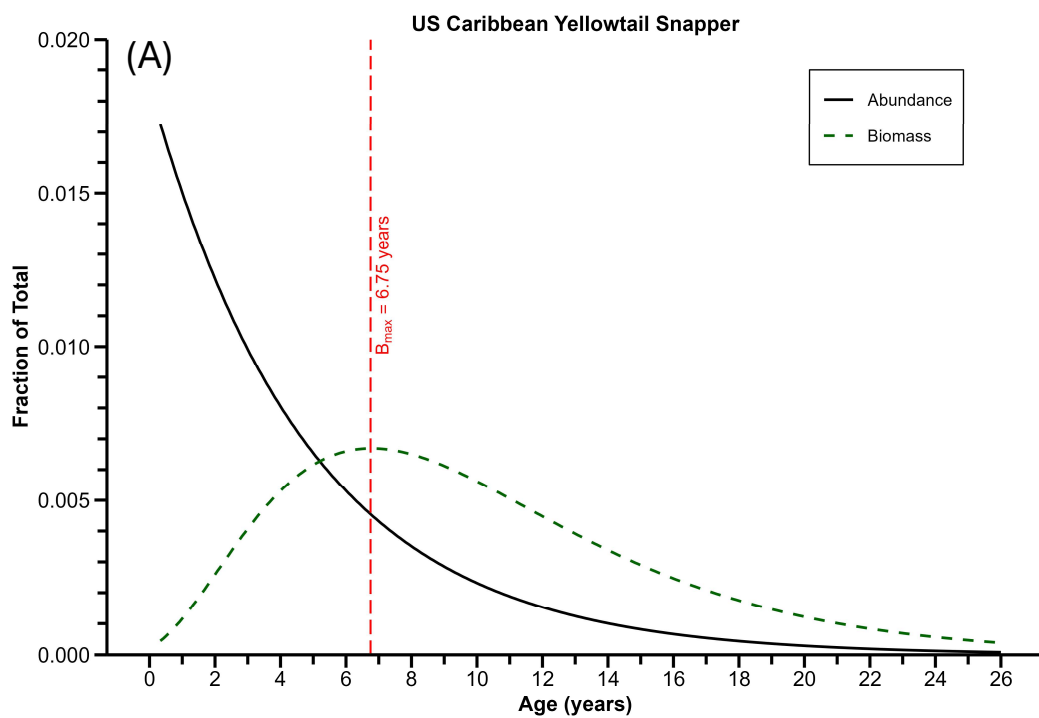
U.S. Caribbean Life History Demographic Data

Yellowtail snapper (YT), Stoplight parrotfish (SLP), Queen Triggerfish (QT)

Symbol	Definition	YT	SLP	QT	Units
L_{∞}	Asymptotic length (Primary)	502.5	470.0	550.0	mm FL
K	Brody growth coefficient (Primary)	0.139	.150	0.1050	per year
a_0	Age at which length equals zero (Primary)	-0.960	-0.4	-0.5850	years
L_{∞}	Asymptotic length	508.0	332.0	430.0	mm FL
K	Brody growth coefficient	0.120	0.390	0.1500	per year
a_0	Age at which length equals zero	-2.730	-0.06	-0.5850	years
W_{∞}	Asymptotic weight	2.181	2.208	1.4662	kg
α	Scalar coefficient of weight-length function	4.000E-05	3.70E-05	5.517808E-08	dimensionless
β	Power coefficient of weight-length function	2.8642	2.9100	2.8690	dimensionless
a_{λ}	Maximum observed age (under exploitative)	26	20	23	years
S_{λ}	Average survivorship probability to a_{λ}	0.0045	0.0045	0.0045	dimensionless
L_r	Length at first recruitment to population	25.0	27.37	36.1	mm FL
L_m	Length at 50% maturity	194.0	153	214.0	mm FL
$\bar{L}$	Average length in the exploited phase	268.727	284.425	305.533	mm FL
L_c	Minimum length at first capture	230.0	230	230	mm FL
L_{λ}	Mean length at maximum age	490.65	447.96	503.77	mm FL
W_{λ}	Mean weight at maximum age	2.0369	1.9201	3.1224	kg
$\bar{R}$	Population recruitment	276,609	66,000	793,000	fish per year
$\hat{M}$	Natural mortality rate estimated from $\hat{a}_{\lambda}$	0.2078	0.2702	0.2349	per year

Yellowtail snapper





Probabilistic Length-at-Age

Mean length at maximum age a_λ

$$L(a_\lambda, t) = L_\lambda = L_\infty(1 - e^{-K(a_\lambda - a_0)})$$

Mean length at midpoint of age interval $a + \Delta a$

$$\bar{L}(a) = L_\infty(1 - e^{-K((a+0.5\Delta a) - a_0)})$$

Probability of being length L given age a

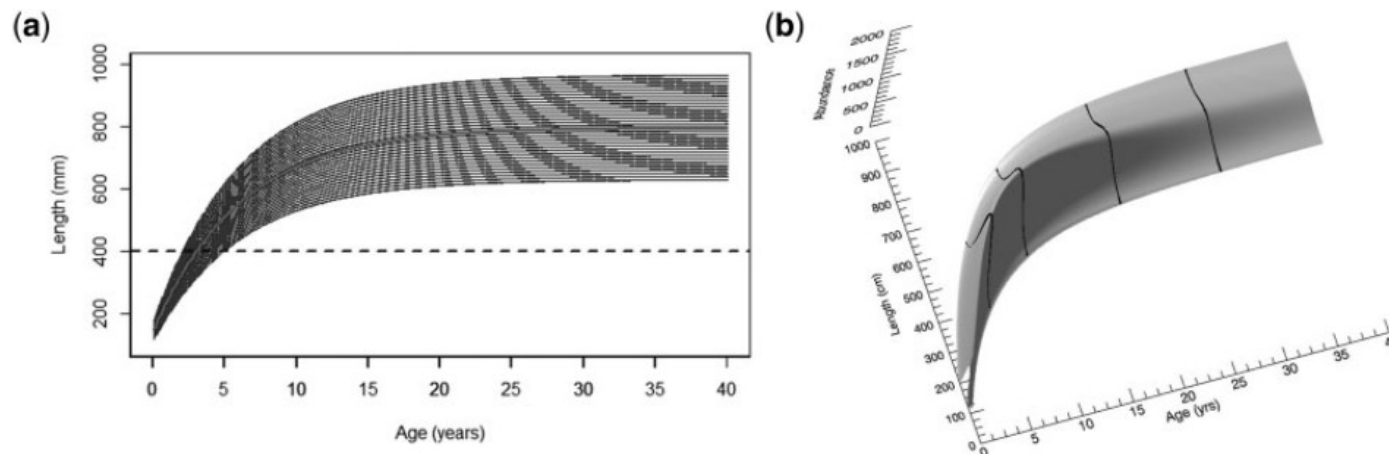
$$p(L|a) = p(c \leq \bar{L}(a) \leq d) = \Phi\left(\frac{d - \bar{L}(a)}{\sigma(a)}\right) - \Phi\left(\frac{c - \bar{L}(a)}{\sigma(a)}\right)$$

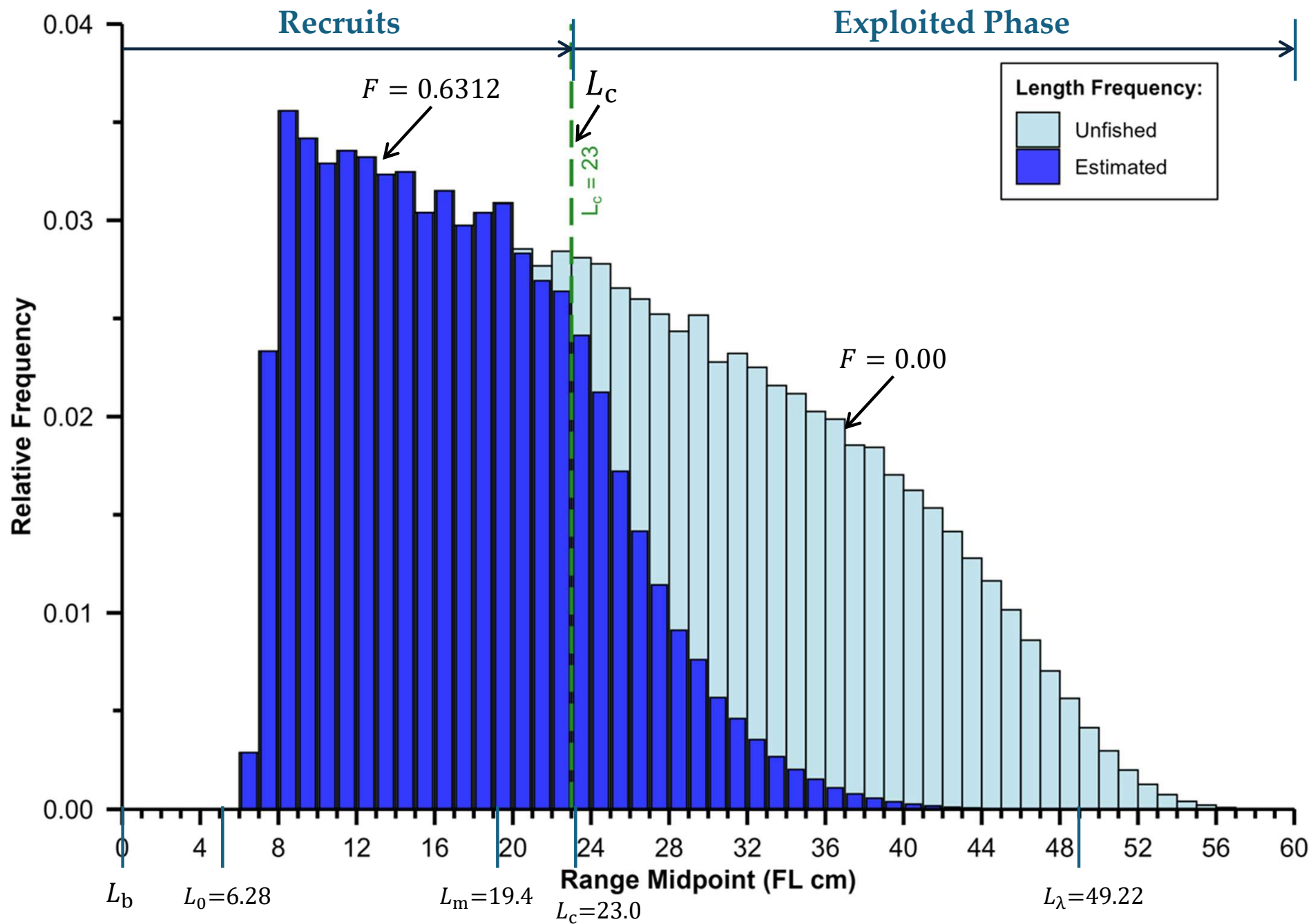
Mean weight at midpoint of age interval $a + \Delta a$

$$\bar{W}(a) = \alpha(\bar{L}(a))^\beta$$

Average population abundance at time t

$$\bar{N}(t) = \int_t^{t+\Delta t} \int_{a_r}^{a_\lambda} N(a, t) (1 - e^{-Z(a, t)}) / Z(a, t) da dt$$



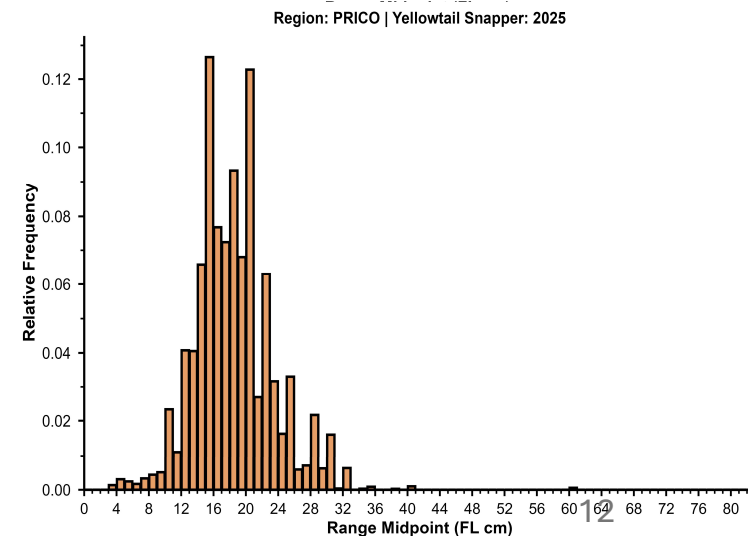
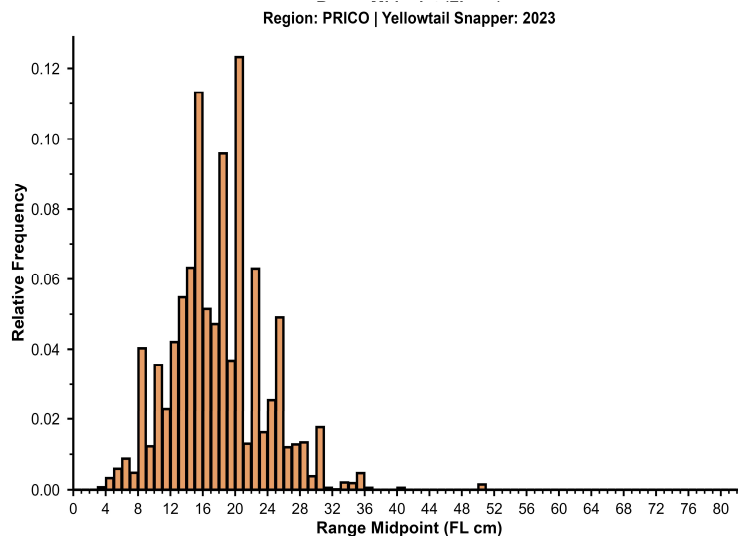
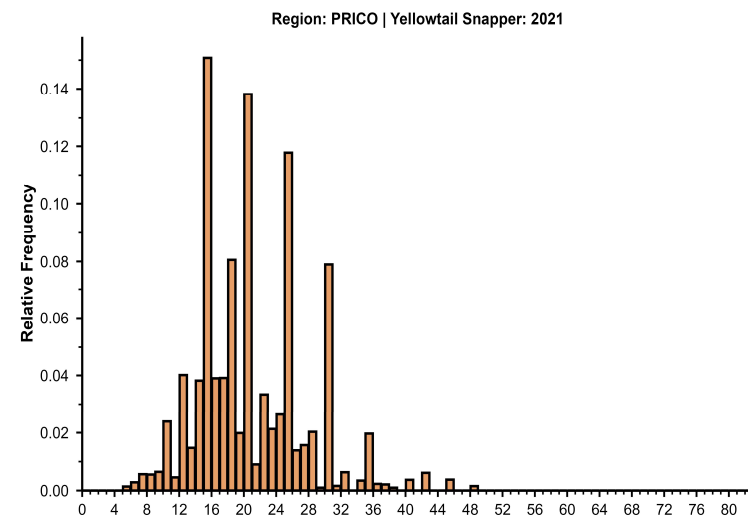
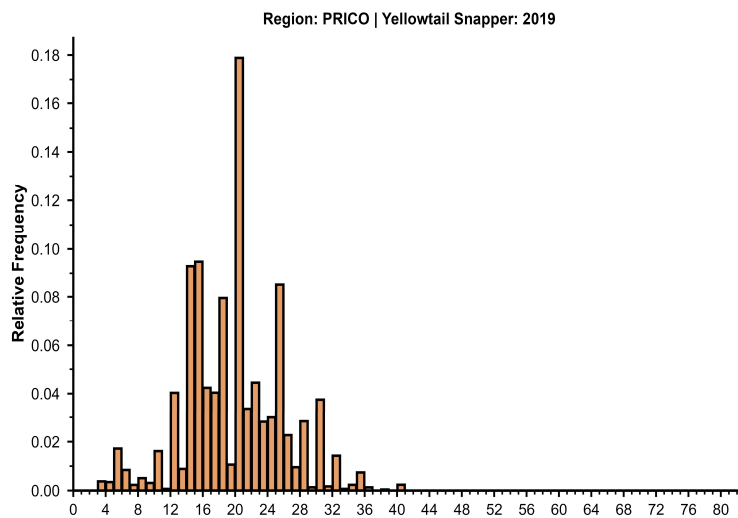


Yellowtail snapper

$$\bar{L}(t) = \frac{F(t) \int_{a_c}^{a_\lambda} N(a, t) L(a, t) da}{F(t) \int_{a_c}^{a_\lambda} N(a, t) da}$$

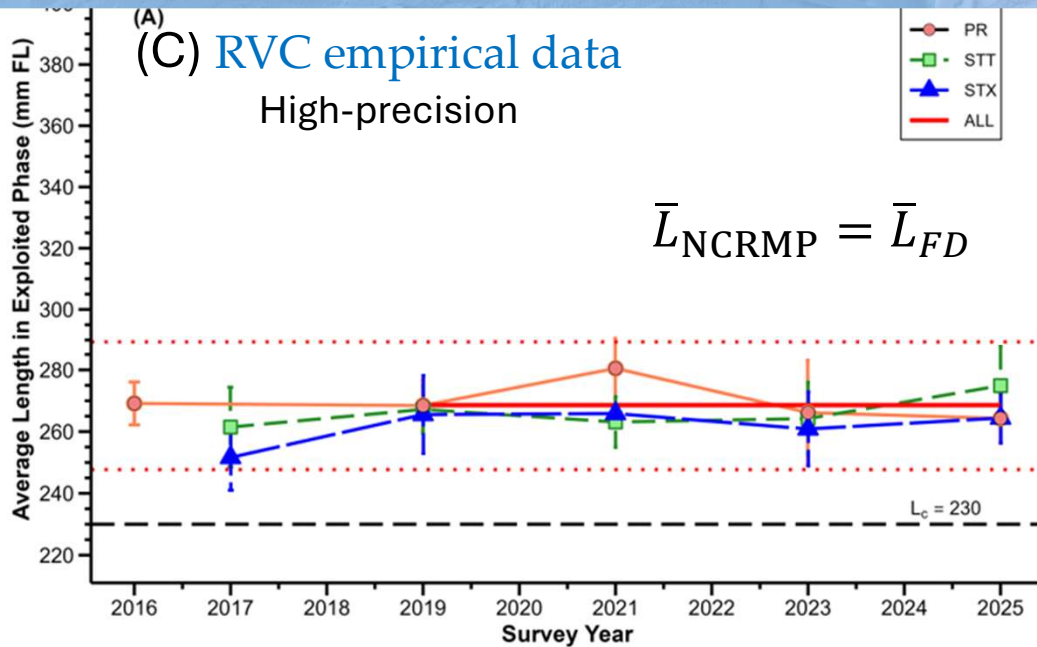
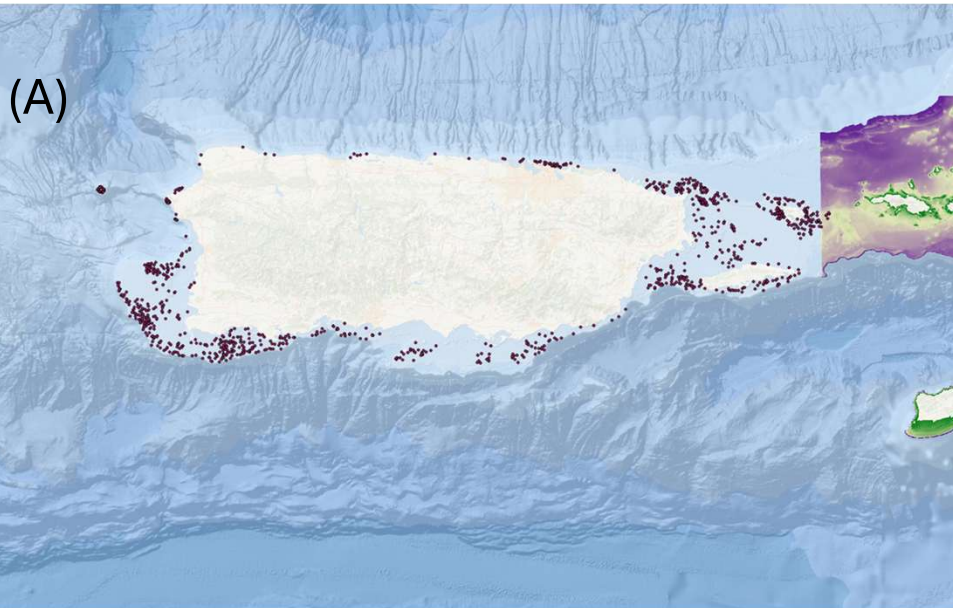
$$\bar{L}(t) = \frac{\bar{y}}{\bar{x}} = \frac{\frac{\sum y_i}{n}}{\frac{\sum x_i}{n}}$$

Fishery-independent
NCRMP

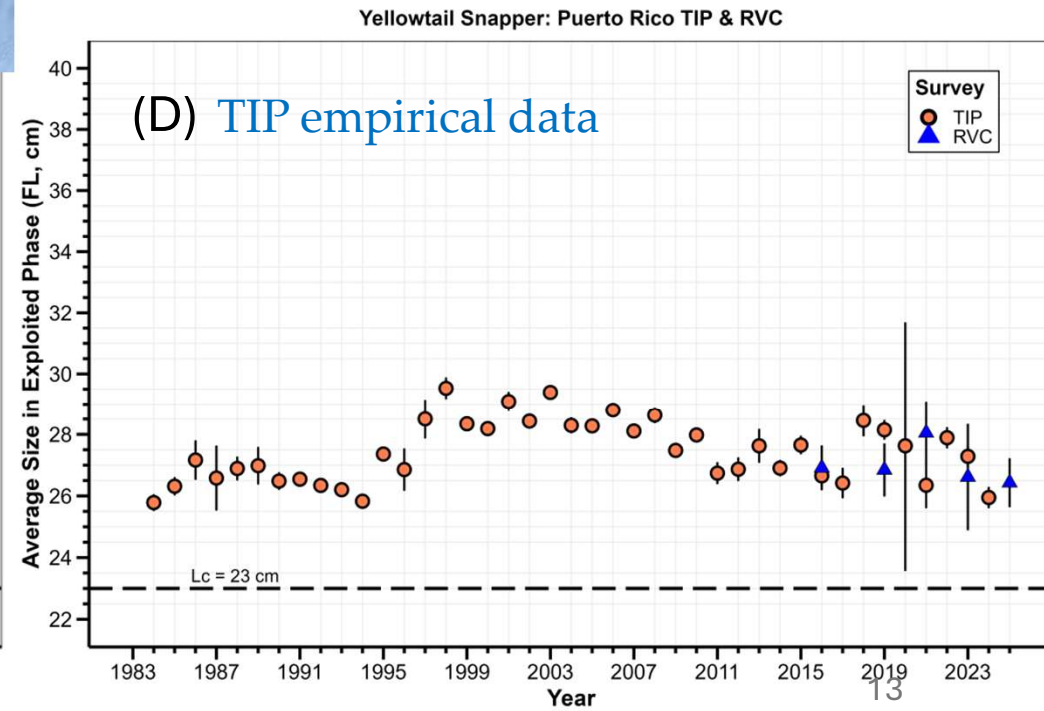
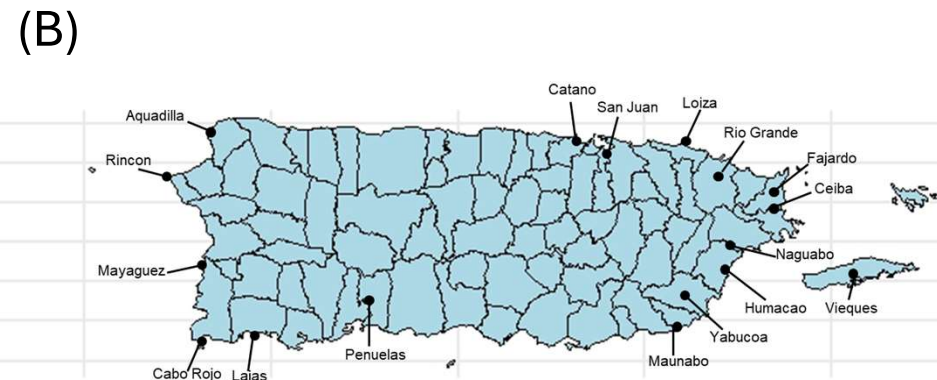


Comparative Data & Metrics

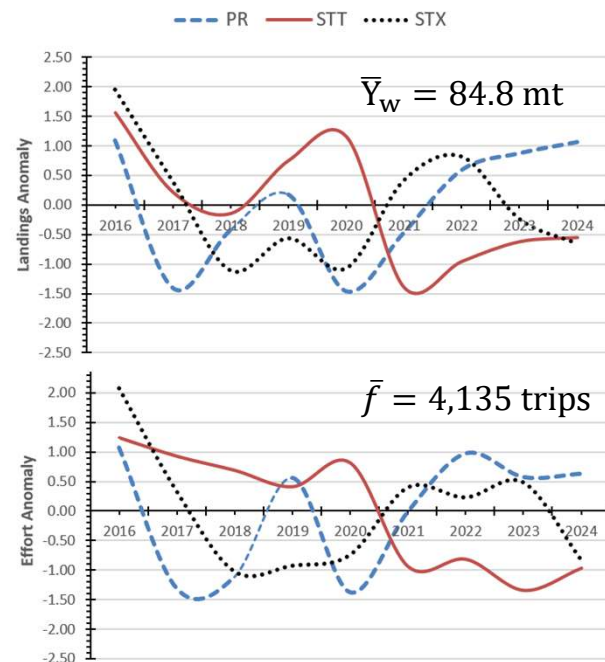
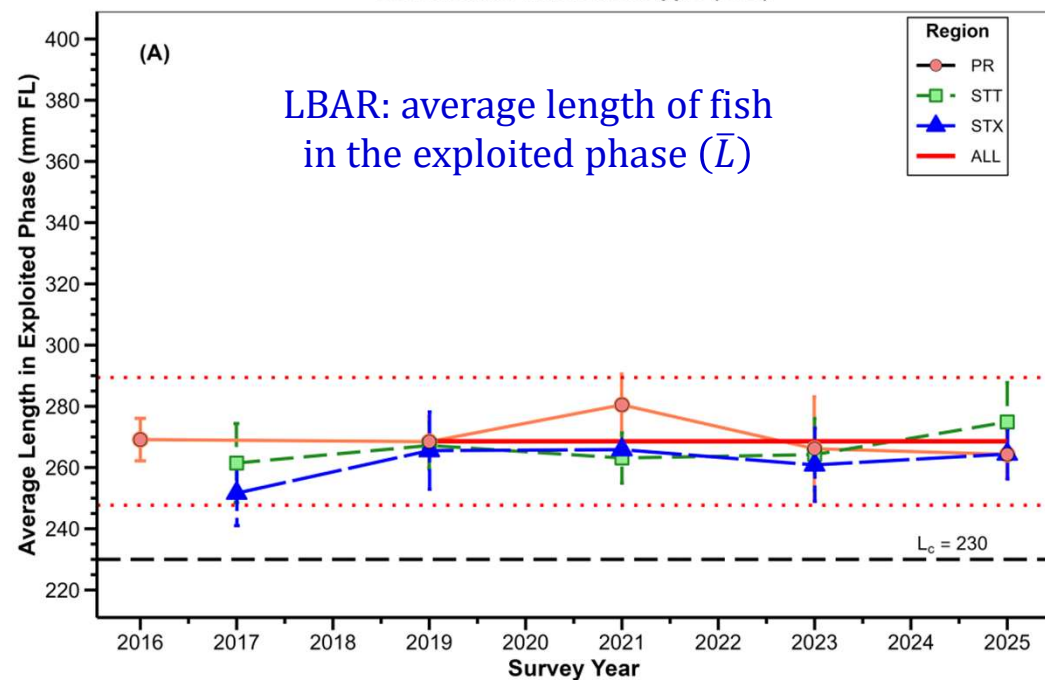
Fishery-independent (RVC)



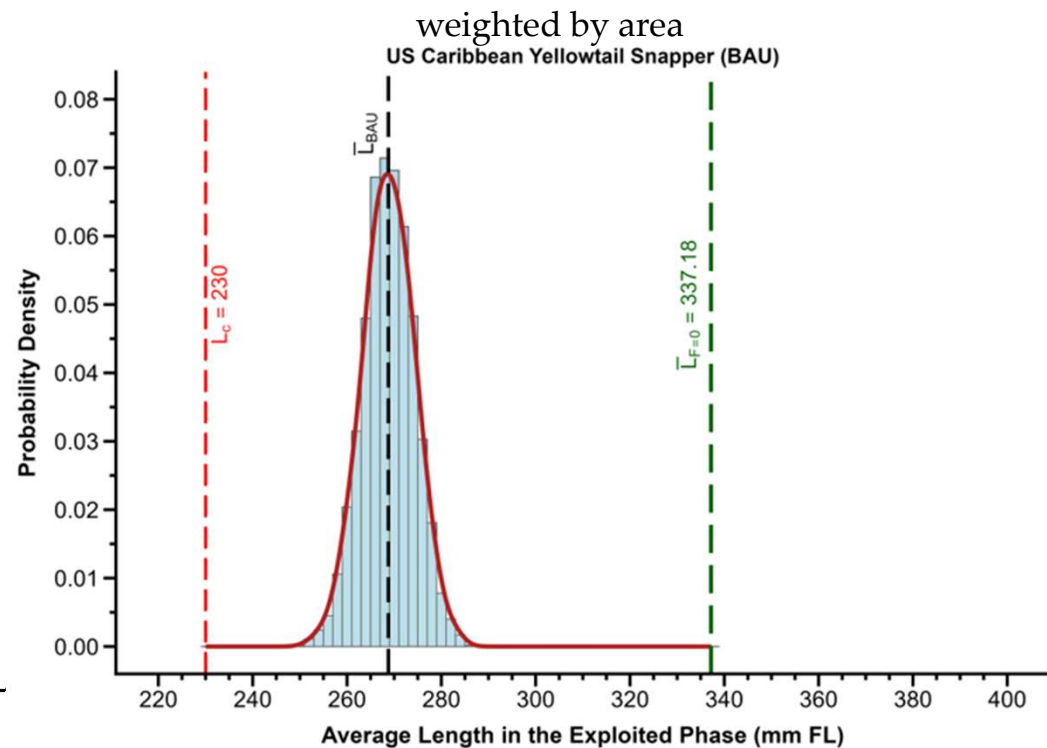
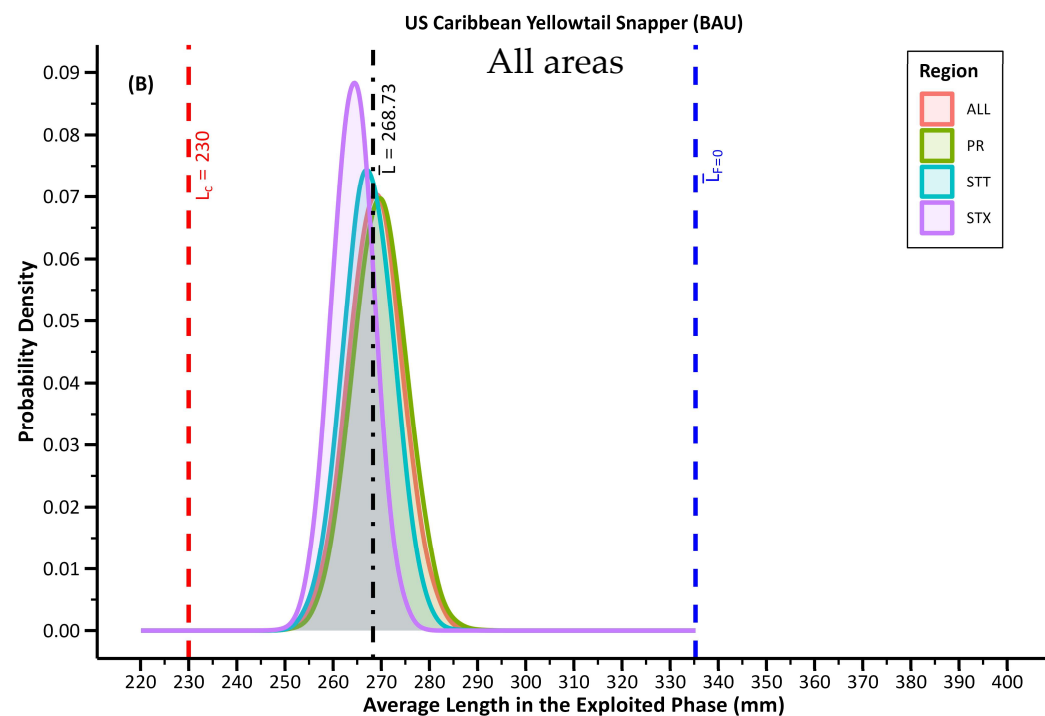
Fishery-dependent (TIP)



US Caribbean Yellowtail Snapper (RVC)



Year	cpue
2016	45.05
2017	43.36
2018	49.30
2019	43.96
2020	44.89
2021	41.73
2022	43.24
2023	47.13
2024	48.49
	45.24



Estimation

$(\bar{L})$: Central method around which overall approach is organized

Bound the life history to observed
maximum age (a_λ) ...
NO Methuselah fish

$$\bar{L}(t) = \frac{\int_{a_c}^{a_\lambda} FN(a, t) L(a, t) da dt}{\int_{a_c}^{a_\lambda} FN(a, t) da dt}$$

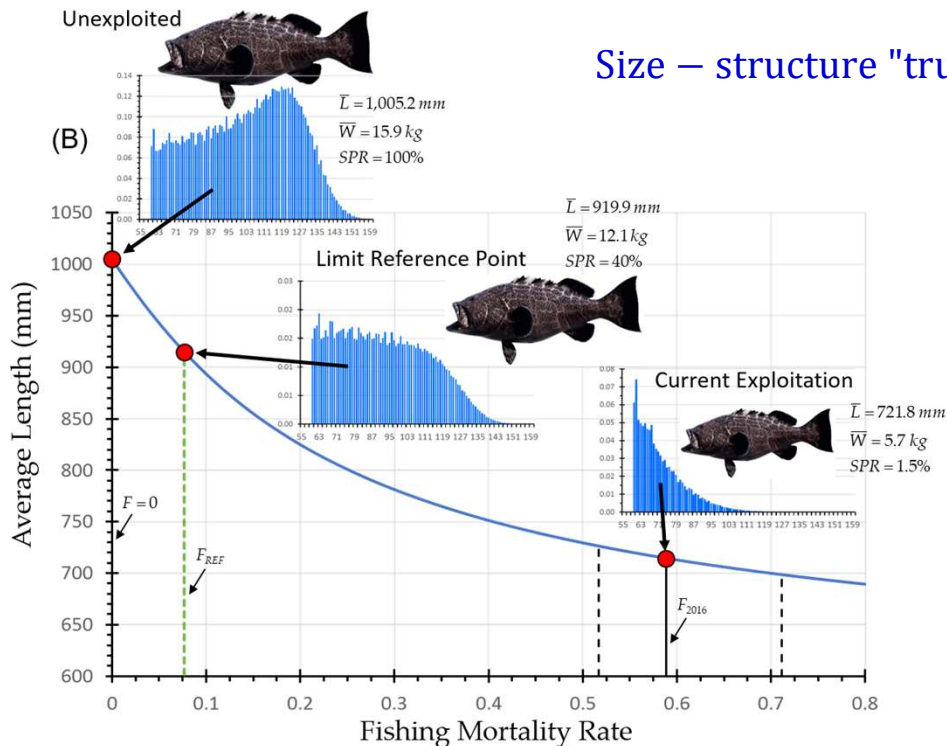
Ault. 1988

Ault & Ehrhardt. 1991

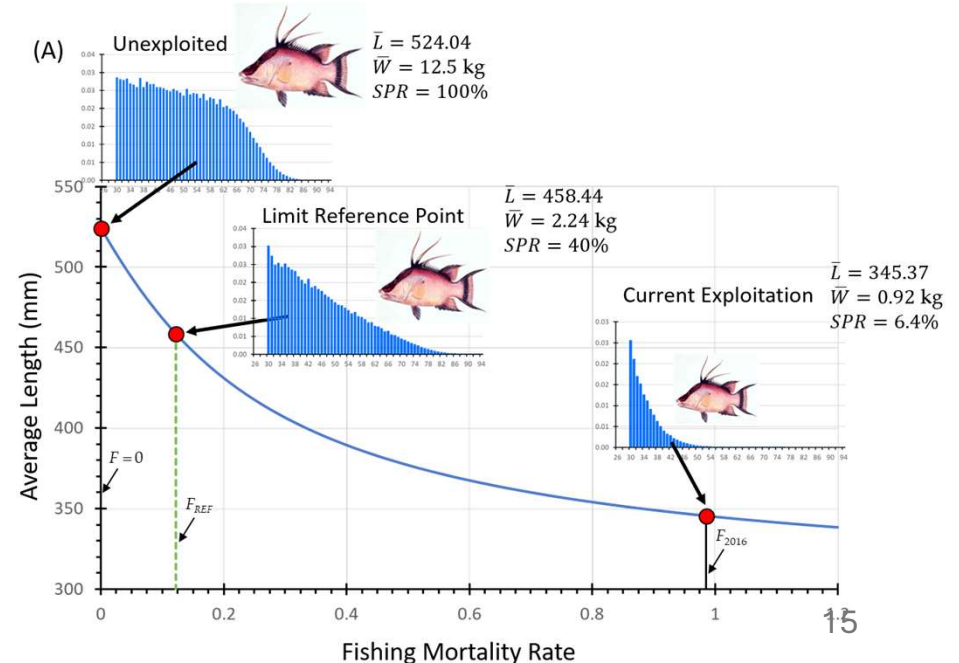
Ehrhardt & Ault. 1992

1 unknown parameter

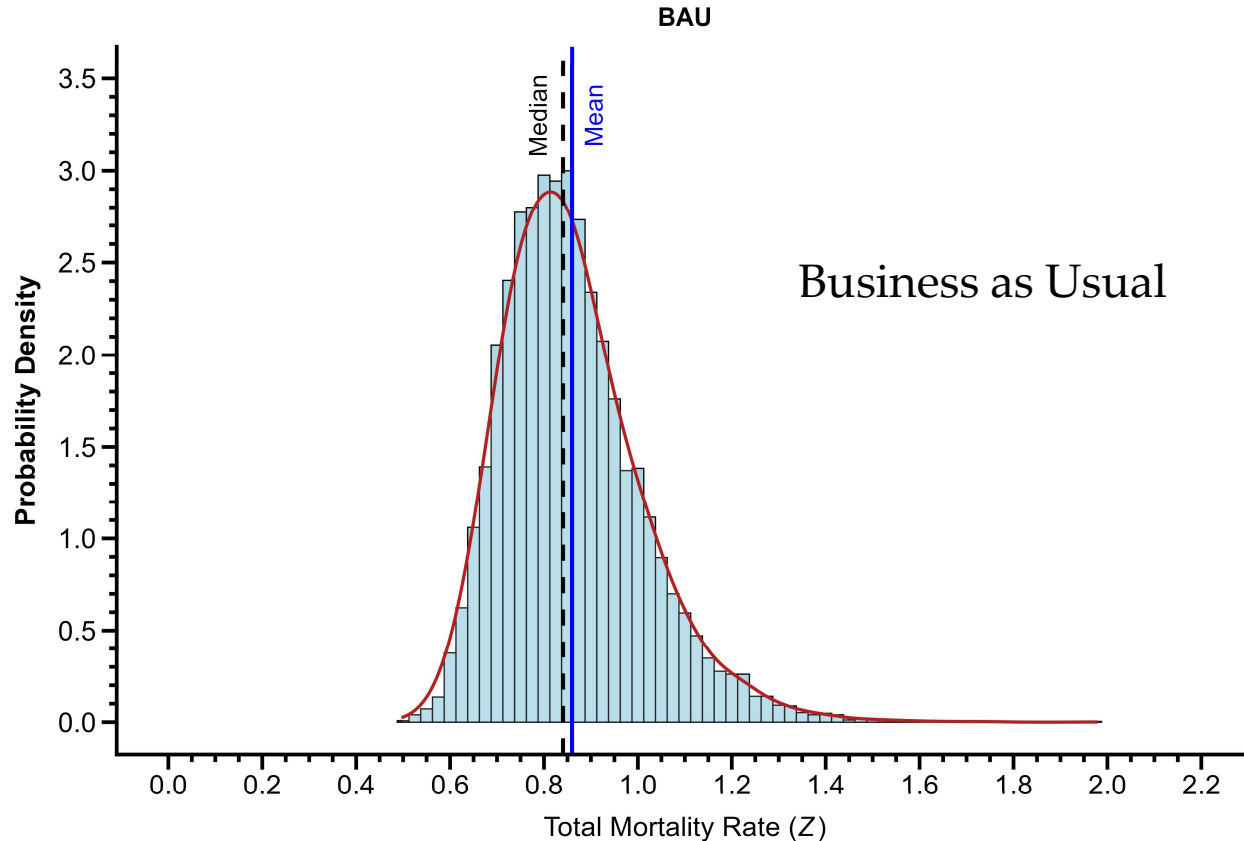
$$\left[\frac{L_\infty - L_\lambda}{L_\infty - L_c} \right]^{\frac{\hat{Z}(t)}{K}} = \frac{\hat{Z}(t)(L_c - \bar{L}(t)) + K(L_\infty - \bar{L}(t))}{\hat{Z}(t)(L_\lambda - \bar{L}(t)) + K(L_\infty - \bar{L}(t))}$$



Size – structure "truncation"

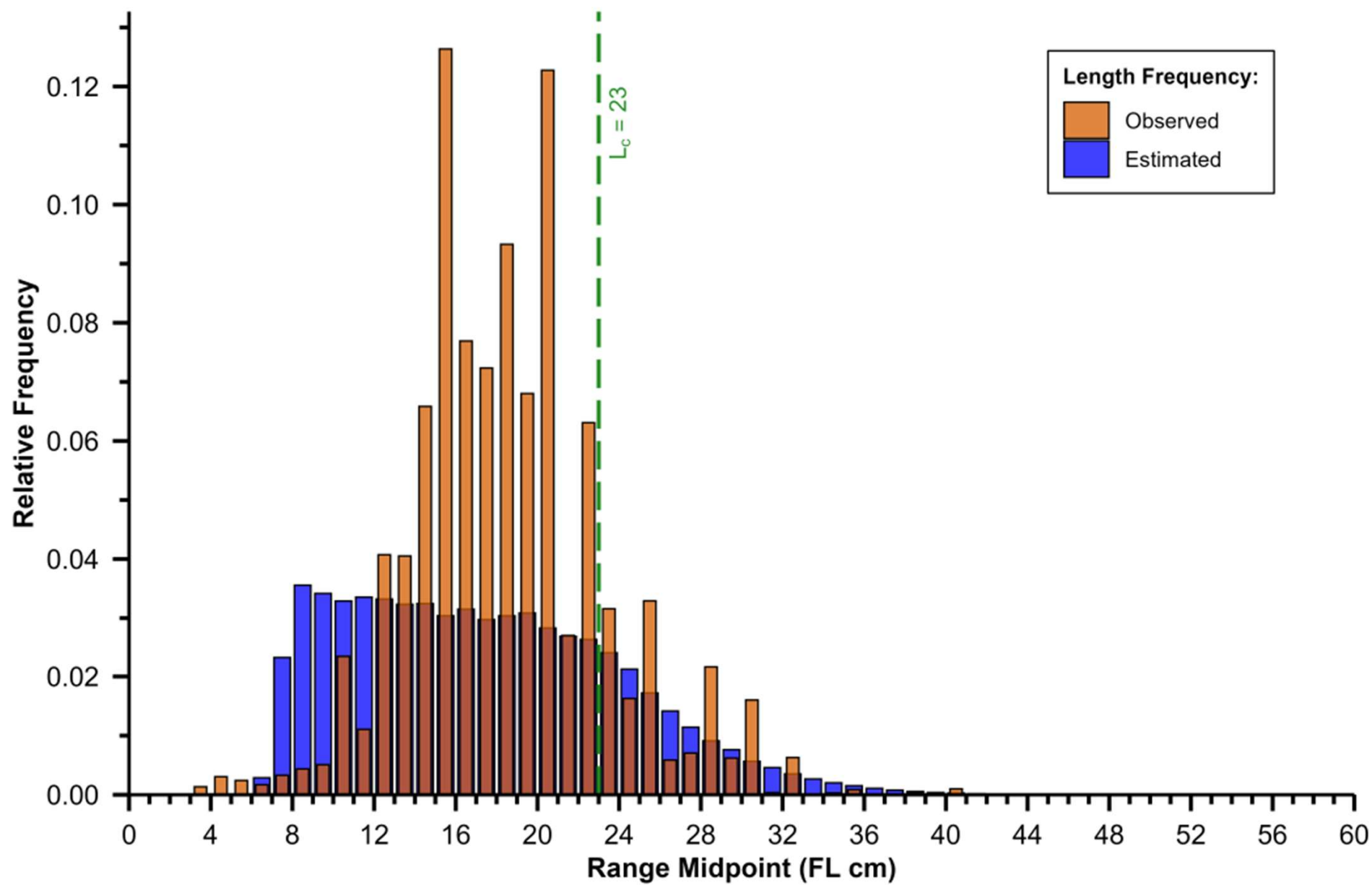


Yellowtail snapper

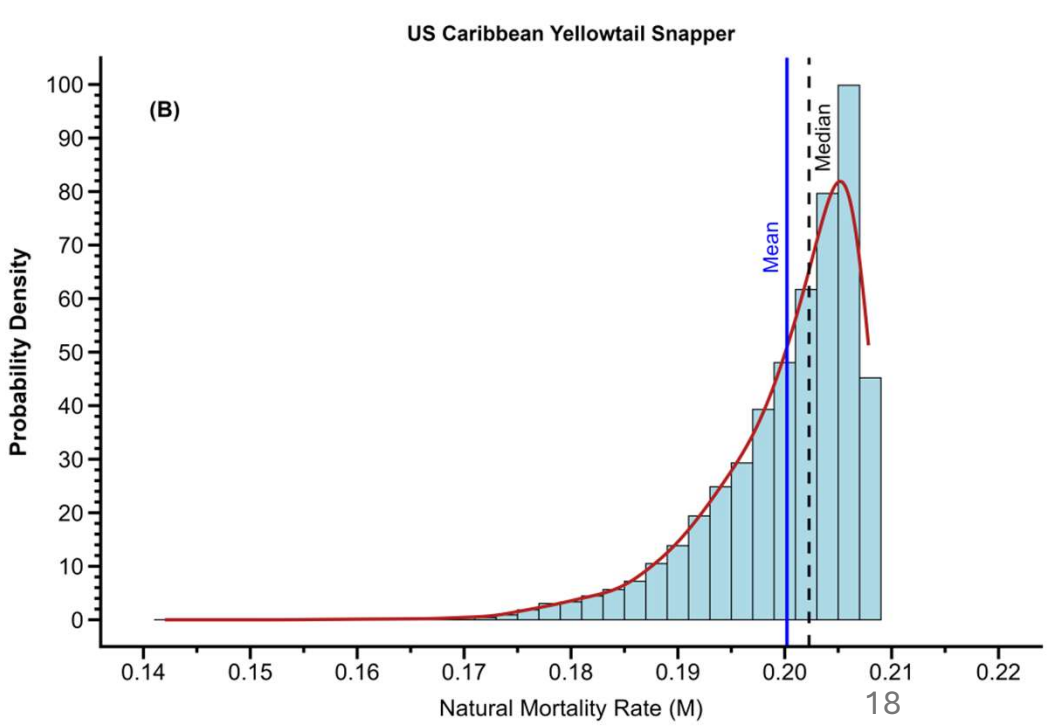
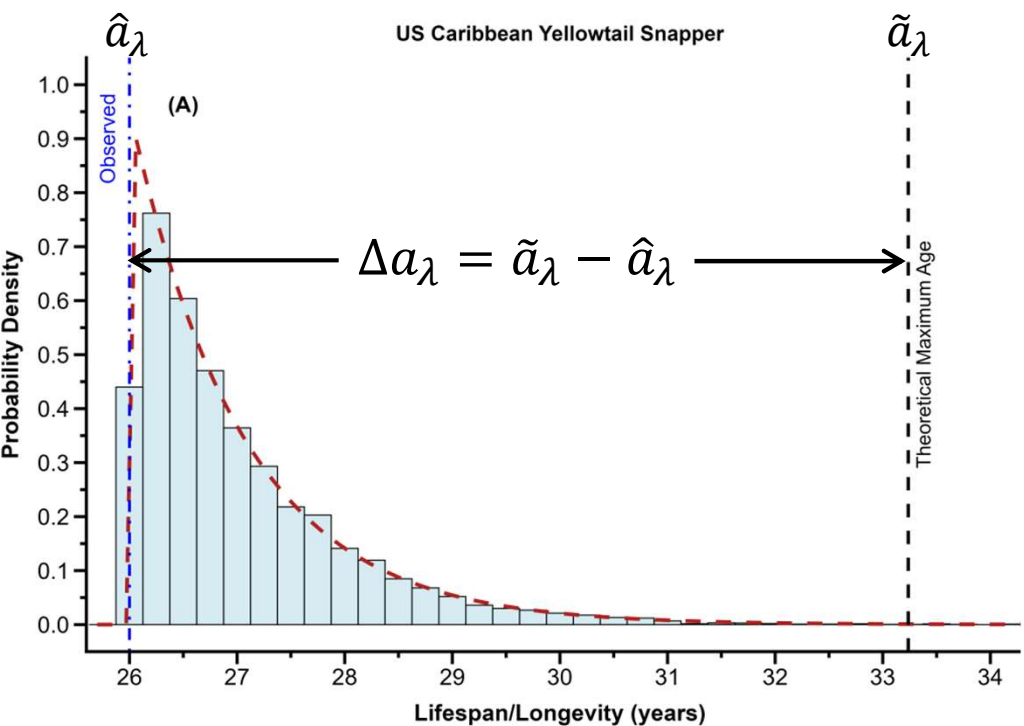


Caveats

- **Requires** representative sampling; reliable life-history parameters; and relatively stable recruitment.
- **Estimates** total mortality ($\hat{Z}$); it does **not** partition fishing mortality among fleets ($f_1 \cdots f_n$).
- **Assumes** the observed length distribution is not substantially biased by gear selectivity or habitat-specific sampling.



	$\hat{a}_\lambda$	M	$\tilde{a}_\lambda$	Δa_λ	$expo(\beta)$
Reef Fish	S_λ	0.0045	0.001		
Yellowtail snapper	26	0.20783	33.24	7.2	0.9545152320
Stoplight parrotfish	20	0.27018	22.03	2.0	3.4022604117
Queen triggerfish	23	0.23494	29.40	6.4	1.0790172187



Probabilistic Mortality Rates

Survivorship to Maximum Age

$$S(a_\lambda) = \frac{N(a_\lambda)}{N(a_c)} = e^{-M(a_\lambda - a_c)}$$

Natural Mortality Rate

$$\hat{M} = \frac{-\ln[S(\hat{a}_\lambda)]}{\hat{a}_\lambda}$$

Theoretical Maximum Age

$$\tilde{a}_\lambda = \frac{-\ln[S(\tilde{a}_\lambda)]}{\hat{M}}$$

Interval from observed to theoretical maximum age

$$\Delta a_\lambda = \tilde{a}_\lambda - \hat{a}_\lambda$$

Exponential Rate parameter

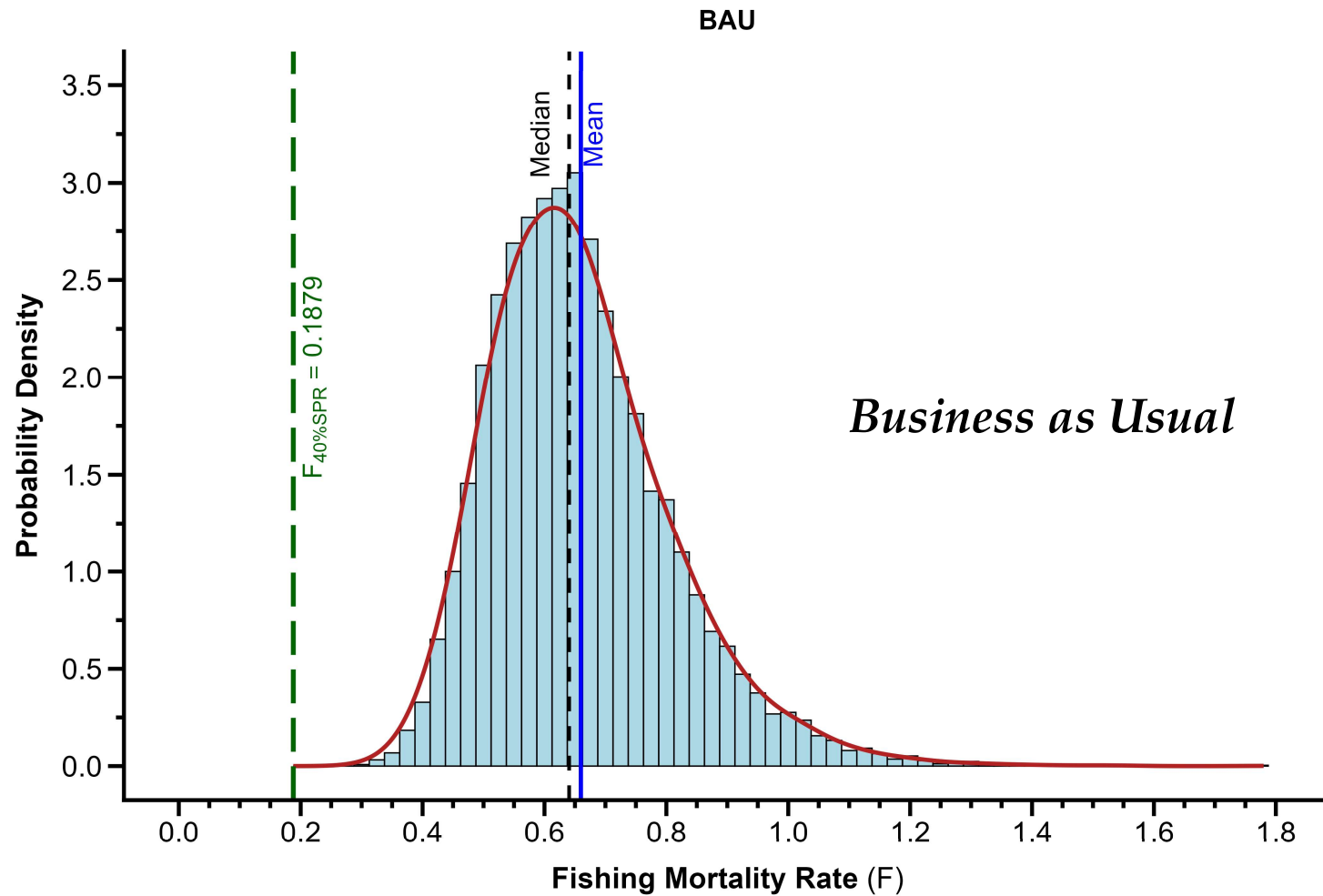
$$\frac{1}{\beta} = \frac{-\ln(0.0045)}{\Delta a_\lambda}$$

For each Monte Carlo iteration, a random draw of total mortality (Z) was paired with a corresponding realization of natural mortality (M), and fishing mortality was calculated

$$\hat{F} = \hat{Z} - \hat{M}$$

Ault et al. 2019

Yellowtail snapper



Length-based Exploited Population Dynamics

Exploitable population biomass

$$B_{EX}(L_c, t) = \int_{L_c}^{L_\lambda} N(L|a, t) W(L|a, t) \phi(L) dL$$

Average abundance (numbers)
at length at time t

$$\bar{N}(L, t) = \sum_{a_r}^{a_\lambda} p(L|a, t) \bar{N}(a, t)$$

Average population biomass

$$\bar{B}(t) = \int_t^{t+\Delta t} \int_{a_r}^{a_\lambda} \bar{N}(a, t) \bar{W}(a, t) da dt$$

Yield in weight at time t

$$Y_w(F, L_c, t) = F(t) B_{EX}(L_c, t)$$

Spawning (mature) stock biomass at time t

$$SSB(t) = \int_{L_m}^{L_\lambda} B(L|a, t) dL$$

Spawning potential ratio

$$SPR = SSB_{\hat{F}} / SSB_{F=0}$$

Fishing mortality rate at 40% SPR

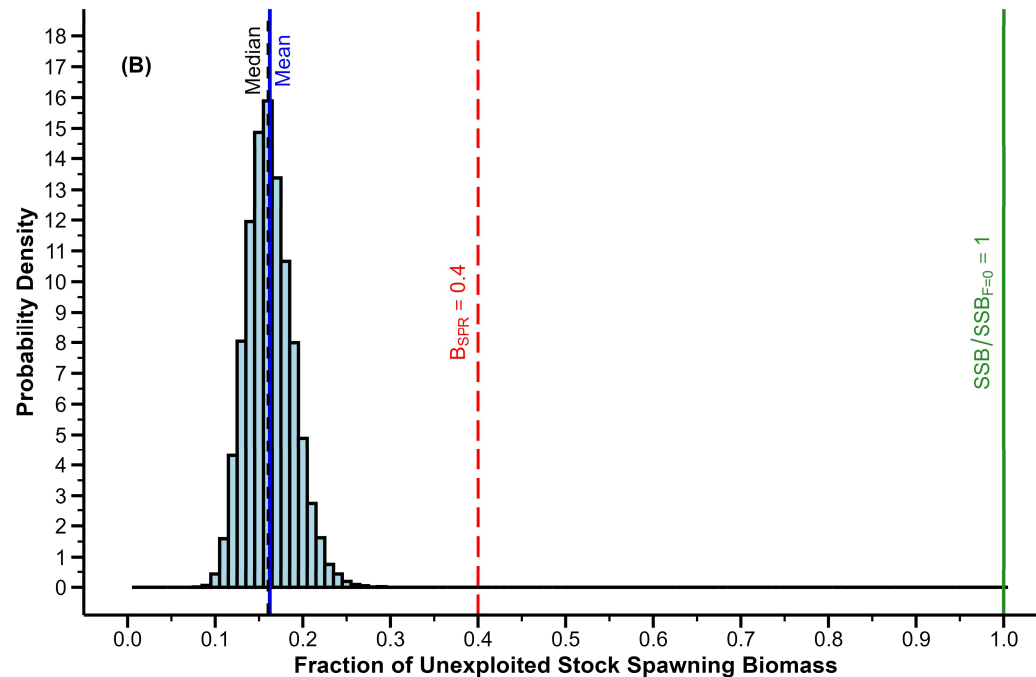
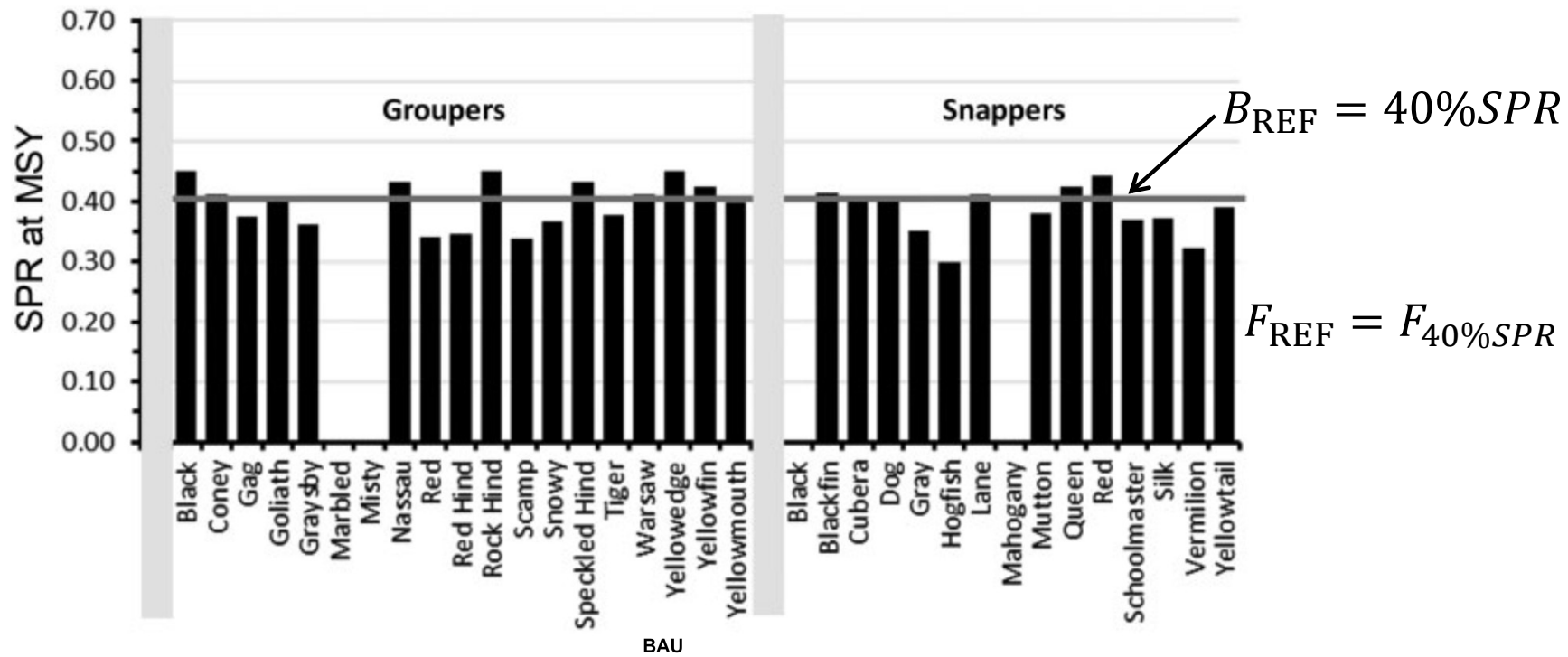
$$F_{REF}$$

Establishing sustainability risk reference points

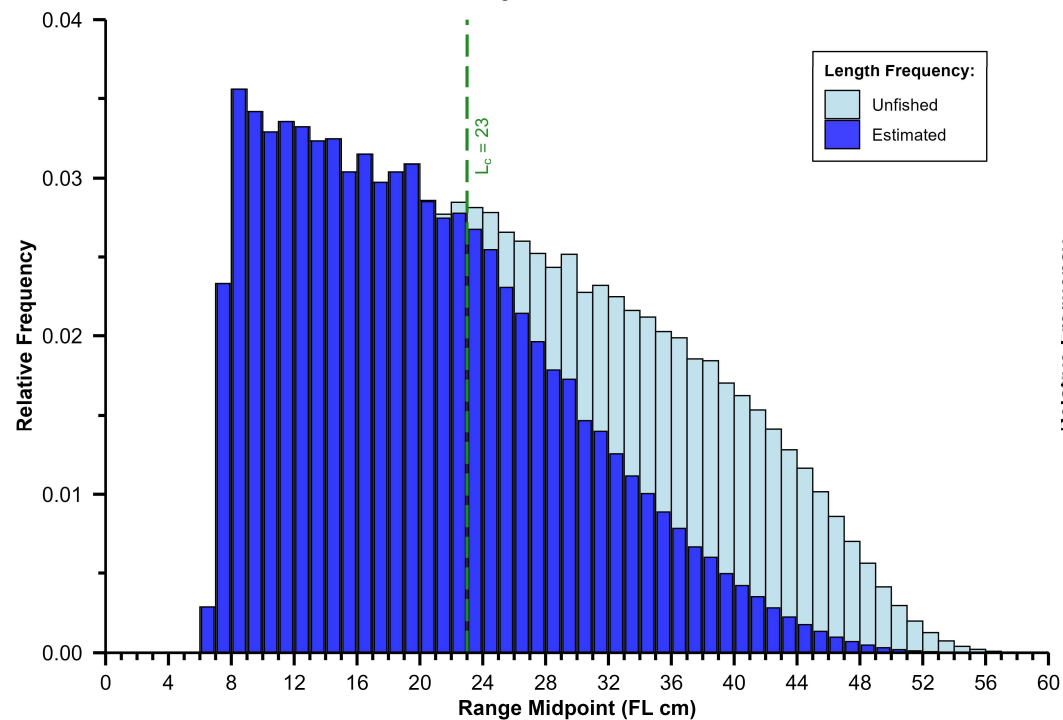
$$F_{MSY} = M$$

$$L_c = L_m$$

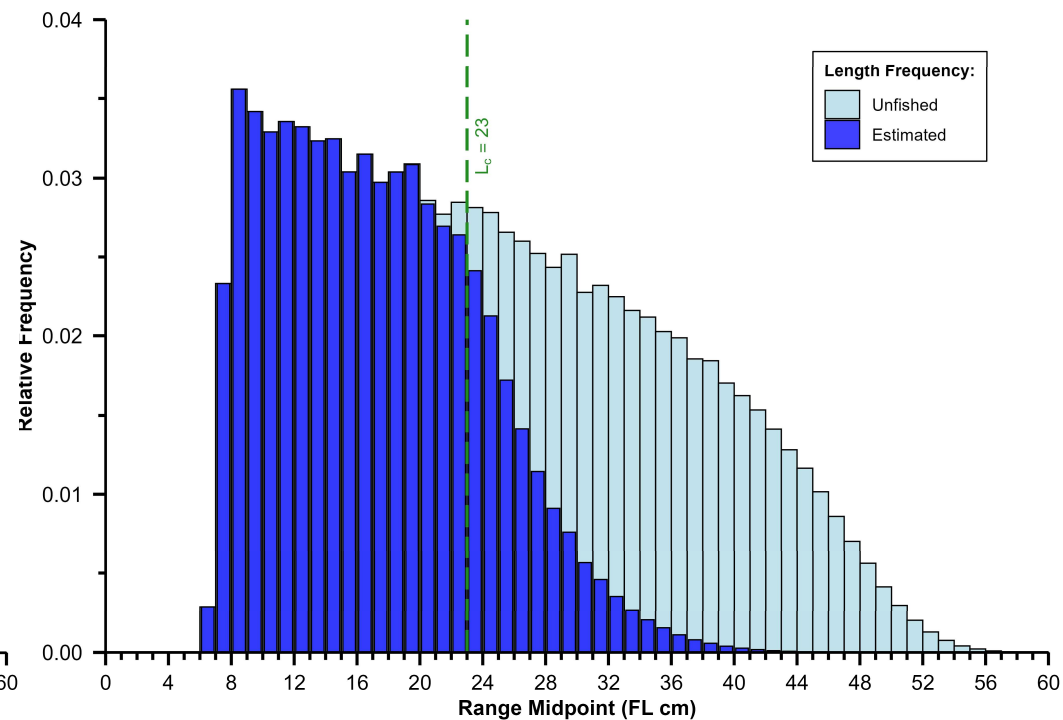
Ault et al. 2019

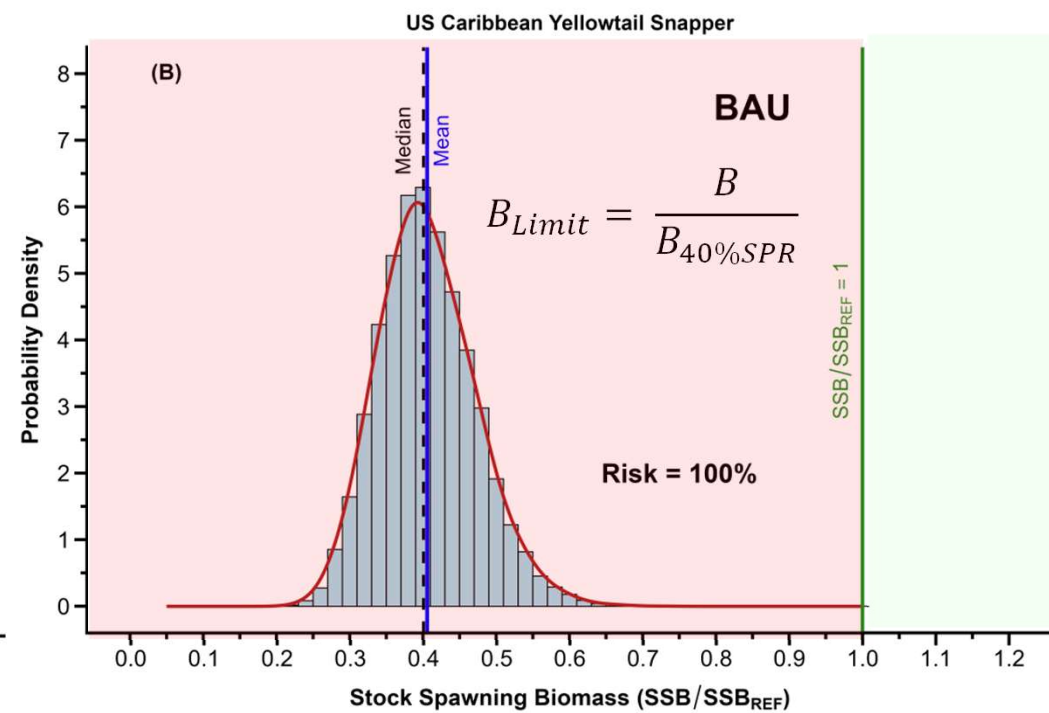
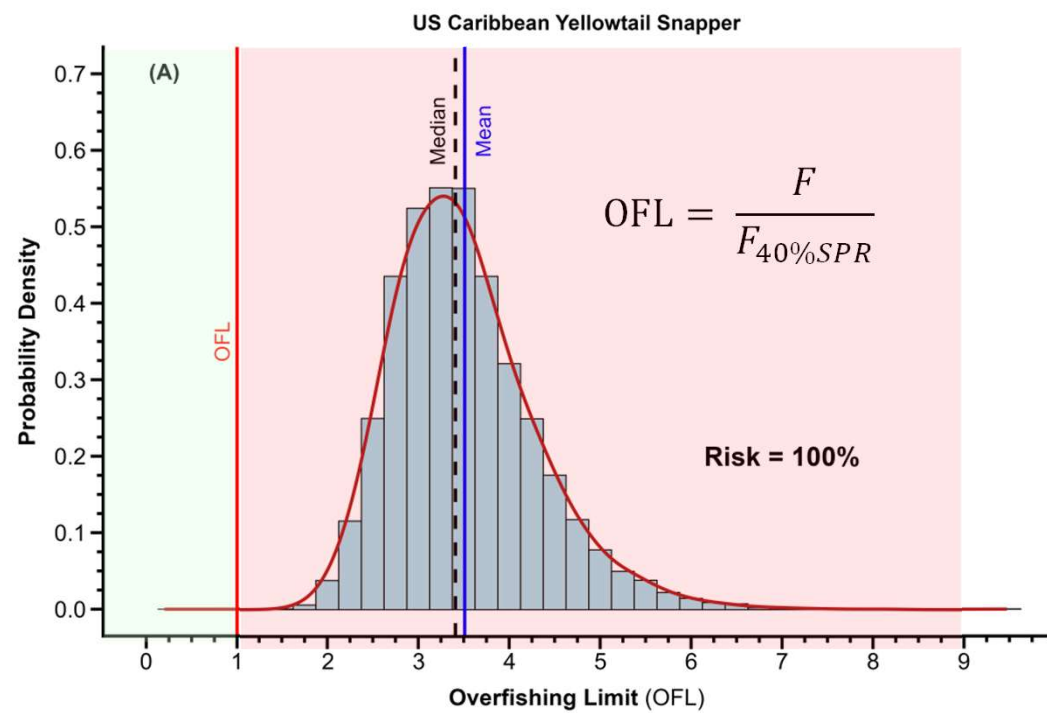


$F_{\text{Limit}} = 40\% \text{ SPR}$

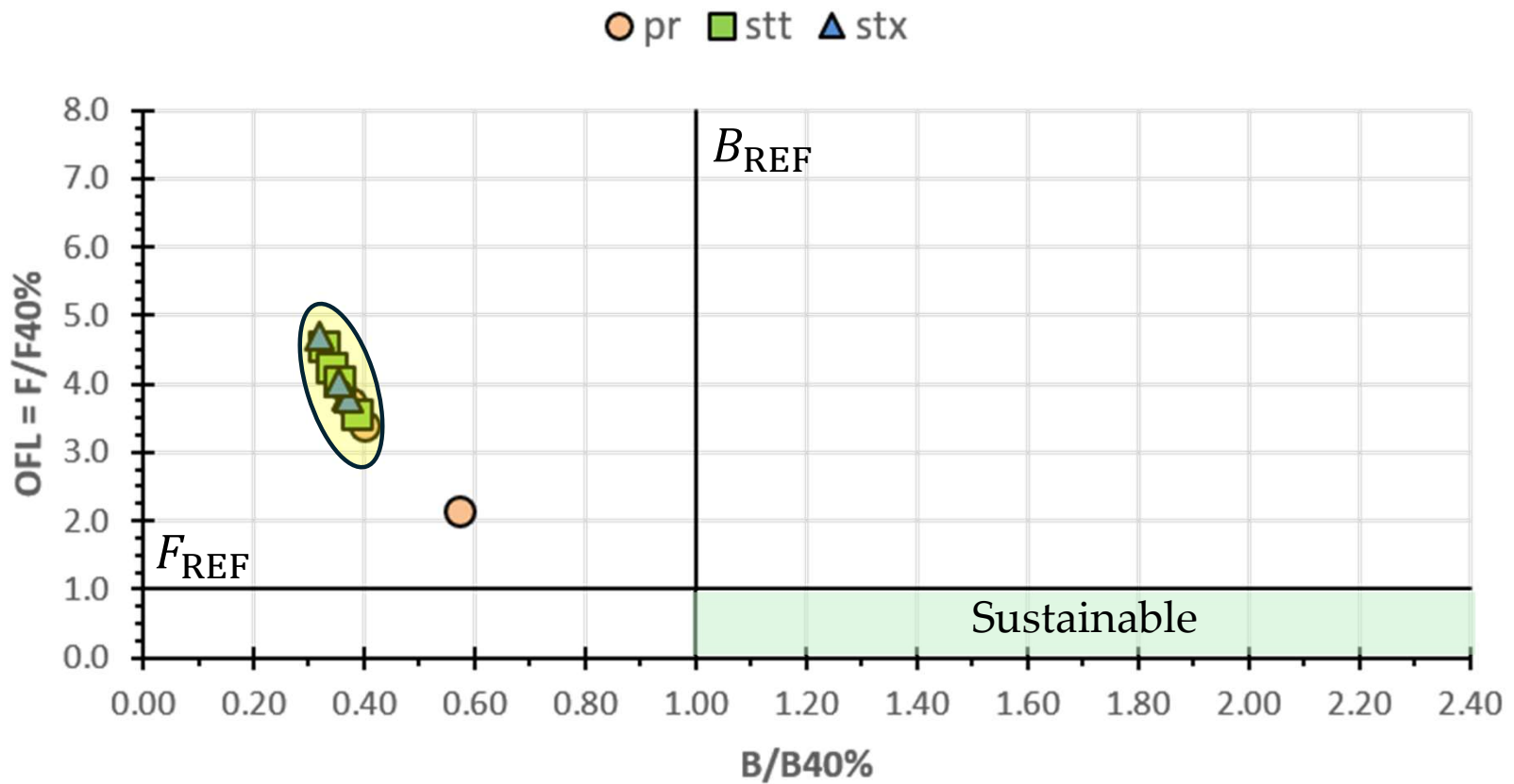


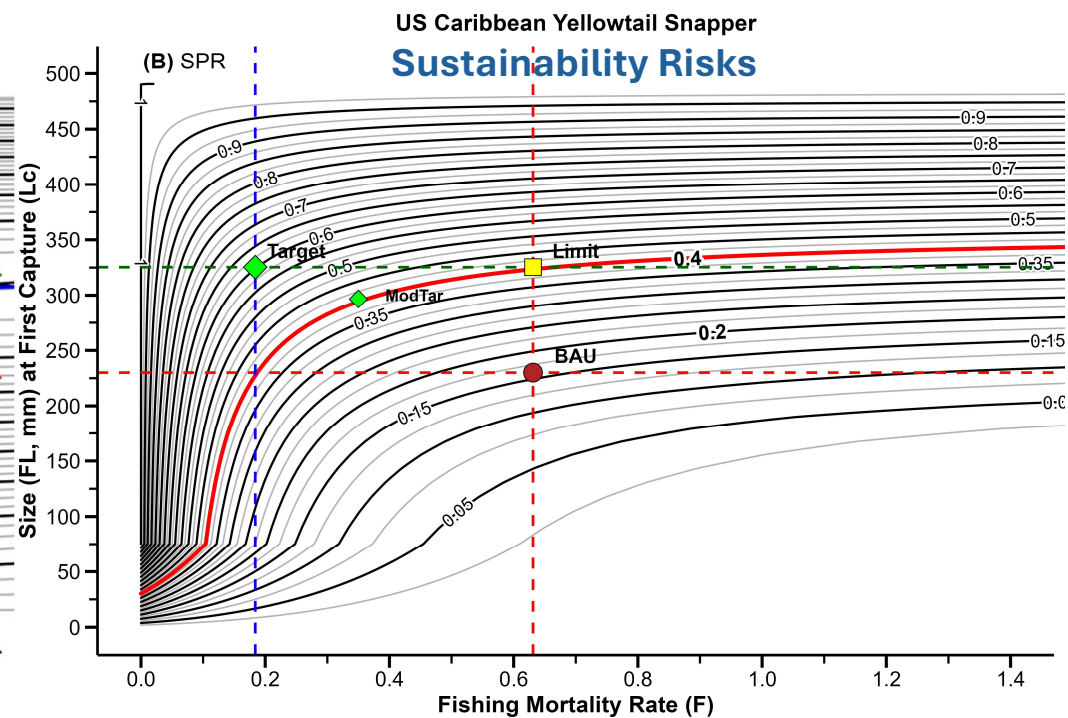
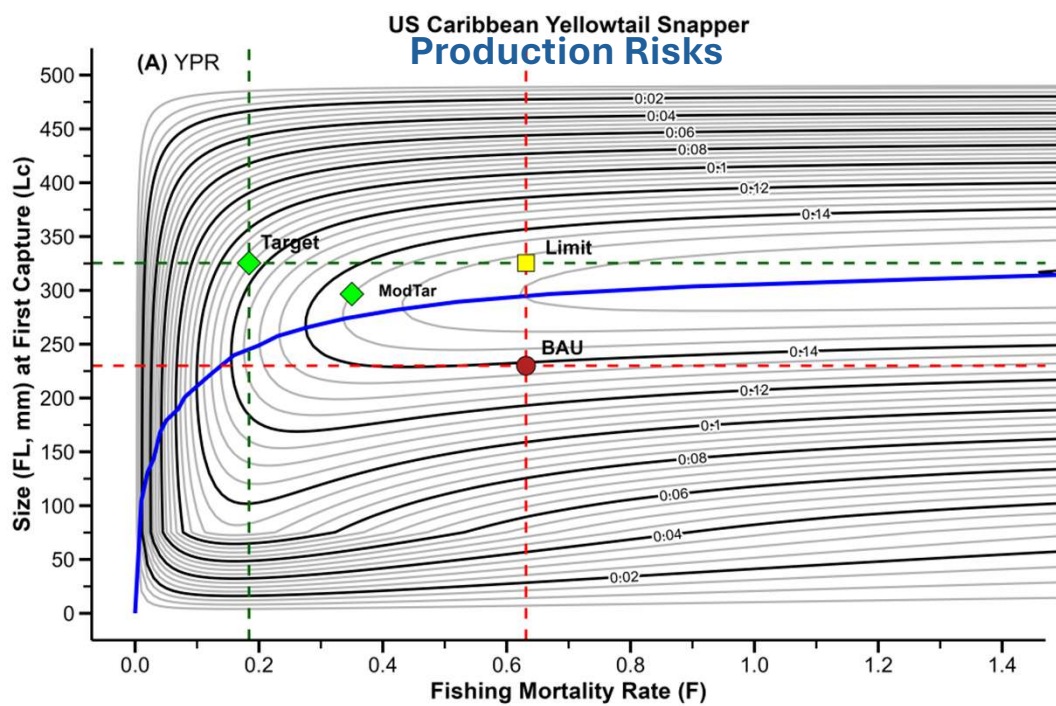
$F = 0.6312: \text{SPR} = 16.21\%$





Yellowtail Snapper: Caribbean Regions

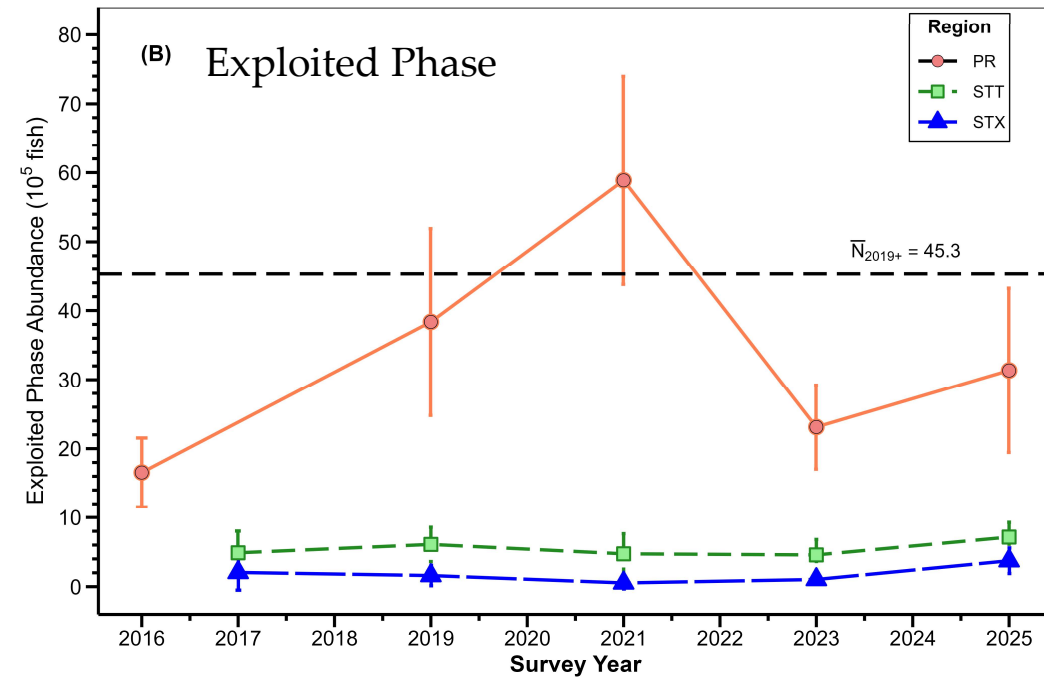
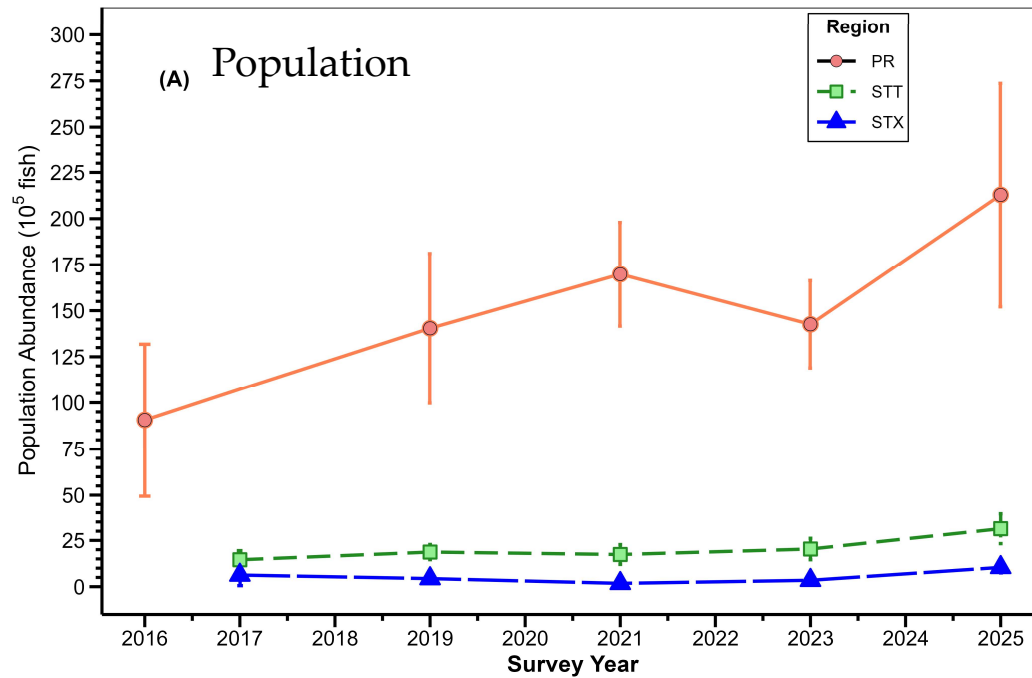




Population Abundance

US Caribbean Yellowtail Snapper (RVC)

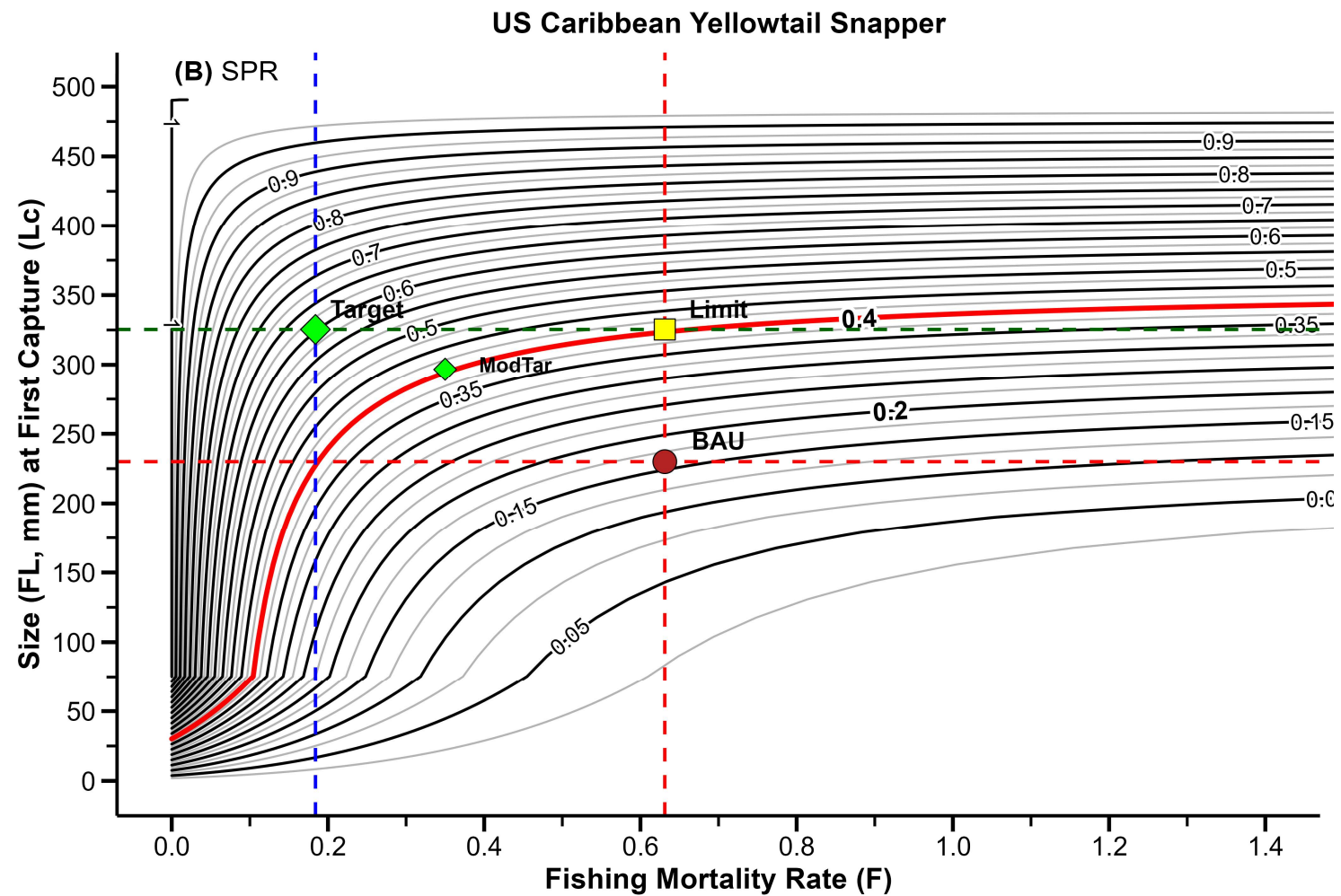
US Caribbean Yellowtail Snapper (RVC)



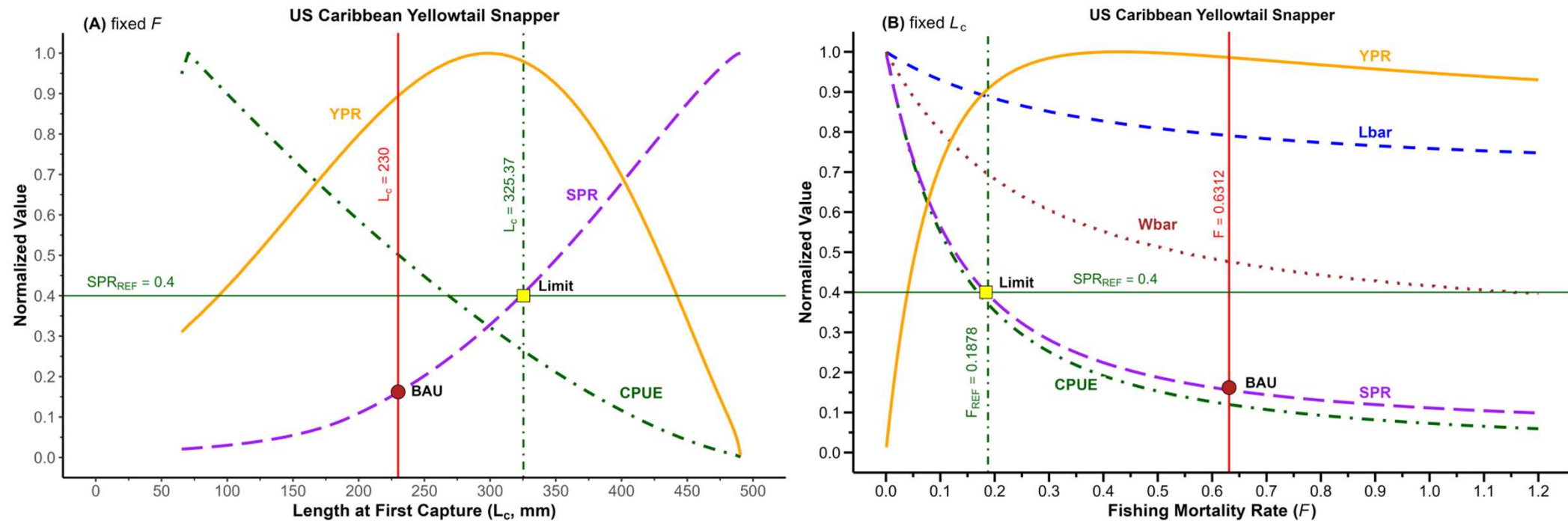
Modulation of Interpretation

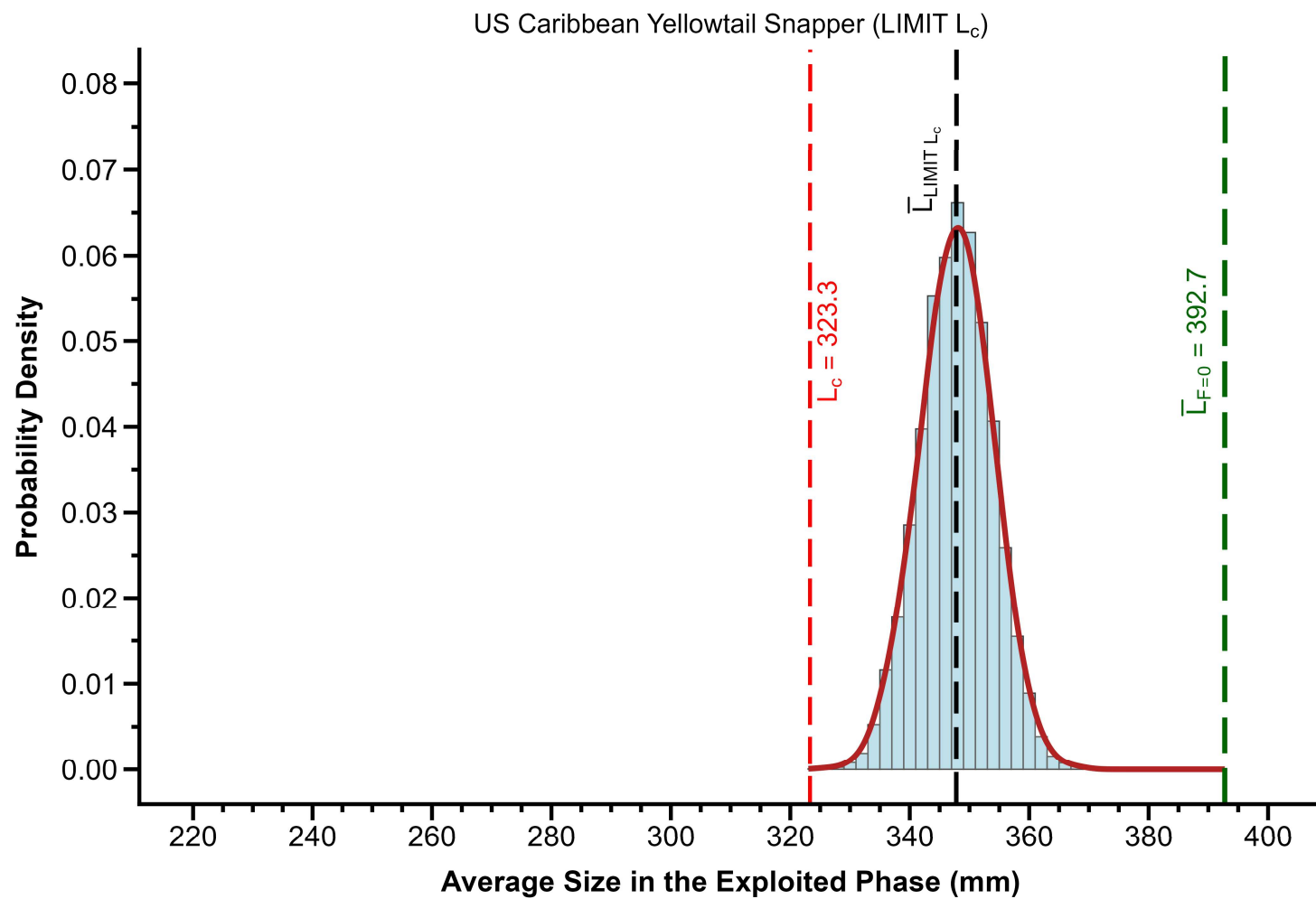
Yellowtail Snapper: Z = 0.8391											
S_λ	M	R	B_0	F	SPR	$\bar{L}$	$\bar{L}_{F=0}$	$\bar{N}_{EX}$	$\bar{B}_{EX}$	Y_N	Y_w
0.0045	0.2078	276,609	547,896	0.6312	0.1621	267.9	337.2	167,779	63,061	105,902	39,804
0.01	0.1771	276,609	754,240	0.6619	0.1296	267.9	346.5	185,393	69,681	122,711	46,122
0.05	0.1152	276,609	1,567,379	0.7238	0.0757	267.9	369.1	226,707	85,210	164,090	61,675

Queen Triggerfish: Z = 0.3374											
S_λ	M	R	B_0	F	SPR	$\bar{L}$	$\bar{L}_{F=0}$	$\bar{N}_{EX}$	$\bar{B}_{EX}$	Y_N	Y_w
0.0045	0.2349	793,000	1,241,907	0.1025	0.5906	303.7	324.6	814,843	688,316	83,521	70,552
0.01	0.2002	793,000	1,812,929	0.1372	0.4727	303.7	333.9	952,679	804,773	130,708	110,415
0.05	0.1302	793,000	4,219,254	0.2071	0.2781	303.7	356.4	1,103,010	1,305,561	270,302	228,433

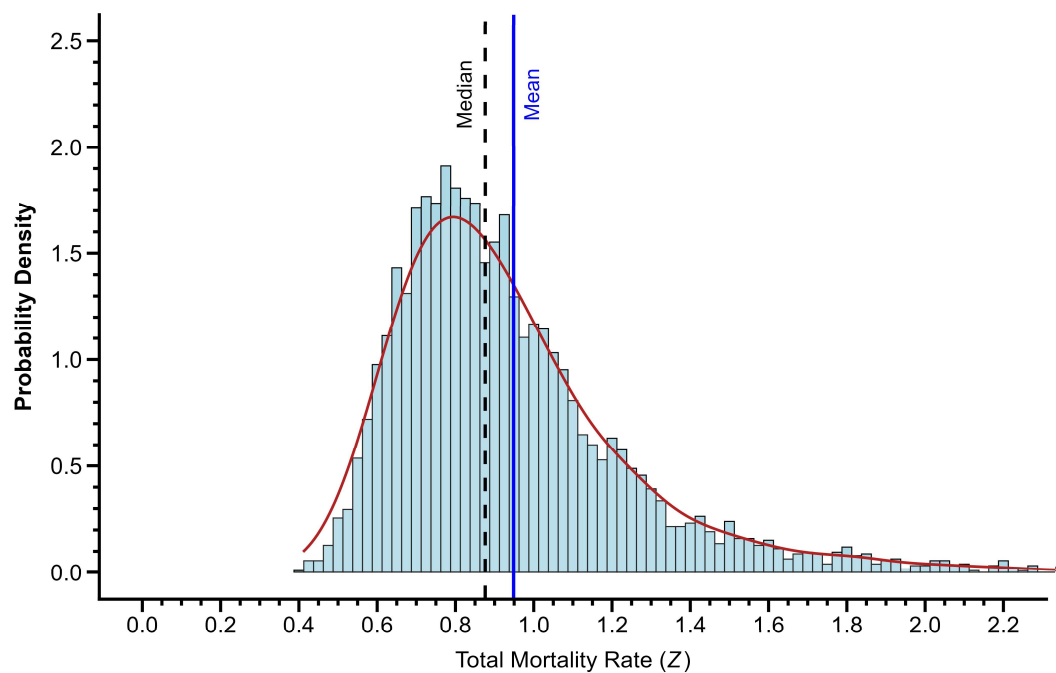


BAU transition to Limit Strategy

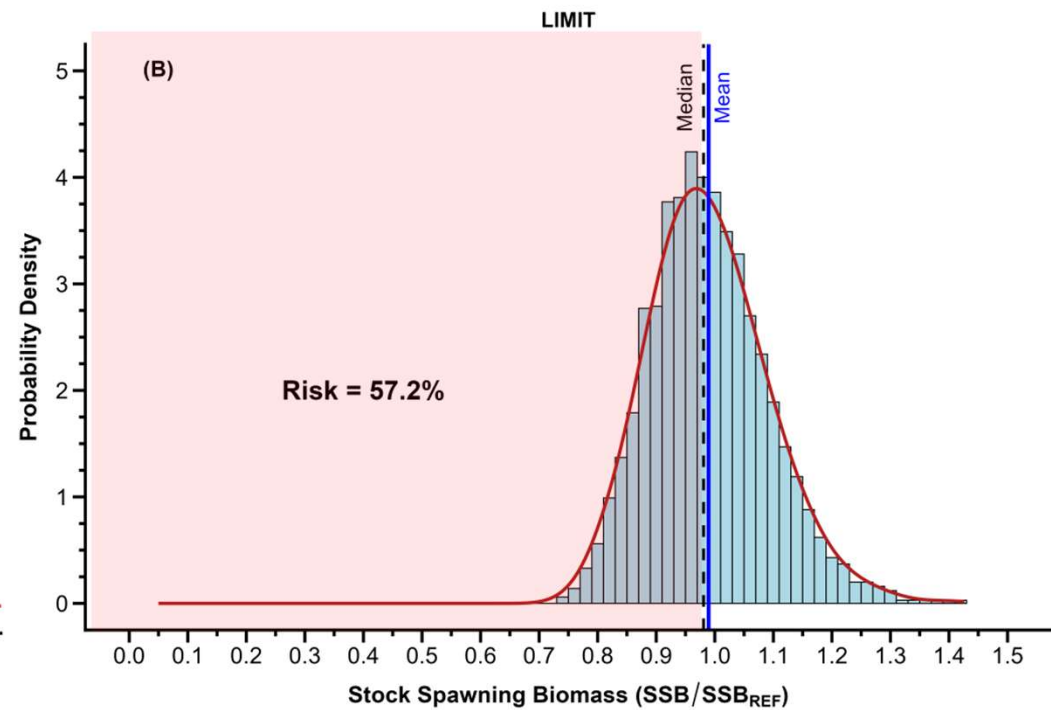
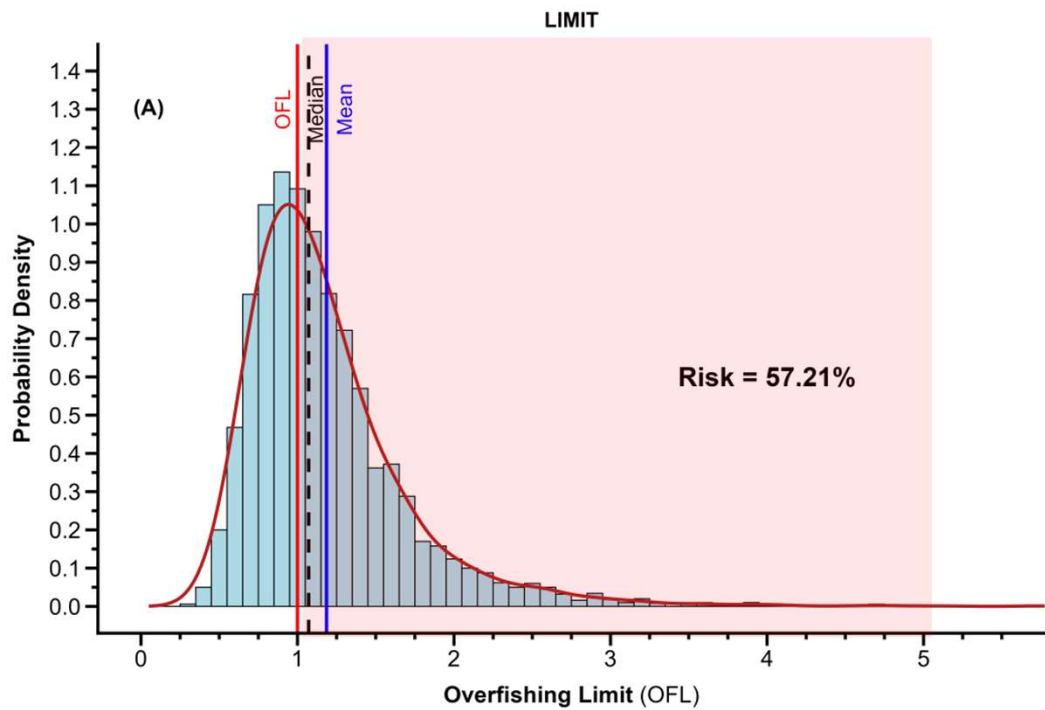
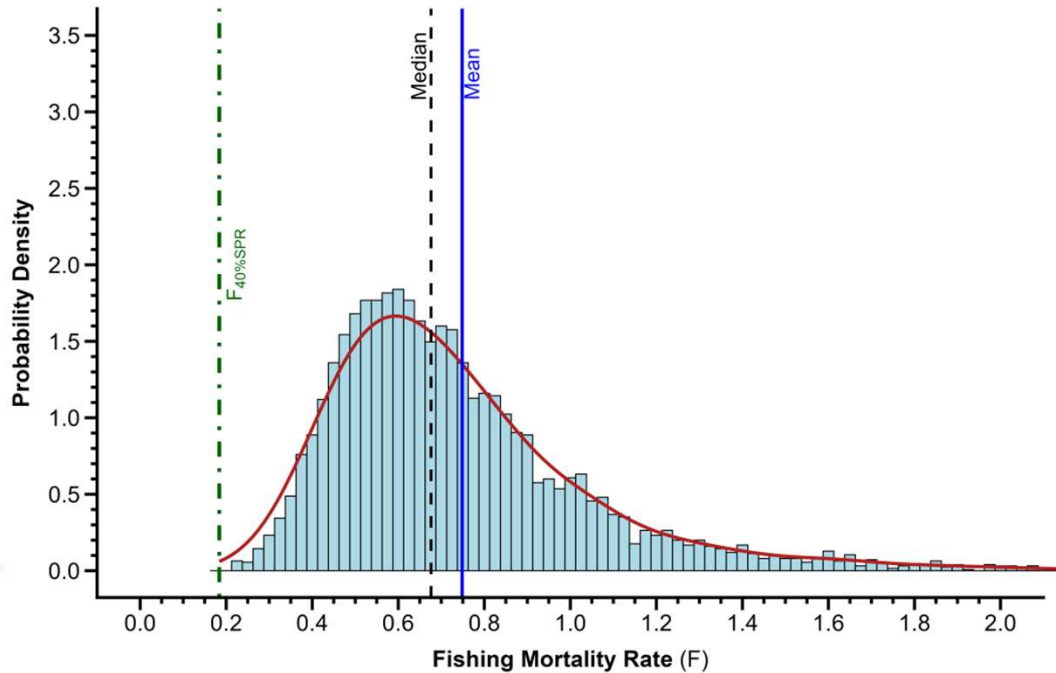




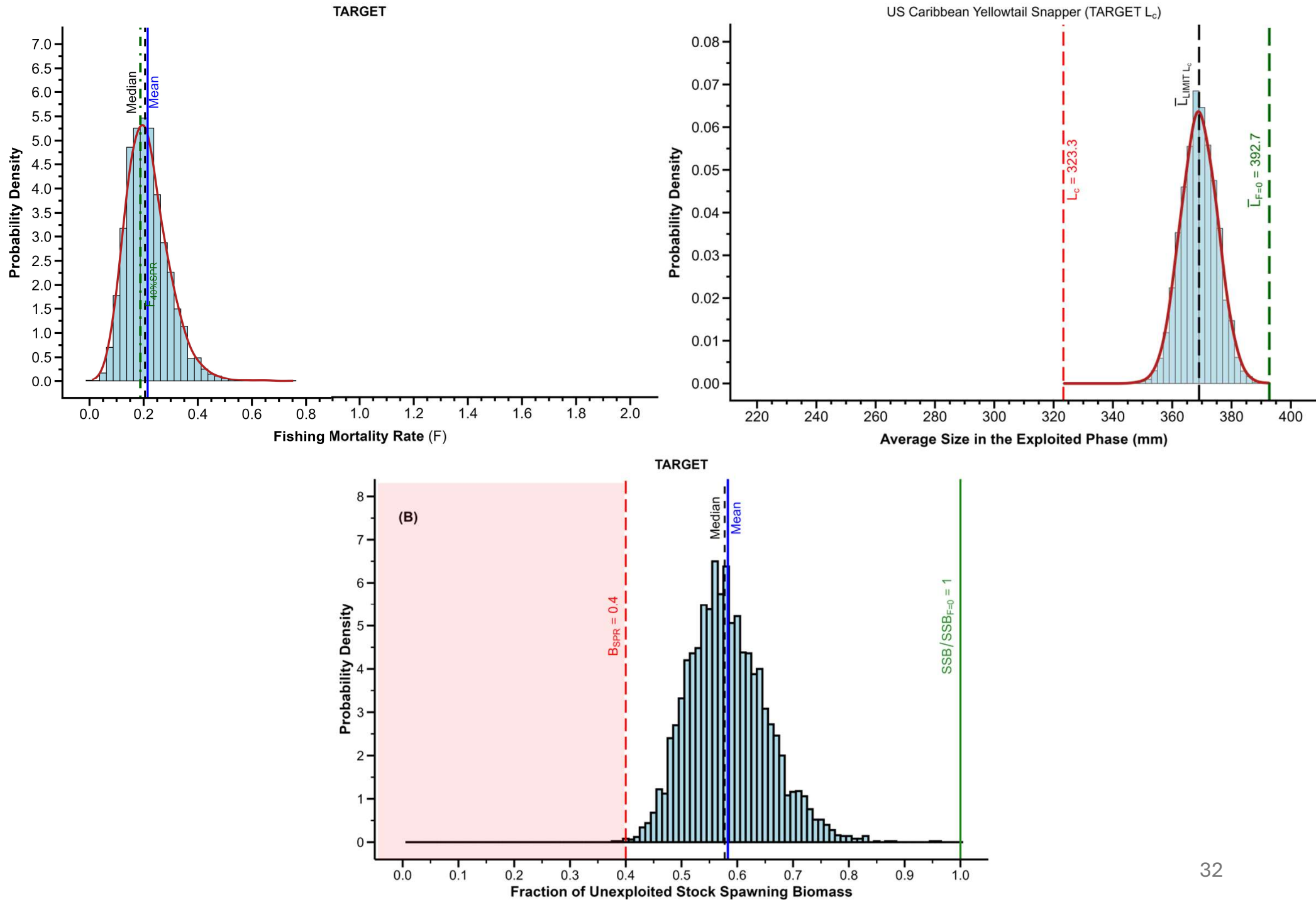
US Caribbean Yellowtail Snapper (LIMIT L_c)



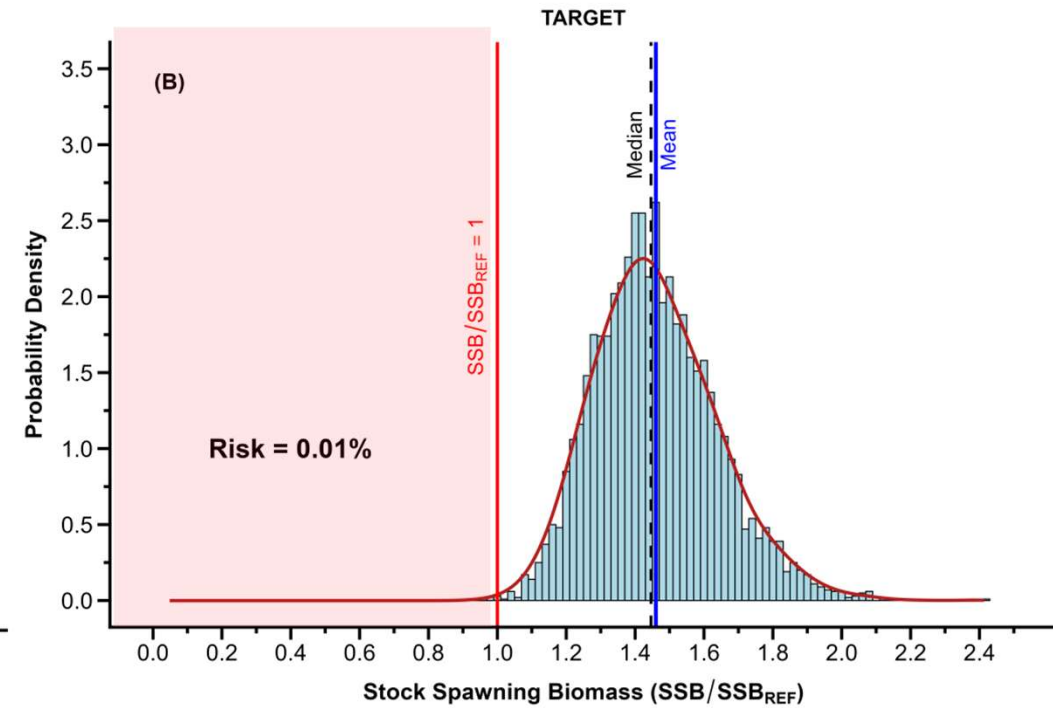
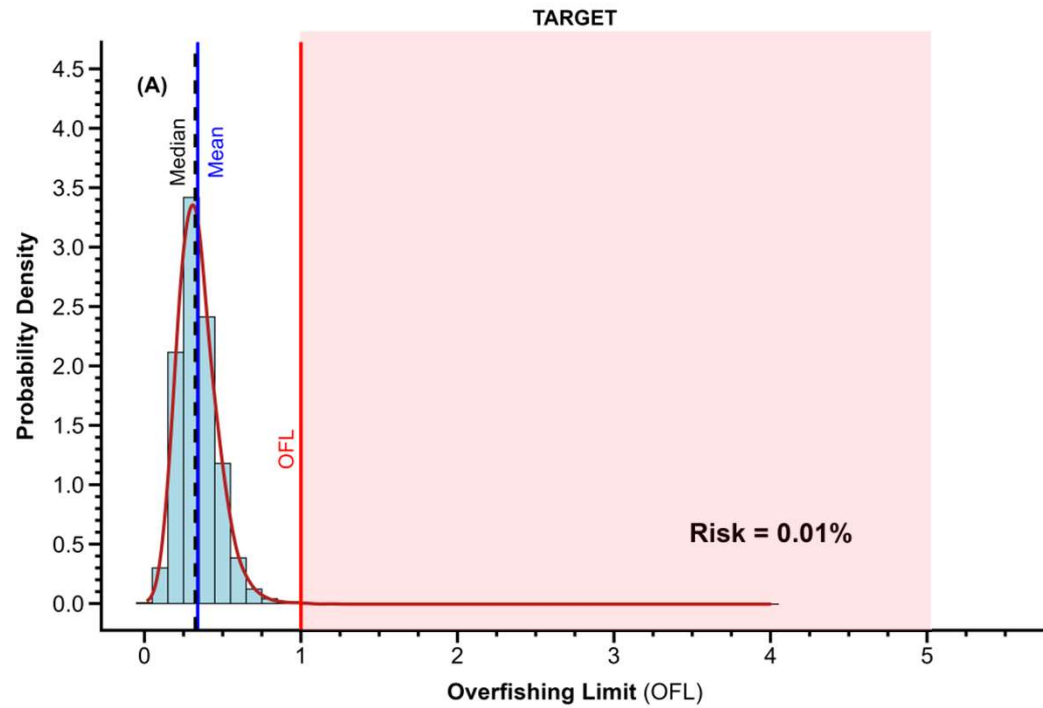
US Caribbean Yellowtail Snapper



Limit transition to Target strategy

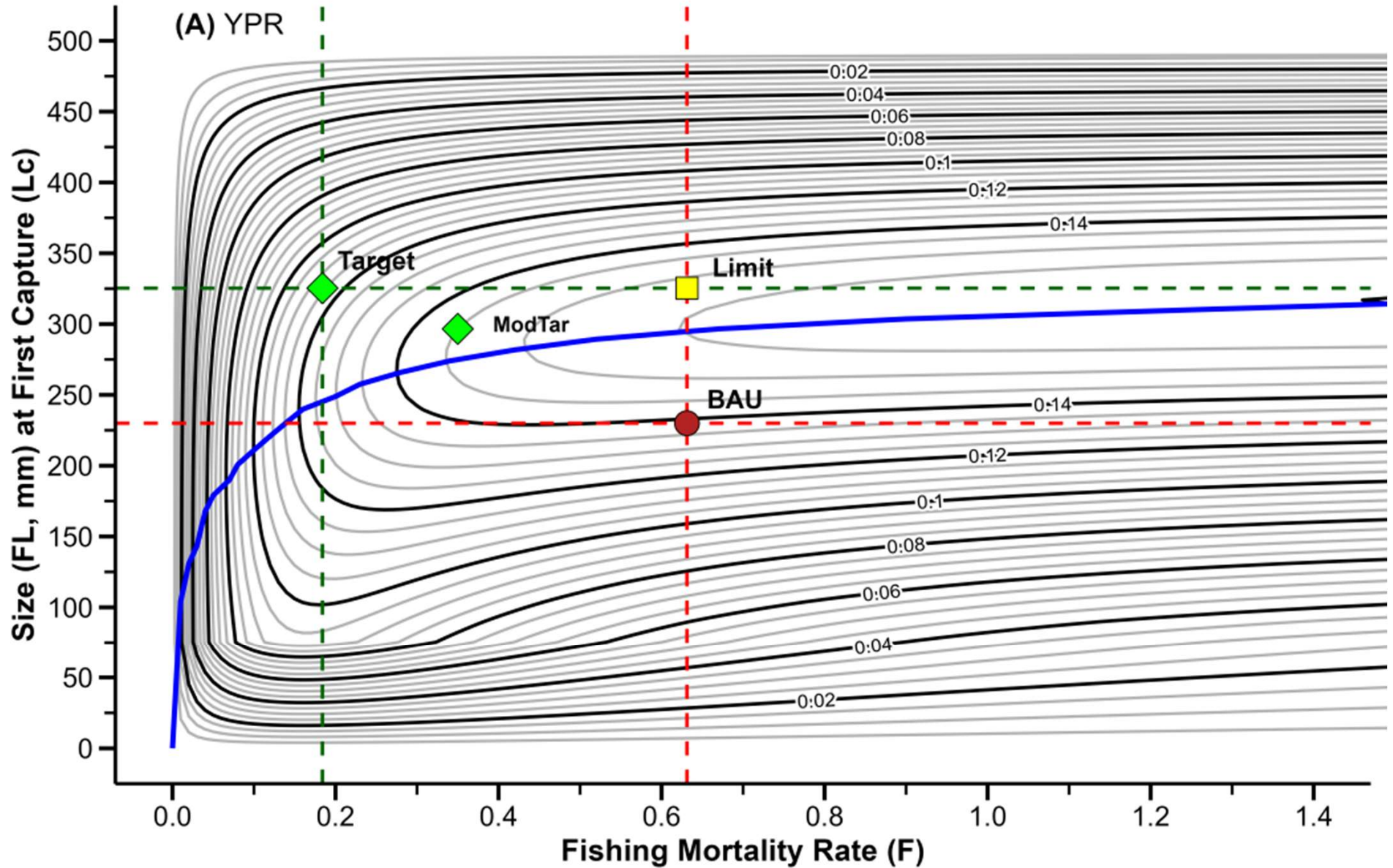


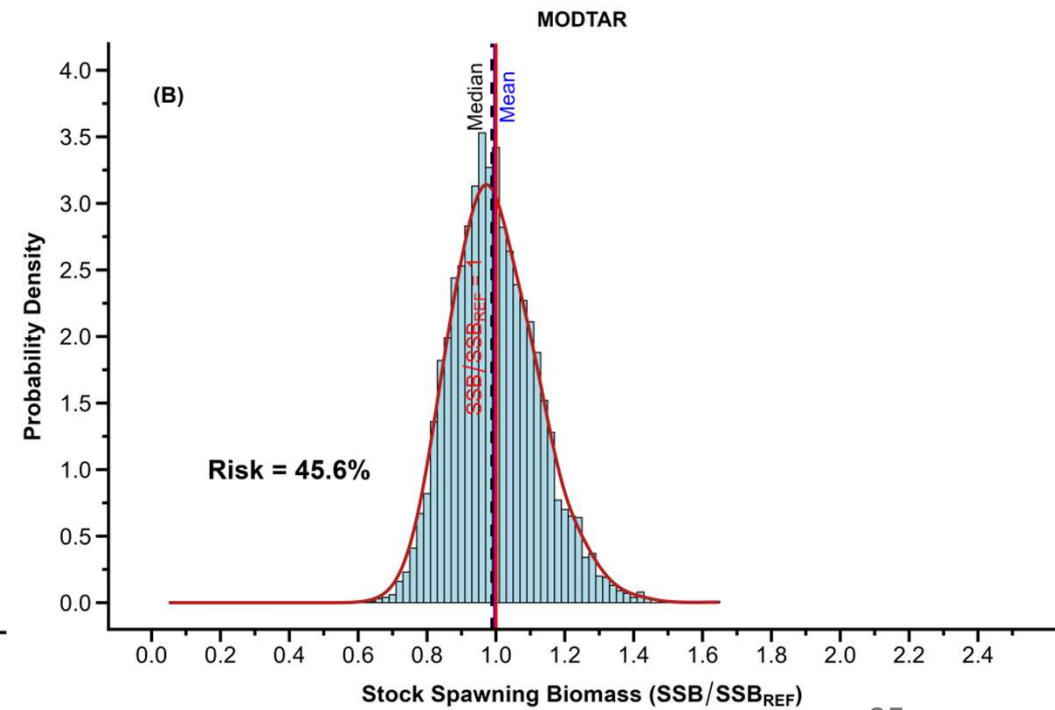
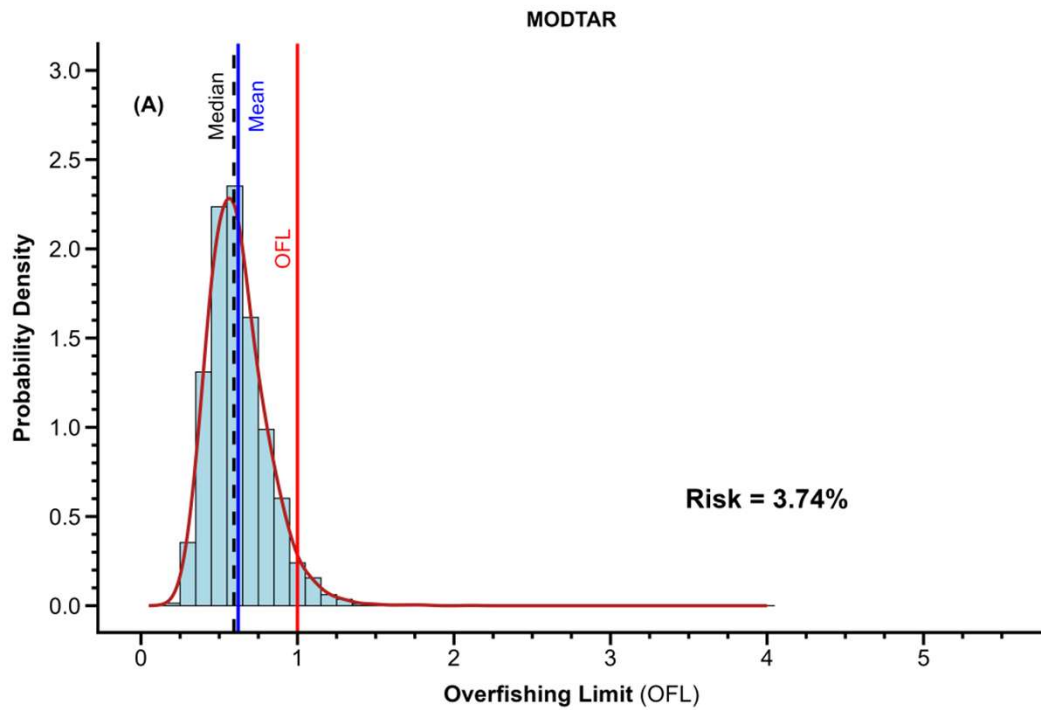
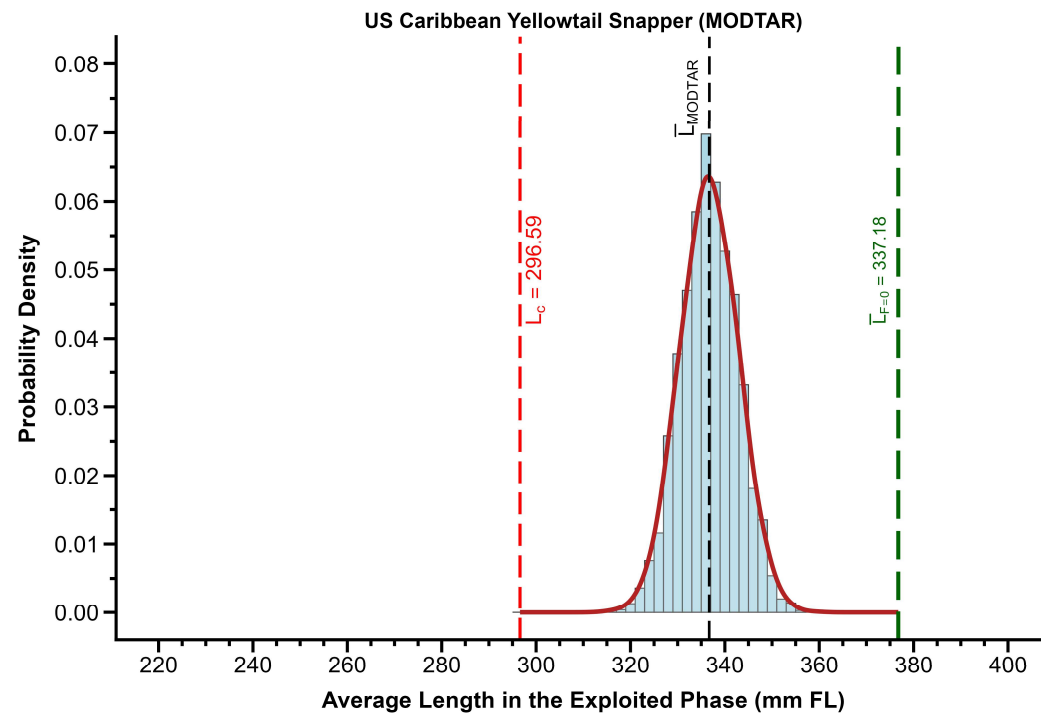
Risk Profile for Target Strategy



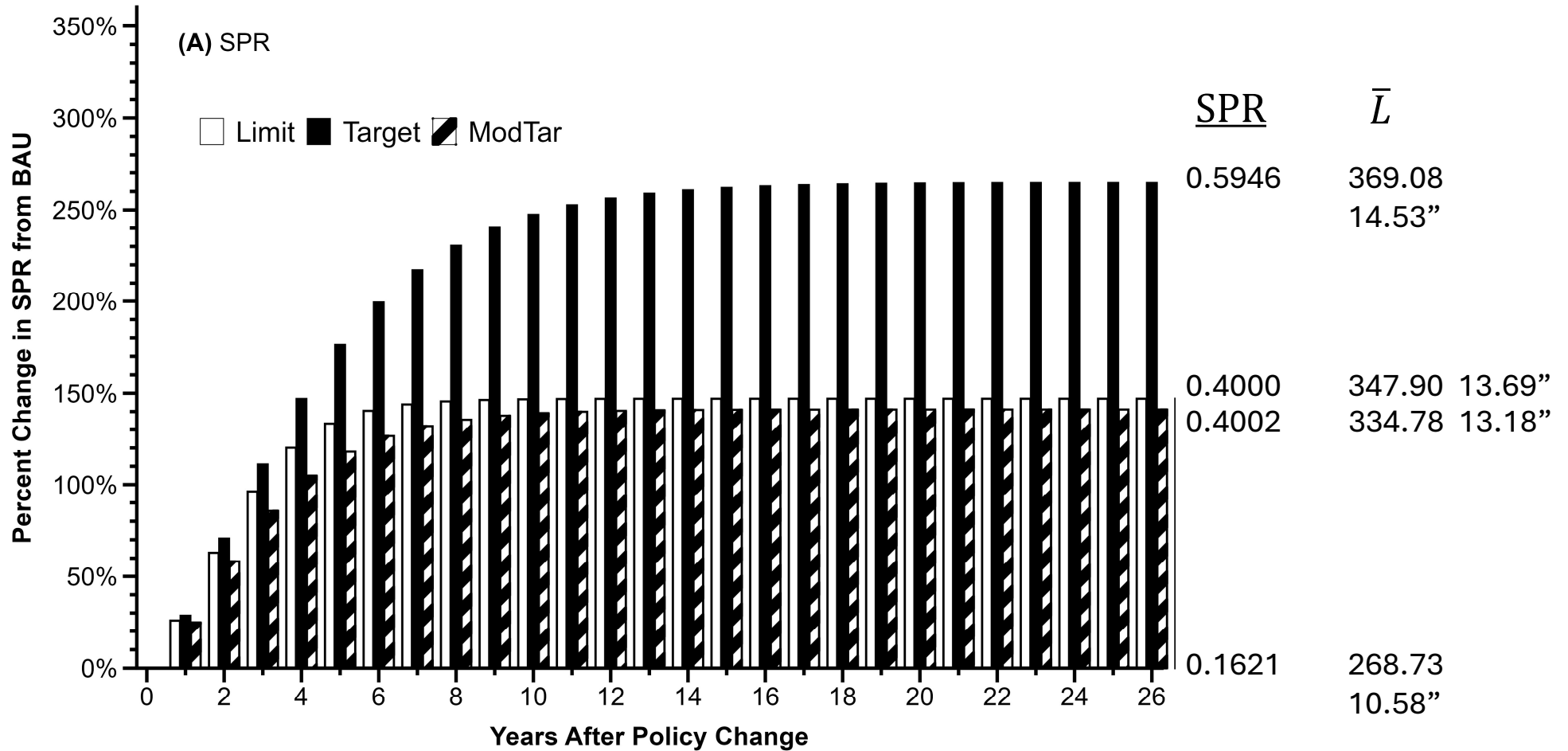
MODTAR

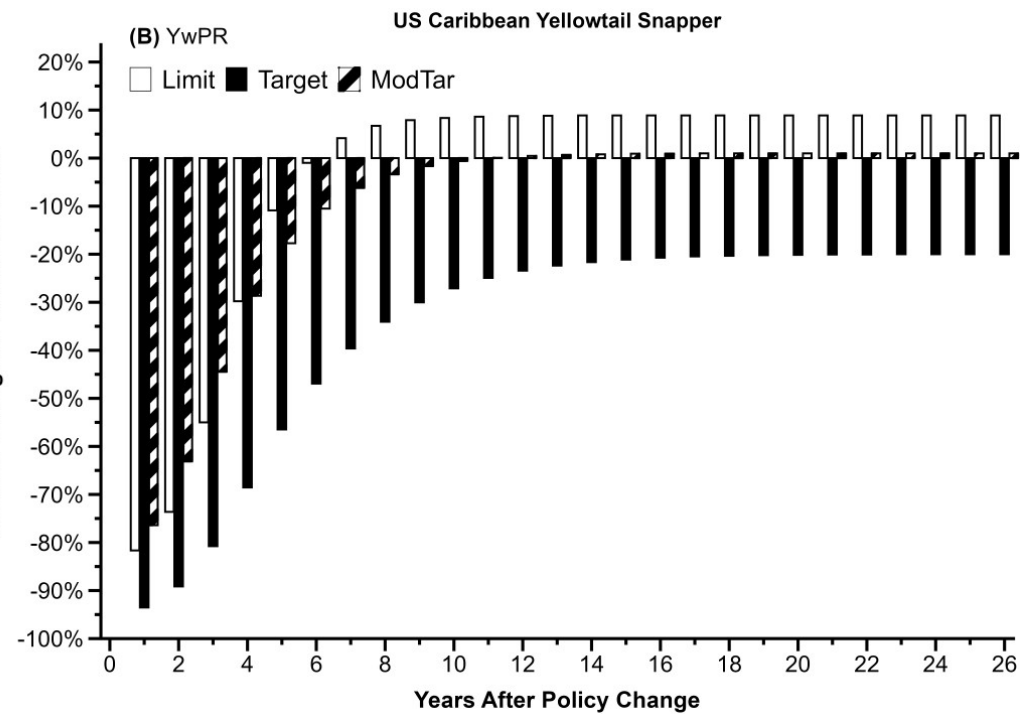
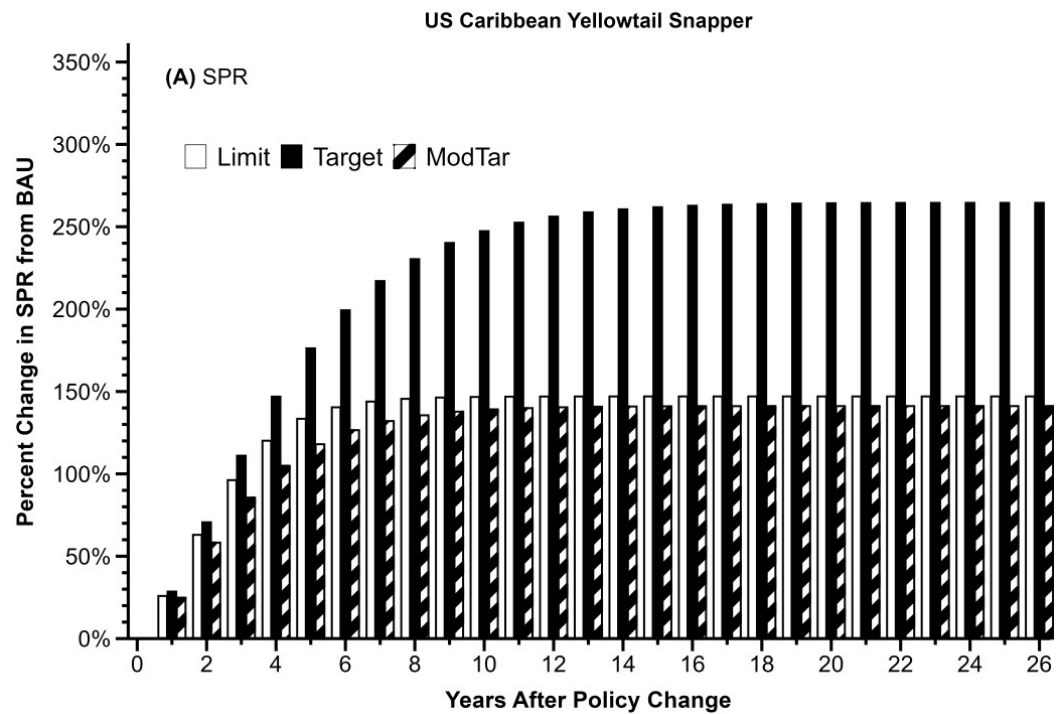
US Caribbean Yellowtail Snapper

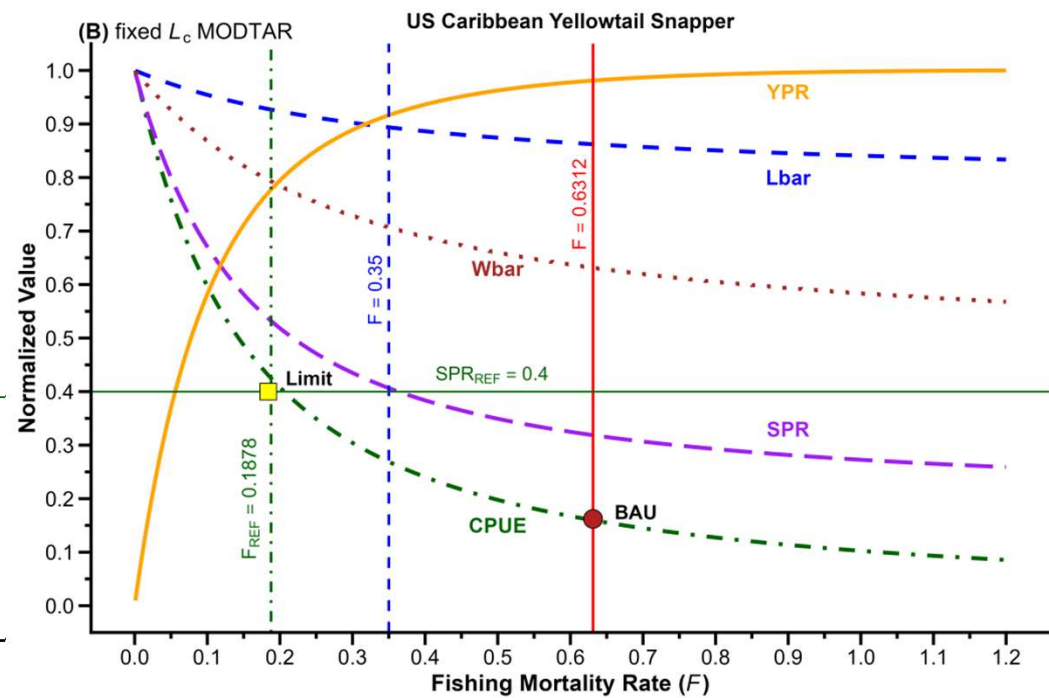
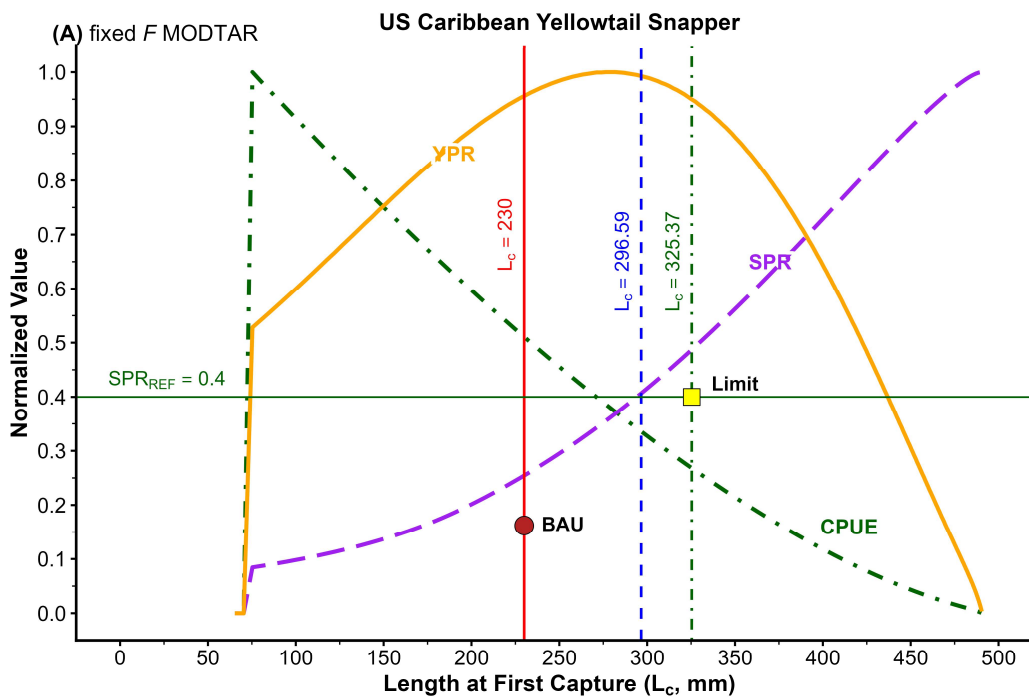




US Caribbean Yellowtail Snapper







Fishery Decision Analysis with Multiple Objectives

A multiattribute utility function (**MUF**) is a formal mathematical representation of a Decision Maker's (DM's) preference structure:

$$\mu(\mathbf{x}) = f[\mu_1(x_1), \mu_2(x_2), \dots, \mu_n(x_n)]$$

$\mu_i(x_i) \equiv i$ -th objective function evaluated over attribute x

$x_i^* \equiv$ most preferred outcome of x_i , where $\mu_i(x_i^*) = 1$

$x_i^0 \equiv$ least preferred outcome of x_i , where $\mu_i(x_i^0) = 0$

Ault & Fox. 1989

$\mu_1(x_1) \equiv$ **Conservation** (maximum population size)

$\mu_2(x_2) \equiv$ **Commercial** (maximum profits and minimum costs)

$\mu_3(x_3) \equiv$ **Recreational** (most fish per trip, biggest fish)

$\mu_4(x_4) \equiv$ **Cultural** (it's God's fish)

$\mu_5(x_5) \equiv$ **Regulatory** (# of management measures), etc.

The MUF is structured:

$$1 + K\mu(x) = \prod_{i=1}^s [Kk_i\mu_i(x_i)]$$

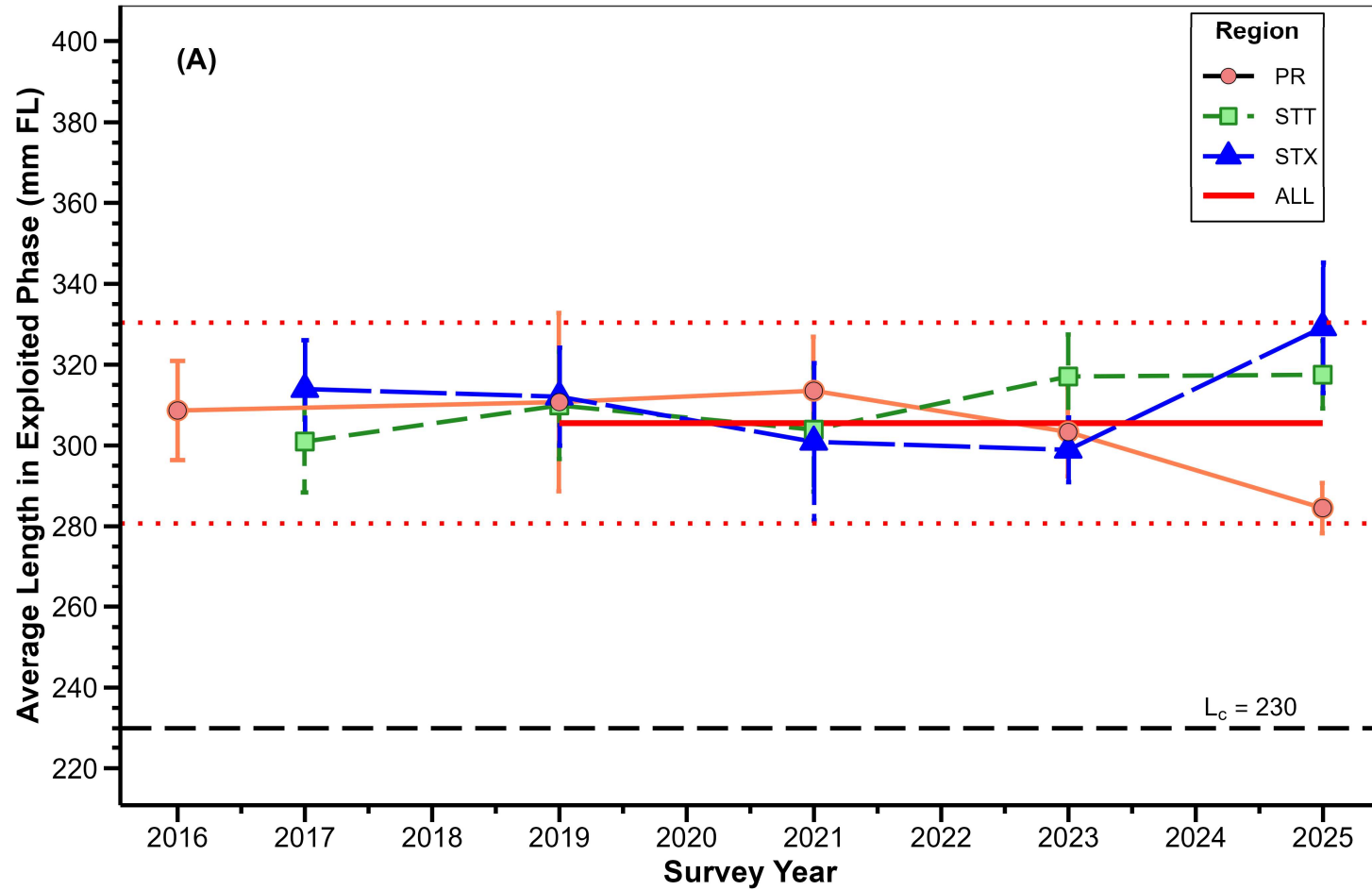
The solution with the highest utility is referred to as the best compromise solution.

Lottery involving x_i^* and x_i^0 : Probability p of receiving x_i^* ; Probability $(1 - p)$ of receiving x_i^0 . What amount of x would be acceptable for certain in place of the above gamble? Once x is determined, the utility of x can be calculated as

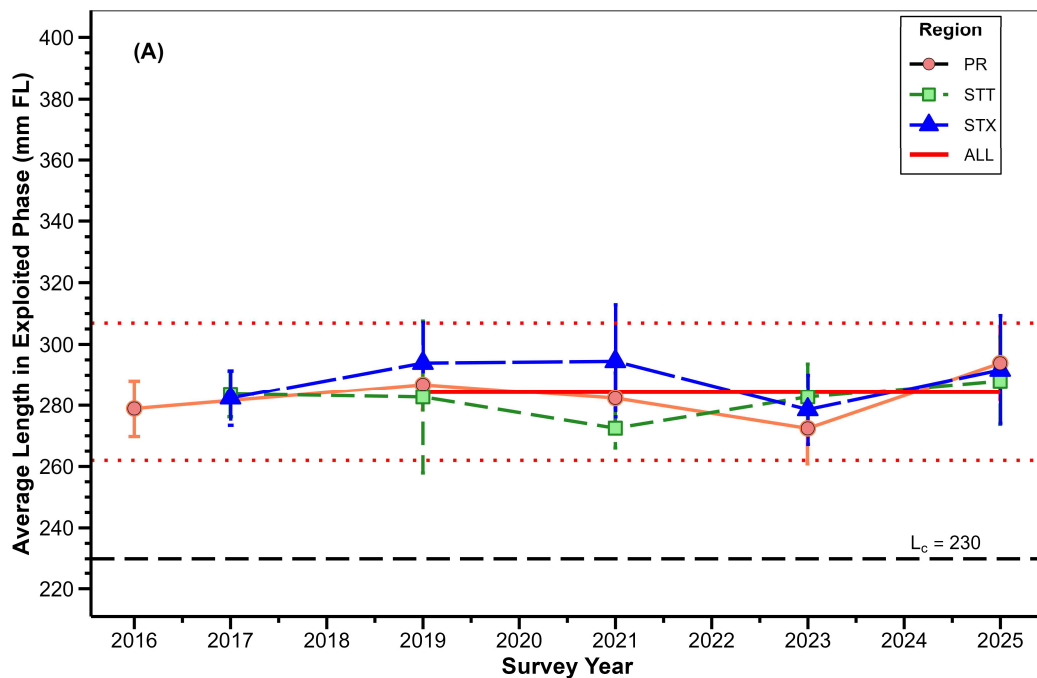
$$\mu(x) = p[\mu(x_i^*)] + (1 - p)[\mu(x_i^0)]$$

By varying p and changing the gamble, any number of points on the DM's utility curve can be generated for each utility function.

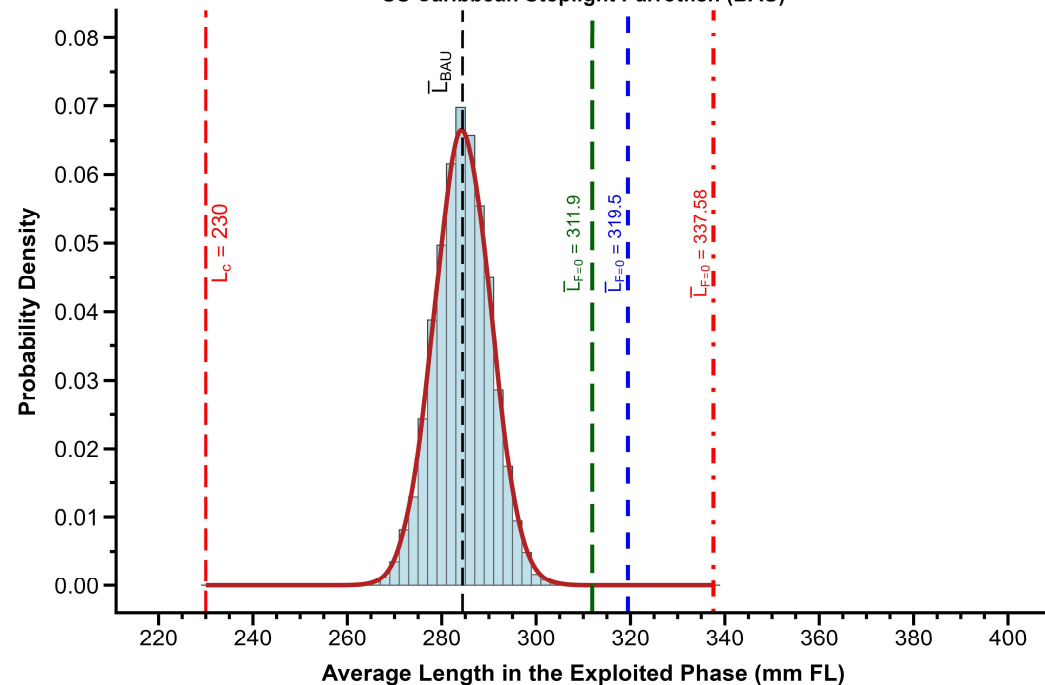
US Caribbean Queen Triggerfish (RVC)



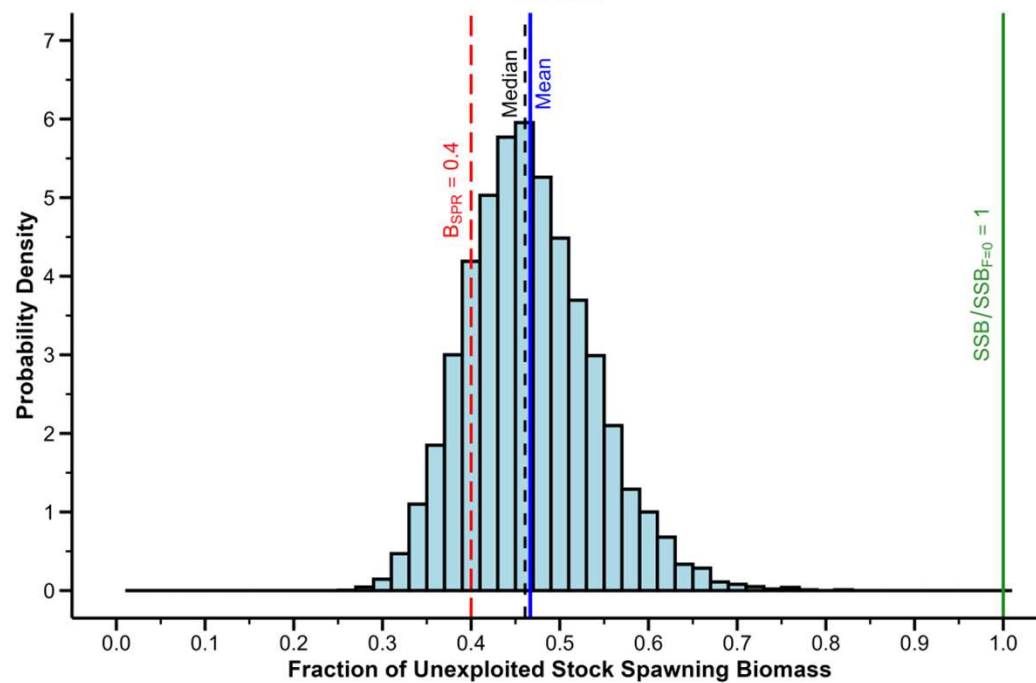
US Caribbean Stoplight Parrotfish (RVC)



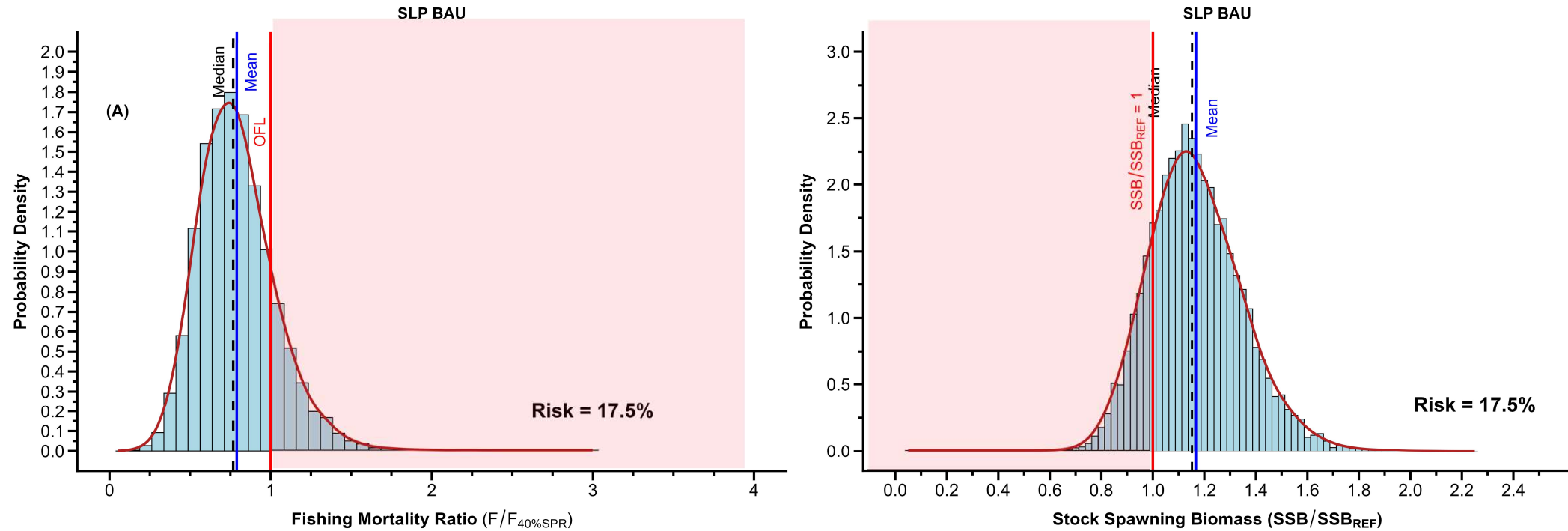
US Caribbean Stoplight Parrotfish (BAU)



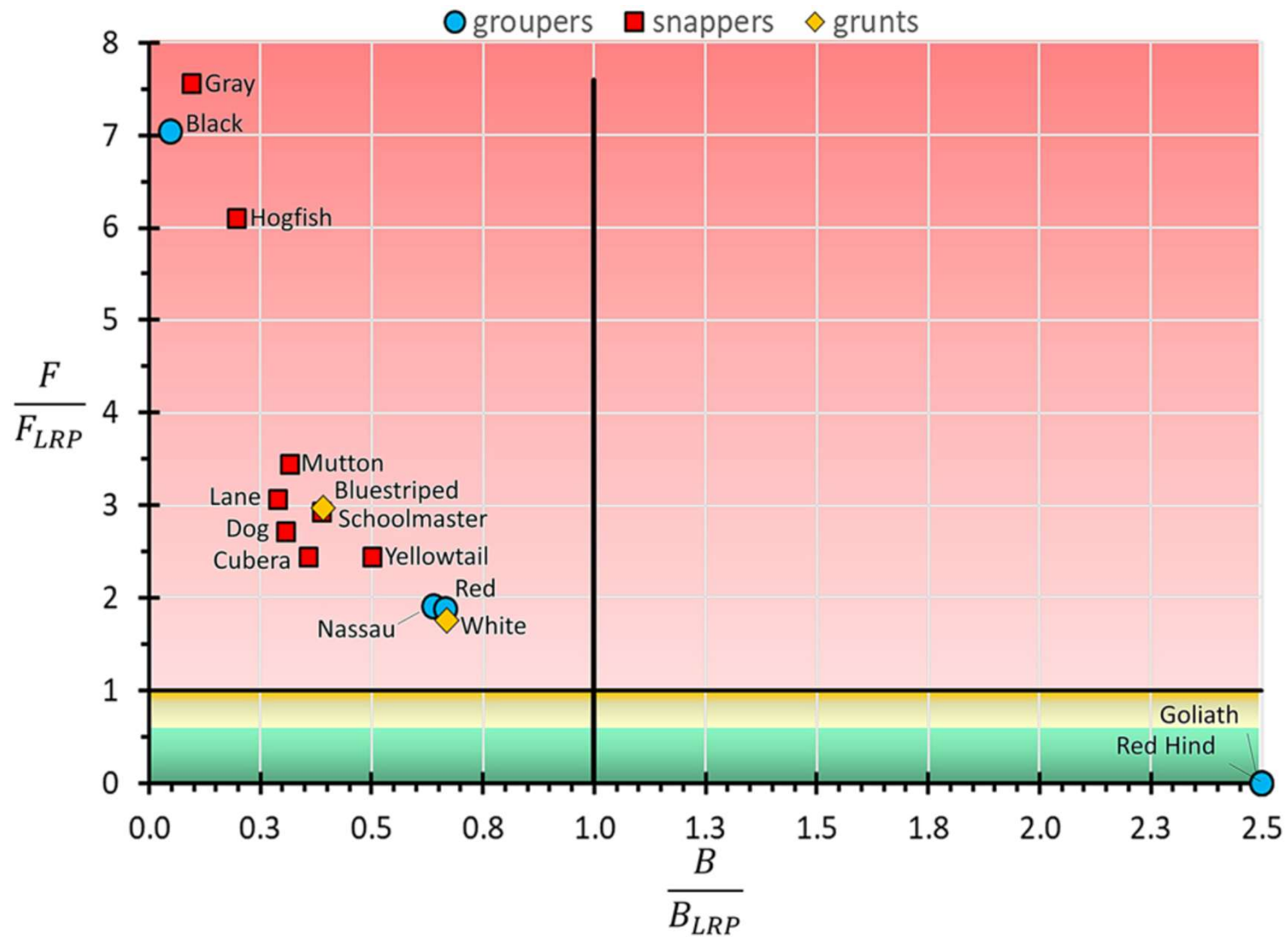
SLP BAU



Stoplight Parrotfish



Fishery “Ecosystem” Components



Ault et al. 2022

How does LBRA inform fisheries management?

- Rapidly **identifies** species at greatest biological risks.
- **Prioritizes** species for stock assessment, monitoring, and management action.
- **Defines the management decision space** by quantifying probabilities of overfishing and other biological risks.
- **Evaluates management scenarios** by forecasting the likelihood and time horizon for achieving biological and harvest objectives.
- **Cross-validates integrated stock assessments** (e.g., Stock Synthesis) using an independent analytical framework based.
- **Facilitates multi-species comparisons** using a consistent analytical framework.
- **Supports precautionary management** in data-limited fisheries by providing biologically informed reference points and quantitative risk estimates.
- **Improves management transparency** by explicitly linking management actions to quantified biological risk.