Another Assessment of Gray Triggerfish (*Balistes capriscus*) in the Gulf of Mexico Using a State-Space Implementation of the Pella-Tomlinson Production Model

Clay E. Porch

Southeast Fisheries Science Center
Miami Laboratory
Sustainable Resource Diviosion
75 Virginia Beach Drive
Miami, Fl 33149-1099

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This paper reports on the application of a state-space implementation of the Pella-Tomlinson (1969) production model to gray triggerfish in the Gulf of Mexico. It is intended to supplement the original assessment by Valle et al. (2001).

METHODS

Brief review of state space models

It is common practice to draw inferences about a population by fitting a proposed model of that population to a set of data. This involves expressing the observed data $\mathbf{Y}$ as a probability function $P(\mathbf{Y} \mid \mathbf{X})$ of certain covariates $\mathbf{X}$ and a parameter vector $\boldsymbol{\Theta}$. According to classical maximum likelihood theory, the values of $\boldsymbol{\Theta}$ that maximize $P(\mathbf{Y} \mid \boldsymbol{\Theta}, \mathbf{X})$ will be efficient unbiased estimates of the true values provided the covariates are known without error. In many cases, however, some of the covariates will be error-prone and alternative methods of estimation are required (Seber and Wild, 1989).

State space models are among the class of models with error prone covariates. Typically the observations $\mathbf{Y}_t$ (e.g., catches) are expressed as functions of time-independent parameters $\boldsymbol{\Theta}$ and time-dependent covariates $\mathbf{X}_t$ that include the unobserved states of the system (e.g., population biomass). A stochastic representation of a state space model is therefore defined by two probability statements: $P(\mathbf{Y} \mid \boldsymbol{\Theta}, \mathbf{X})$, quantifying the likelihood of observation errors in the data, and $P(\mathbf{X} \mid \boldsymbol{\Theta})$, quantifying the likelihood of process errors in the covariates (states).

A statistically rigorous Bayesian treatment of state space models is straightforward (see Schnute, 1994). By Bayes theorem, $P(\boldsymbol{\Theta}, \mathbf{X} \mid \mathbf{Y})\, P(\mathbf{Y}) = P(\mathbf{Y} \mid \boldsymbol{\Theta}, \mathbf{X})\, P(\boldsymbol{\Theta}, \mathbf{X})$. Inasmuch as $P(\mathbf{Y})$ is a constant and $P(\boldsymbol{\Theta}, \mathbf{X})$ may be expressed as $P(\mathbf{X} \mid \boldsymbol{\Theta})\, P(\boldsymbol{\Theta})$,

$$P(\Theta, \mathbf{X} \mid \mathbf{Y}) \propto P(\mathbf{Y} \mid \Theta, \mathbf{X})\, P(\mathbf{X} \mid \Theta)\, P(\Theta)\,. \tag{1}$$

Here $P(\boldsymbol{\Theta})$ is the so-called 'Bayes prior', the analyst's best guess of the probability density for $\boldsymbol{\Theta}$.

Estimates for $\boldsymbol{\Theta}$ (and therefore also $\mathbf{X}$) may be obtained by maximizing (1), known as the method of highest posterior density (HPD), which of course is equivalent to minimizing

$$\mathcal{L} = -\log P(\mathbf{Y} \mid \Theta, \mathbf{X}) - \log P(\mathbf{X} \mid \Theta) - \log P(\Theta) \tag{2}$$

Alternatively, one may employ the classical Bayes moment estimator

$$\hat{\theta}_i = \int \theta_i\, P(\mathbf{Y} \mid \Theta, \mathbf{X})\, P(\mathbf{X} \mid \Theta)\, P(\Theta)\, d\theta_i \quad , \qquad \theta_i \in \left\{\Theta, \mathbf{X}\right\}\ . \tag{3}$$

Covariances may be obtained from the classical Bayes estimator

$$\hat{\sigma}_{ij} = \int (\theta_i - \hat{\theta}_i)(\theta_j - \hat{\theta}_j)\, P(\mathbf{Y} \mid \Theta, \mathbf{X})\, P(\mathbf{X} \mid \Theta)\, P(\Theta)\, d\theta_i \theta_j\,. \tag{4}$$

Notice that when process errors and priors are ignored equation (2) reduces to the classical negative log-likelihood expression and the HPD estimates are equivalent to the maximum likelihood solution. In that case the covariance matrix may be obtained from the inverse of the Hessian matrix of second derivatives with respect to Θ.

State space implementation of Pella-Tomlinson model

The Pella-Tomlinson (1969) generalized production model may be written in the form

$$\frac{dB}{dt} = rB\left(1 - \left(B/k\right)^{m-1}\right) - FB \tag{5}$$

where B denotes biomass, r is the intrinsic rate of increase, k is the carrying capacity, F is the fishing mortality rate, and m is the exponent controlling the inflection point of the production curve. There is no general analytical solution for this differential equation, although analytic solutions exist for specific values of m (e.g., the classic Schaeffer model with m=2). The present algorithm uses the semi-implicit difference approximation suggested by Otter Research Ltd. (2000),

$$B_{t+\delta} = \frac{B_t(1 + r\delta)}{1 + (r(B_t / k)^{m-1} + F_t)\delta} \qquad (6).$$

Tests comparing this approximation with the exact solution for m=2 indicate it is accurate to several significant digits with δ= 1/16 yr.

The process and observation equations are summarized in Table 1 and the parameter vector in Table 2. Process errors in the state variables and observation errors in the data variables are accommodated using the first-order autoregressive (AR1) model

$$\begin{aligned} g_{t+1} &= \mathrm{E}[g_{t+1}]\, e^{\varepsilon_{t+1}} \\ \varepsilon_{t+1} &= \rho\varepsilon_t + a_{t+1} \end{aligned} , \tag{7}$$

where g represents any given state or observation variable, a is a normal-distributed random error with mean 0 and standard deviation σ_g, and E[g] denotes the expected value of g given by the deterministic components of the process or observation equations in Table 1. In the case of data, the g_t in (7) correspond to observed quantities, but in the case of states the g_t are unobserved and must be estimated along with the parameter vector.

For stability reasons, it is assumed that ε_0 = 0, leading to the negative log-density

$$-\log \mathrm{P}(g\,|\,\Theta, \mathbf{X}) = \frac{1}{2\sigma_g^2}\left[(\ln g_1 - \ln \mathrm{E}[g_1])^2 + \sum_{t=1}^{N-1}(\ln g_{t+1} - \ln \mathrm{E}[g_{t+1}] - \rho \ln g_t + \rho \ln \mathrm{E}[g_t])^2\right] + N \log \sigma_g , \tag{8}$$

where ρ_g is the correlation coefficient and σ_g^2 is the variance of log(a). In the present model, the variances of the process and observation errors are parameterized as multiples of an overall variance parameter σ^2, i.e., $\sigma_g^2 = V_g \sigma^2$. Note that the 'random walk' model of Fournier et al. (1998) is merely a special case of (8) with $\rho = 1$ and $E[g_t] = g_0$ (a time-invariant parameter).

The model was implemented using the nonlinear optimization package AD Model Builder (Otter Research Ltd., 2000), which provides facilities for estimating the mode and shape of posterior distributions formed by (8) and the negative logarithms of the priors.

Table 1. Stochastic equations used to define the state space Pella-Tomlinson model.

Variables	Description
Process functions for state variables	
$m_t = m_0\, e^{-\varepsilon_{m,t}},\ \ \varepsilon_{m,t} = \rho_m\, \varepsilon_{m,t-1} + a_{m,t}$	exponent controlling inflection point of production curve
$r_t = r_0\, e^{-\varepsilon_{r,t}},\ \ \varepsilon_{r,t} = \rho_r\, \varepsilon_{r,t-1} + a_{r,t}$	intrinsic rate of production
$k_t = \dfrac{B_1}{\alpha}\, e^{-\varepsilon_{k,t}},\ \ \varepsilon_{k,t} = \rho_k\, \varepsilon_{k,t-1} + a_{k,t}$	carrying capacity of environment
$q_{f,t} = q_{f,0}\, e^{-\varepsilon_{q,f,t}},\ \ \varepsilon_{q,f,t} = \rho_{q,f}\, \varepsilon_{q,f,t-1} + a_{q,f,t}$	catchability for fishery f
$E_{f,t} = E_{f,0}\, e^{-\varepsilon_{E,f,t}},\ \ \varepsilon_{E,f,t} = \rho_{E,f}\, \varepsilon_{E,f,t-1} + a_{E,f,t}$	effort expended by fishery f
Observation functions for data variables	
$C_{ft} = \left(\delta\, q_{ft} E_{ft} \sum_{j=1}^{16} B_{t+j\delta}\right) e^{-\varepsilon_{C,f,t}},\ \ \varepsilon_{C,f,t} = \rho_{C,f}\, \varepsilon_{C,f,t-1} + a_{C,f,t}$	catch of fishery *f*
$I_{ft} = \left(\delta q_{ft} \sum_{j=1}^{16} B_{t+j\delta}\right) e^{-\varepsilon_{I,f,t}},\ \ \varepsilon_{I,f,t} = \rho_{I,f}\, \varepsilon_{I,f,t-1} + a_{I,f,t}$	CPUE of fishery *f*
State moments	
$B_{t+\delta} = \dfrac{B_t(1 + r_t\delta)}{1 + (r_t(B_t / k_t)^{m_t - 1} + F_t)\delta}$	biomass
$F_t = \sum_{f=1}^{n} q_{ft} E_{ft}$	fishing mortality rate

Table 2. Time-independent parameters of the state-space Pella-Tomlinson model and their use in the analyses of gray triggerfish. The term expectation refers to the long term mean of a quantity.

Parameter	Value for gray triggerfish	Description
B_1	estimated	biomass at start of first year (1986 or 1990)
α	estimated	carrying capacity scale factor ($K = B_1/\alpha$)
m_0	estimated	production curve exponent expectation
r_0	estimated	intrinsic production rate expectation
$q_{f,0}$	estimated	catchability coefficient expectation for fishery f
$E_{f,0}$	geometric mean of series	effort expectation for fishery f
ρ_k	0	process correlation for carrying capacity
ρ_m	0	process correlation for production exponent
ρ_r	0	process correlation for rate of production
$\rho_{q,f}$	1.0	process correlation for catchability for fishery f
$\rho_{E,f}$	0.5	process correlation for effort for fishery f
$\rho_{C,f}$	0	process correlation for catch of fishery f
$\rho_{I,f}$	0	process correlation for CPUE of fishery f
V_k	0	relative process variance in carrying capacity
V_m	0	relative process variance in production exponent
V_r	0	relative process variance in rate of production
$V_{q,f}$	0.1 or 10.0	relative process variance in catchability for fishery f
$V_{E,f}$	50.0	relative process variance in effort for fishery f
$V_{C,f}$	1.0	relative observation variance for catch of fishery f
$V_{I,f}$	1.0 or 10.0	relative observation variance for CPUE of fishery f
σ^2	estimated	variance scale (controls absolute magnitude)

Application to gray triggerfish

The three fisheries considered by Valle et al. (2001) in their analyses were recreational, head boat, and commercial (mostly handline). The corresponding catch and CPUE data were taken directly from their Table 12. Two sets of analyses were conducted. The first used all of the available data from 1986 to 1998 whereas the second used only data from 1990 forward.

The reason for using the truncated data set is obvious from Figure 1. The recreational and head boat CPUE series both indicate a consistent increase during the 1980's despite the fact that the total yield was also increasing. This implies either increased catchability (q) or increased production (r) over that time span. The former appears more likely inasmuch as gray triggerfish have been increasingly targeted with the decline of more desirable stocks (Valle et al., 2001). Therefore, while the catchabilities of the short series were time-invariant, the catchabilities of the recreational and head boat fisheries in the long (1986-98) series were allowed to vary from year to year as a random walk ($\rho_q = 1$). Two levels of process variance, high (V_q=10) and low (V_q=0.1), were used in separate runs to bracket the possibilites. The catchability coefficient for the commercial fishery was held constant because the time series was short.

Effort was allowed to vary from year to year essentially as a free parameter by allowing a huge relative process variance (V_E = 50) and moderate correlation (ρ = 0.5). The non-zero correlation made little difference because the variance was so large. No process error was allowed for the other state variables (*m, r, k*) owing to the relatively short time series.

There was little basis upon which to formulate the priors for the estimated parameters, therefore I used uniform priors defined over a plausible range of values (see Table 3). The only exception was the exponent *m,* for which a lognormal prior with low log-scale standard deviation (0.2) was imposed. This was done because *m* is notoriously difficult to estimate from brief time series (in this context even the 'long' 1986-1998 series is brief) and the usual assumption is that *m* equals 2.

The catches and MRFSS CPUE data were assumed to have similar log-scale variances (i.e., similar CV's) such that $V_{I,MRFSS} = V_{C,MRFSS} = V_{C,headboat} = V_{C,comm} = 1$. The headboat and commercial CPUE indices were treated as much less precise and assigned a relative variance of 10 ($V_{I,headboat} = V_{I,comm} = 10$). In effect, this is equivalent to assuming that CV's of the headboat and commercial CPUE series are about three times larger than the CV's of the other data. Additional analyses were conducted assuming a high relative variance for the MRFSS CPUE index as well (i.e., $V_{I,MRFSS} = V_{I,headboat} = V_{I,comm} = 10$) in order to examine the sensitivity of the results to the relative variances assumed. Thus, a total of six runs were made (short and long data series, high and low relative variance for the MRFSS index, and high and low process variance for the catchability coefficients in the long series).

Table 3. Priors used to constrain estimated parameters. Note that C_1 denotes the total catch in the first year (1986 or 1990) and $\bar{I}_f$ denotes the geometric mean of the CPUE indices for each fishery.

Parameter	Prior	Rationale
B_1	uniform(C_1/5.0, C_1/0.01)	Since $C_1 \approx F_1 B_1$, and probably $0.01 << F_1 << 5$
α	uniform(0.1, 1.0)	Do not expect K (=B_1/α) < B_1, and since fishery was relatively new, probably $K < 10*B_1$
m_0	lognormal(2.0, 0.2)	Difficult to estimate, the usual assumption is that *m*=2
r_0	uniform(0.1, 5.0)	Relatively uninformative prior
q_f	uniform$\left(\frac{\bar{I}_f}{100\Omega}, \frac{\bar{I}_f}{0.1\Omega}\right)$	Probably $0.1\Omega < B_t < 100\ \Omega$ (Ω greatest observed annual catch.)

RESULTS

Each of the six models provided reasonably good fits to the data (Figure 2) and generally yielded parameter estimates with reasonably low estimated CV's (Table 4). However, the magnitudes of the parameter estimates and management benchmarks *(MSY,* F_{MSY} and B_{MSY}) were very sensitive to the use of the short or long data series and to the relative weight given to the MRFSS catch and CPUE data (Tables 4 and 5, Figure 3). Moreover, in five of the six cases the estimates for one or more parameters were at or near the boundary constraints.

Short series (1990-1998)

The results were sensitive to the variance attributed to the MRFSS index (Table 4). Both runs indicate that overfishing has been occurring throughout the time series and that the stock is currently in a severely overfished condition (Figure 4a, Table 5). The main differences between the two analyses are in the predictions of *MSY* and F_{MSY} and in the historical perception of stock status: The run with a lower variance for the MRFSS index predicted that MSY was less than the average catch during the time series and that the 1990-1991 biomass was above B_{MSY}.

Long series (1986-1998)

The runs for the long series were analogous to those of the short series except that the catchability coefficients for the MRFSS and headboat fisheries were allowed to vary through time as a random walk. When the variance of the MRFSS CPUE index was set equal to that of the catch (low variance scenario), the perception of stock status relative to the benchmarks was similar to those of the short series (Figure 4b), but the magnitude of the historical *F'*s was unbelievably high. When the variance of the MRFSS CPUE was assumed to be ten times greater than the variance of the catch, the estimated magnitude of the historical F's remained high, but the prognosis of stock status depended on the variance allowed for the catchability (V_q). When V_q was low the model predicted the stock has been overfished throughout the time series, but when V_q was high the model predicted the stock had recovered from an initially overfished condition. In the case of the latter, however, the fit to the commercial index was much poorer than for the other models (Figure 4c).

The added flexibility provided by allowing large inter-annual variations in catchability (V_q=10) permitted a better fit to the headboat index than was possible with only small variations (V_q=0.1) and led to reduced standard errors for the population parameters and management benchmarks (see Tables 4 and 5). With a low MRFSS index variance the catchabilities of the MRFSS and headboat fisheries were estimated to increase during the first part of the time series and level out towards the end (Figure 5a), just as one might expect with increased targeting of gray triggerfish. The same was true for the run with high MRFSS index variance and low V_q (Figure 5b). In the case of the high MRFSS index variance and high V_q, however, catchabilities were estimated to decrease (Figure 5b); contrary to anecdotal expectations.

DISCUSSION

The estimates from the two models using the short series suggest an unproductive stock ($r < 0.4$) with a very high carrying capacity ($k >>$ MSY) that is currently overfished. The estimates of current F, although only about 0.3, were still much greater than F_{MSY} and the biomass at the beginning of 1999 was estimated to be considerably less than B_{MSY} (Figure 4a). The estimates from the four models involving the long series depict just the opposite-- a highly productive stock ($r > 2.0$) with a very low carrying capacity (in 3 cases lower than MSY). Three of the four models also suggest that the stock has undergone extremely heavy fishing pressure (> 2.0) and is currently overfished (Figure 4b,c). The only exception was the model that assigned a high relative variance to the MRFSS CPUE index as well as the catchability coefficients, which predicted the stock had been overfished early in the time period, but was now recovered. This model, however, provided the poorest fit to the commercial CPUE series and suggested that q has decreased despite anecdotal evidence to the contrary.

The fishing mortality rates estimated by the four long-series models seem unrealistic (Figure 4b,c), being much higher than the historical estimates for more highly prized species in the Gulf of Mexico such as red snapper and red grouper (Schirripa and Legault, 1999; Schirripa et al, 1999). On the other hand, the very unproductive stock suggested by the short series model with high variance assigned to the MRFSS CPUE index also seems unrealistic (Figure 6). More importantly, the estimates of MSY from these five models are much higher than the observed catches, yet the stocks are estimated to have been severely overfished at the start of the time series. This implies that the catches prior to 1986 were much higher than presently observed, which seems very unlikely inasmuch as gray triggerfish were not considered a desirable catch until recently. The only model formulation that did not exhibit these shortcomings was the short series with the lower variance assigned to the MRFSS CPUE index. This model is also the only formulation for which none of the parameter estimates hit the boundary constraints, therefore I consider it to be the most plausible of the six models examined.

Five of the six models indicate the stock is currently overfished and will remain so under the current rate of fishing (Figure 6), yielding estimates for the ratios B_{1999}/B_{MSY} and F_{1999}/F_{MSY} that were similar to the estimates from models 1 and 2 of Valle et al. (2001). Thus, it appears that the results of this study corroborate the conclusions of Valle et al. (2001) that gray triggerfish have been overfished in the Gulf of Mexico. Nevertheless, this assessment must be regarded as uncertain owing to the sensitivity of the results to the length of the time series and choice of relative variances.

There are several areas where the present state-space model could be improved. First, the formulation of the Bayes priors for initial biomass and carrying capacity might benefit from discussions with fishermen and others relating to the perceived historical abundance of the stock and the possibility of environmental changes affecting carrying capacity. Further examination of the known biological characteristics of the stock and comparisons with production model estimates for species with similar traits might also improve our perception of the production parameters m and r. There is also the issue of the relative weights of the catch and CPUE indices. Absolute estimates of the variances are available from the MRFSS, and these should probably be used. However, comparable variances are not available for the other sources and these may have to be determined subjectively. Finally, it may be that a production model is not the most appropriate assessment tool for this species owing to changes in the average weight and reproductive characteristics of the stock with increased fishing pressure. It would be instructive to employ alternatives, such as delay-difference models, which implicitly take changes in age-structure into account (albeit crudely).

ADDENDUM

The 2001 reef fish stock assessment panel agreed that the short series model with the lower variance assigned to the MRFSS CPUE index was the most plausible of the formulations presented. They requested an additional projection of that model under a constant yield scenario that allowed the stock to recover to B_{MSY} within 10 years (by 2011). The results from that exercise are summarized in Figure 7 and Table 6.

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Table 4. Parameter estimates from the various model formulations. The labels 'short' and 'long' refer to the length of the data series (1990-1998 versus 1986-1998). Shaded cells indicate values at or near the limits imposed on the search algorithm.

Variable	Model configuration	Point estimates		Standard error	CV (%)
		HPD (mode)	Bayes mean		
B_1	Short, V_{MRFSS} = 1	9449.40	8881.41	2852.51	32
	Short, V_{MRFSS} = 10	13406.00	14160.60	1175.96	8
	Long, V_{MRFSS} = 1, V_q=0.1	364.82	375.77	103.20	27
	Long, V_{MRFSS} = 1, V_q=10	739.36	757.78	162.95	22
	Long, V_{MRFSS} = 10, V_q=0.1	313.92	358.69	68.25	19
	Long, V_{MRFSS} = 10, V_q=10	292.97	302.31	17.80	6
m_0	Short, V_{MRFSS} = 1	1.99	1.99	0.34	17
	Short, V_{MRFSS} = 10	2.02	1.99	0.45	22
	Long, V_{MRFSS} = 1, V_q=0.1	2.41	2.44	0.51	21
	Long, V_{MRFSS} = 1, V_q=10	2.33	2.33	0.20	9
	Long, V_{MRFSS} = 10, V_q=0.1	2.65	2.52	0.36	14
	Long, V_{MRFSS} = 10, V_q=10	3.53	3.36	0.58	17
r_0	Short, V_{MRFSS} = 1	0.37	0.41	0.14	33
	Short, V_{MRFSS} = 10	0.10	0.10	0.01	6
	Long, V_{MRFSS} = 1, V_q=0.1	5.00	4.98	0.04	1
	Long, V_{MRFSS} = 1, V_q=10	2.14	2.12	0.33	16
	Long, V_{MRFSS} = 10, V_q=0.1	4.65	4.19	0.76	18
	Long, V_{MRFSS} = 10, V_q=10	4.82	4.75	0.32	7
k_0	Short, V_{MRFSS} = 1	12944.00	13154.20	4461.35	34
	Short, V_{MRFSS} = 10	133850.00	60510.80	68681.10	114
	Long, V_{MRFSS} = 1, V_q=0.1	1689.70	1743.25	652.35	37
	Long, V_{MRFSS} = 1, V_q=10	7392.80	7461.12	1664.41	22
	Long, V_{MRFSS} = 10, V_q=0.1	3139.10	3570.51	669.69	19
	Long, V_{MRFSS} = 10, V_q=10	1554.70	1637.33	286.26	17

Table 5. Estimates of management benchmarks from the various model formulations. The labels 'short' and 'long' refer to the length of the data series (1990-1998 versus 1986-1998).

Variable	Model configuration	Point estimates HPD (mode)	Point estimates Bayes mean	Standard error	CV (%)
MSY	Short, $V_{\text{MRFSS}} = 1$	1182.70	1309.89	529.48	40
	Short, $V_{\text{MRFSS}} = 10$	3385.60	1543.02	1842.30	119
	Long, $V_{\text{MRFSS}} = 1$, V_q=0.1	2644.40	2690.67	559.05	21
	Long, $V_{\text{MRFSS}} = 1$, V_q=10	4769.10	4725.24	450.98	10
	Long, $V_{\text{MRFSS}} = 10$, V_q=0.1	5039.00	4857.58	610.11	13
	Long, $V_{\text{MRFSS}} = 10$, V_q=10	3259.80	3244.81	440.35	14
B_{MSY}	Short, $V_{\text{MRFSS}} = 1$	6456.50	6526.56	1938.93	30
	Short, $V_{\text{MRFSS}} = 10$	67144.00	30222.80	34940.10	116
	Long, $V_{\text{MRFSS}} = 1$, V_q=0.1	904.96	930.92	290.99	31
	Long, $V_{\text{MRFSS}} = 1$, V_q=10	3912.40	3946.85	836.34	21
	Long, $V_{\text{MRFSS}} = 10$, V_q=0.1	1739.50	1939.48	320.31	17
	Long, $V_{\text{MRFSS}} = 10$, V_q=10	944.13	977.23	145.58	15
F_{MSY}	Short, $V_{\text{MRFSS}} = 1$	0.18	0.20	0.08	38
	Short, $V_{\text{MRFSS}} = 10$	0.05	0.05	0.01	23
	Long, $V_{\text{MRFSS}} = 1$, V_q=0.1	2.92	2.91	0.38	13
	Long, $V_{\text{MRFSS}} = 1$, V_q=10	1.22	1.21	0.21	17
	Long, $V_{\text{MRFSS}} = 10$, V_q=0.1	2.90	2.52	0.53	21
	Long, $V_{\text{MRFSS}} = 10$, V_q=10	3.45	3.33	0.31	9
B_{1999}/ B_{MSY}	Short, $V_{\text{MRFSS}} = 1$	0.42	0.40	0.15	38
	Short, $V_{\text{MRFSS}} = 10$	0.06	0.18	0.14	75
	Long, $V_{\text{MRFSS}} = 1$, V_q=0.1	0.32	0.36	0.22	60
	Long, $V_{\text{MRFSS}} = 1$, V_q=10	0.12	0.14	0.06	41
	Long, $V_{\text{MRFSS}} = 10$, V_q=0.1	0.15	0.16	0.05	32
	Long, $V_{\text{MRFSS}} = 10$, V_q=10	1.56	1.59	0.08	5
F_{1998}/ F_{MSY}	Short, $V_{\text{MRFSS}} = 1$	1.71	1.70	0.35	21
	Short, $V_{\text{MRFSS}} = 10$	3.85	3.72	0.98	26
	Long, $V_{\text{MRFSS}} = 1$, V_q=0.1	1.36	1.30	0.37	28
	Long, $V_{\text{MRFSS}} = 1$, V_q=10	1.58	1.50	0.37	24
	Long, $V_{\text{MRFSS}} = 10$, V_q=0.1	1.39	1.38	0.19	14
	Long, $V_{\text{MRFSS}} = 10$, V_q=10	0.17	0.17	0.03	16

Table 6. Projections of future yields (millions of pounds) based on the short time series model with low variance attributed to the MRFSS CPUE index and assuming (a) the fishing mortality rate stays the same as estimated for 1999, (b) the fishing mortality rate is reduced to F_{MSY}, (c) the fishing mortality rate is reduced to the level that will allow recovery to B_{MSY} by 2011, and (d) the allowable catch in weight is fixed to the level that will allow recovery to B_{MSY} by 2011.

Year	*(a)* Status quo	*(b)* F_{MSY}	*(c)* $F_{RECOVER}$	*(d)* $Y_{RECOVER}$
1990	2.875	2.875	2.875	2.875
1991	2.721	2.721	2.721	2.721
1992	2.033	2.033	2.033	2.033
1993	1.997	1.997	1.997	1.997
1994	1.830	1.830	1.830	1.830
1995	1.479	1.479	1.479	1.479
1996	0.951	0.951	0.951	0.951
1997	0.919	0.919	0.919	0.919
1998	0.854	0.854	0.854	0.854
1999	0.836	0.836	0.836	0.836
2000	0.820	0.820	0.820	0.820
2001	0.804	0.804	0.804	0.804
2002	0.790	0.494	0.367	0.558
2003	0.777	0.547	0.425	0.558
2004	0.765	0.600	0.487	0.558
2005	0.754	0.653	0.549	0.558
2006	0.744	0.706	0.612	0.558
2007	0.735	0.756	0.673	0.558
2008	0.727	0.804	0.731	0.558
2009	0.719	0.849	0.784	0.558
2010	0.711	0.891	0.832	0.558
2011	0.705	0.929	0.875	0.558

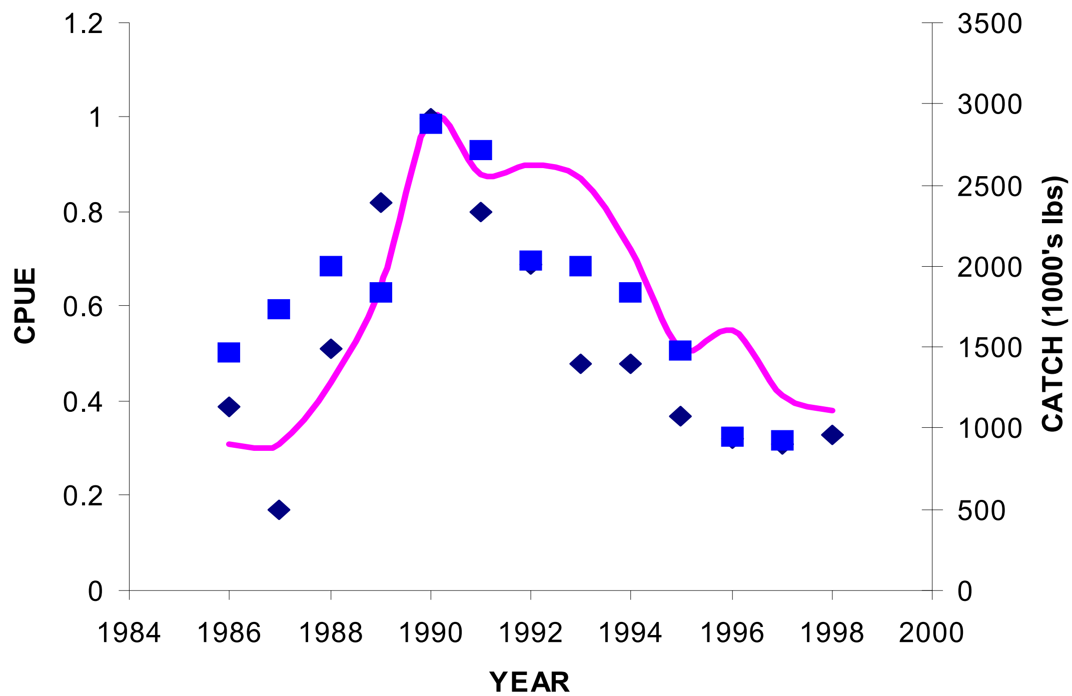

Figure 1. Trends in total catch (line) compared with the trends in catch per unit effort (squares = MRFSS, diamonds = headboat).

SHORT SERIES, $V_{I,\,MRFSS} = 1.0$

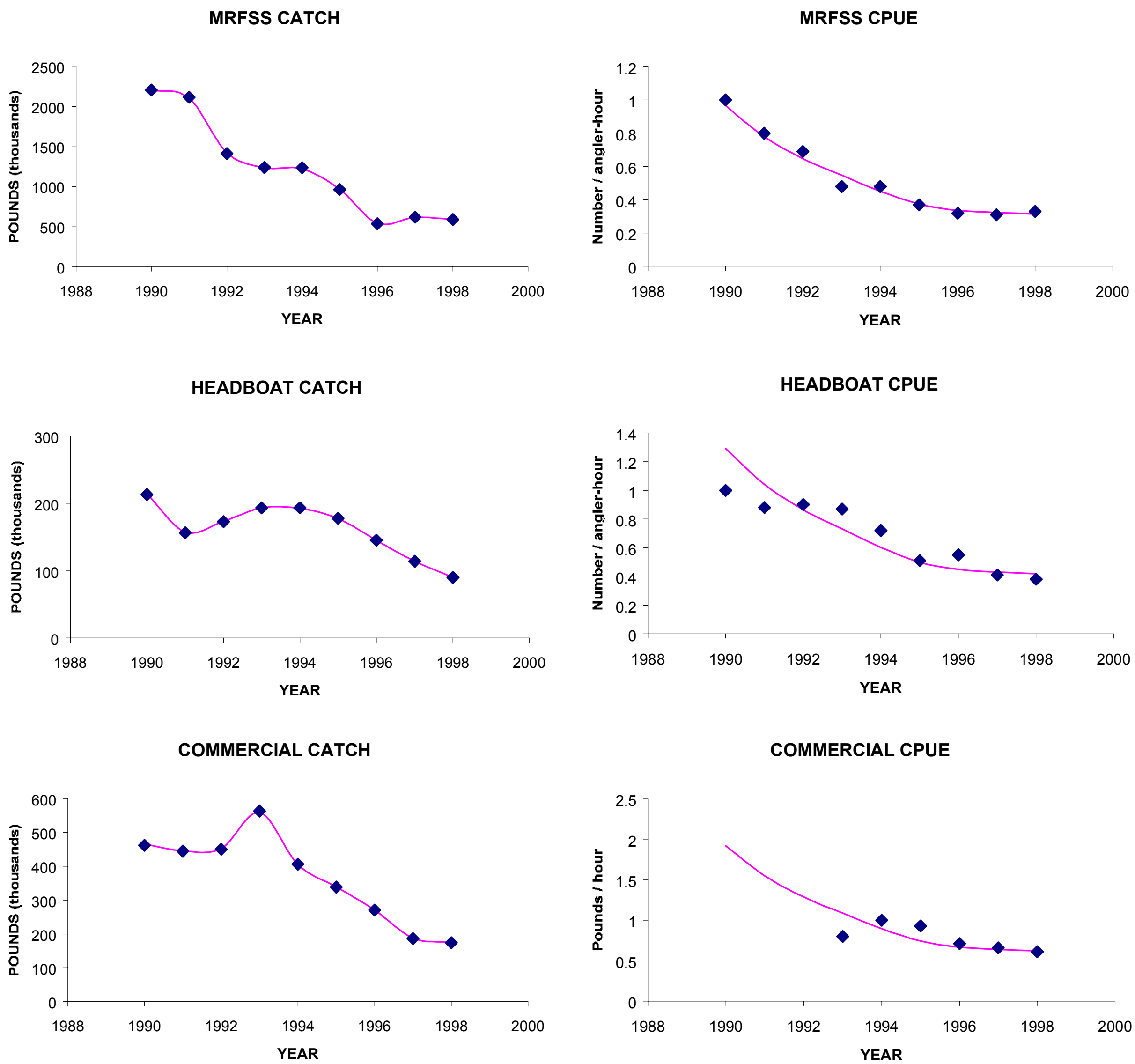

Figure 2a. Model fits (lines) to data (points) from 1990 to 1998 when the variance of the MRFSS CPUE index is assumed to be the same as the variance of the MRFSS catch.

SHORT SERIES, $V_{I,\,MRFSS} = 10$

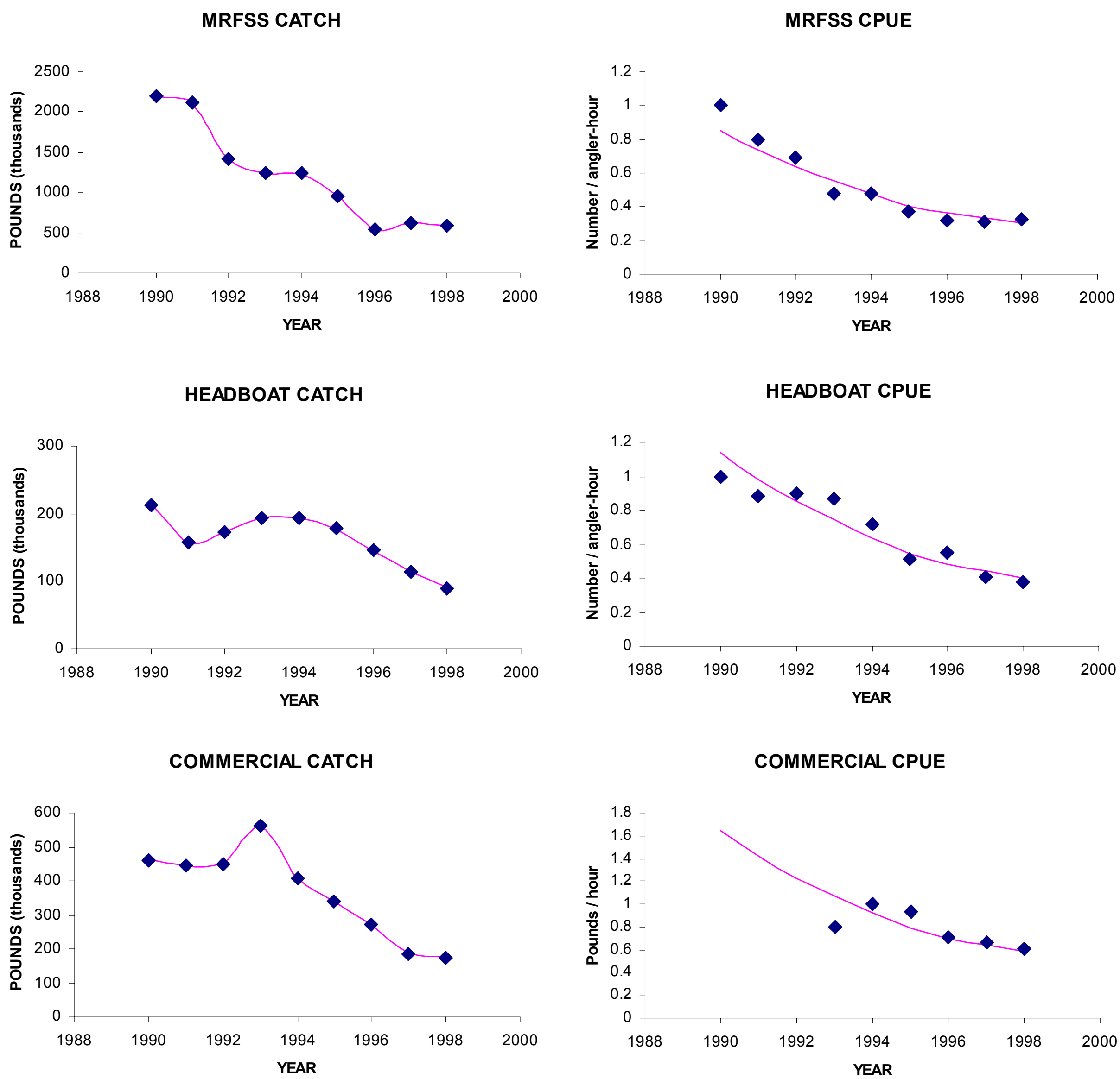

Figure 2b. Model fits (lines) to data (points) from 1990 to 1998 when the variance of the MRFSS CPUE index is assumed to be the same as the variance of the headboat and commercial CPUE indices.

LONG SERIES, $V_{I,\,MRFSS} = 1.0$, $V_q = 0.1$

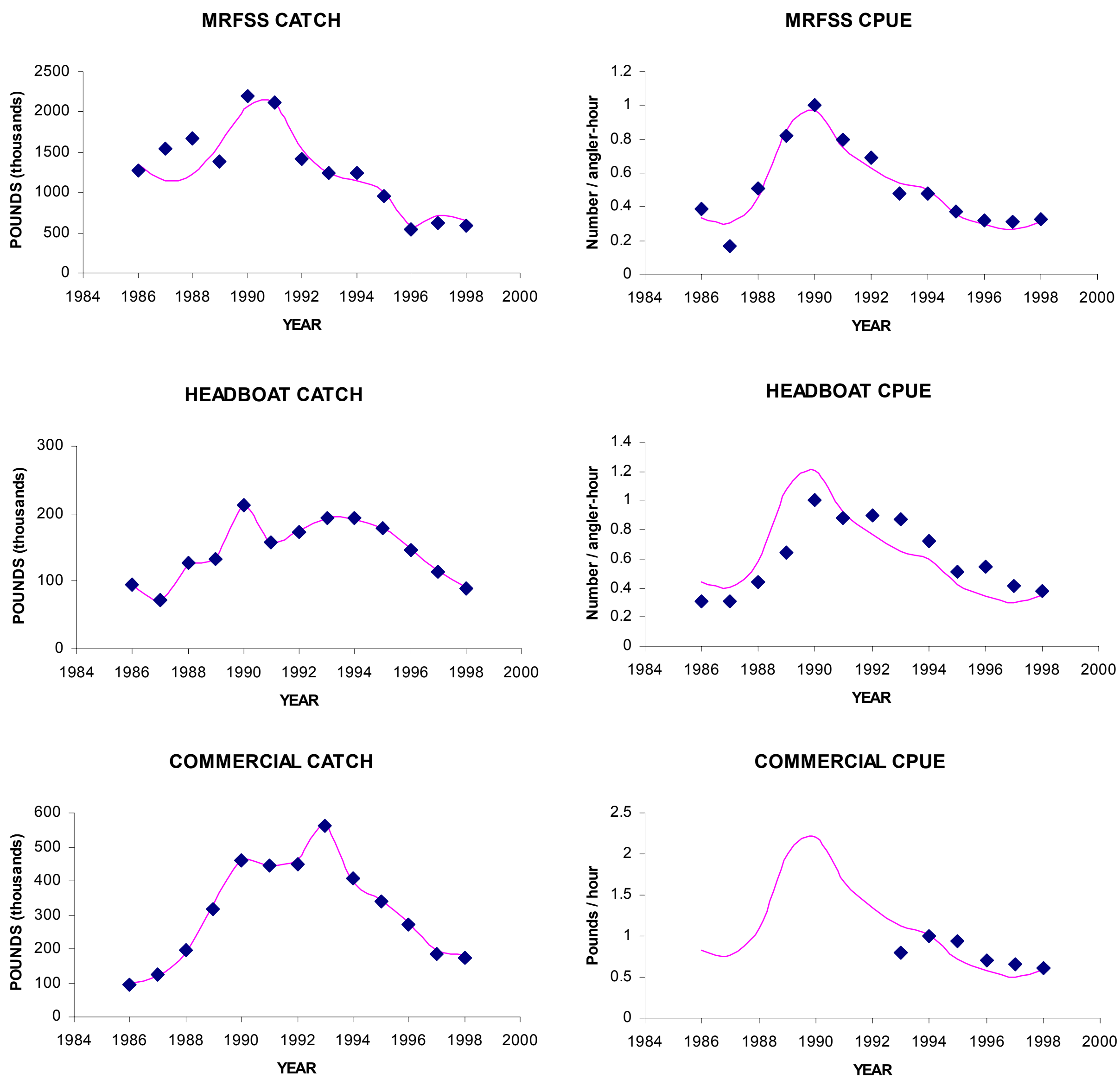

Figure 2c. Model fits (lines) to data (points) from 1986 to 1998 when the variance of the MRFSS CPUE index is assumed to be the same as the variance of the MRFSS catch and only small inter-annual variations in the catchability of the MRFSS and headboat fleets are allowed.

LONG SERIES, $V_{I,\,MRFSS} = 1.0$, $V_q = 10$

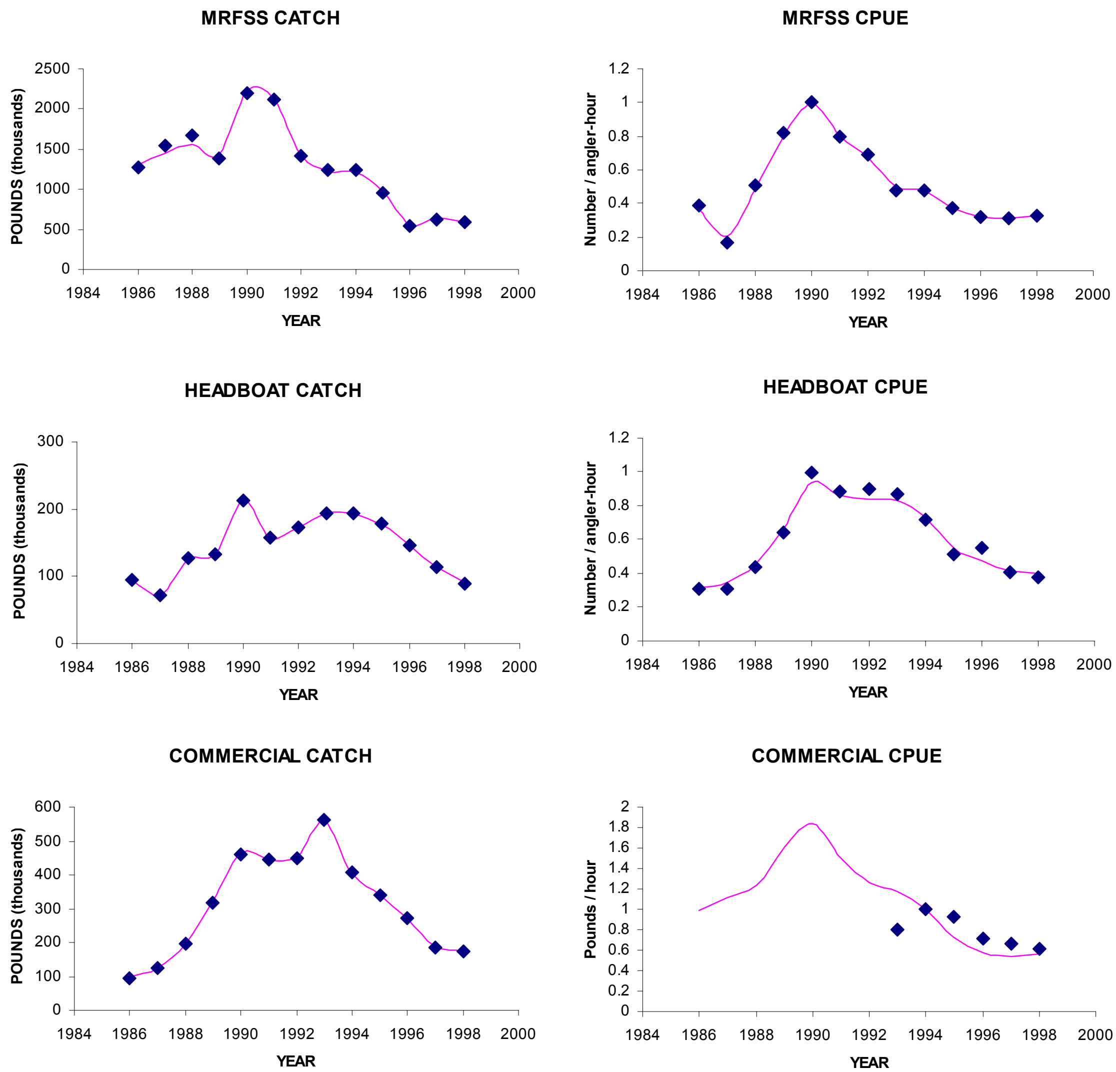

Figure 2d. Model fits (lines) to data (points) from 1986 to 1998 when the variance of the MRFSS CPUE index is assumed to be the same as the variance of the MRFSS catch and large inter-annual variations in the catchability of the MRFSS and headboat fleets are allowed.

LONG SERIES, $V_{I,\ MRFSS} = 10$, $V_q = 0.1$

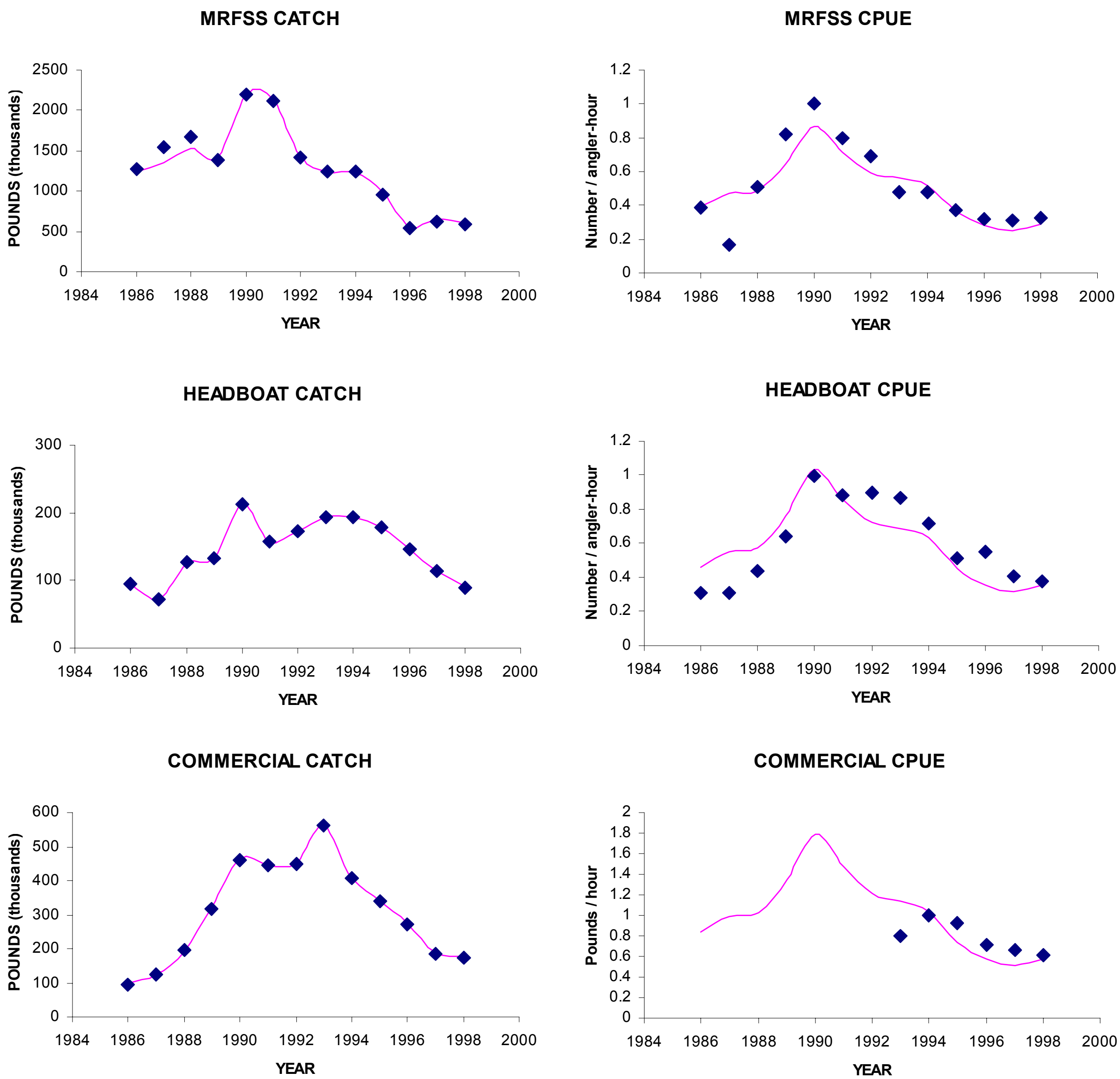

Figure 2e. Model fits (lines) to data (points) from 1986 to 1998 when the variance of the MRFSS CPUE is assumed to be the same as the headboat and commercial CPUE indices ($V_{I,f} = 10$) and only small inter-annual variations in the catchability of the MRFSS and headboat fleets are allowed.

LONG SERIES, $V_{I,\,MRFSS} = 10$, $V_q = 10$

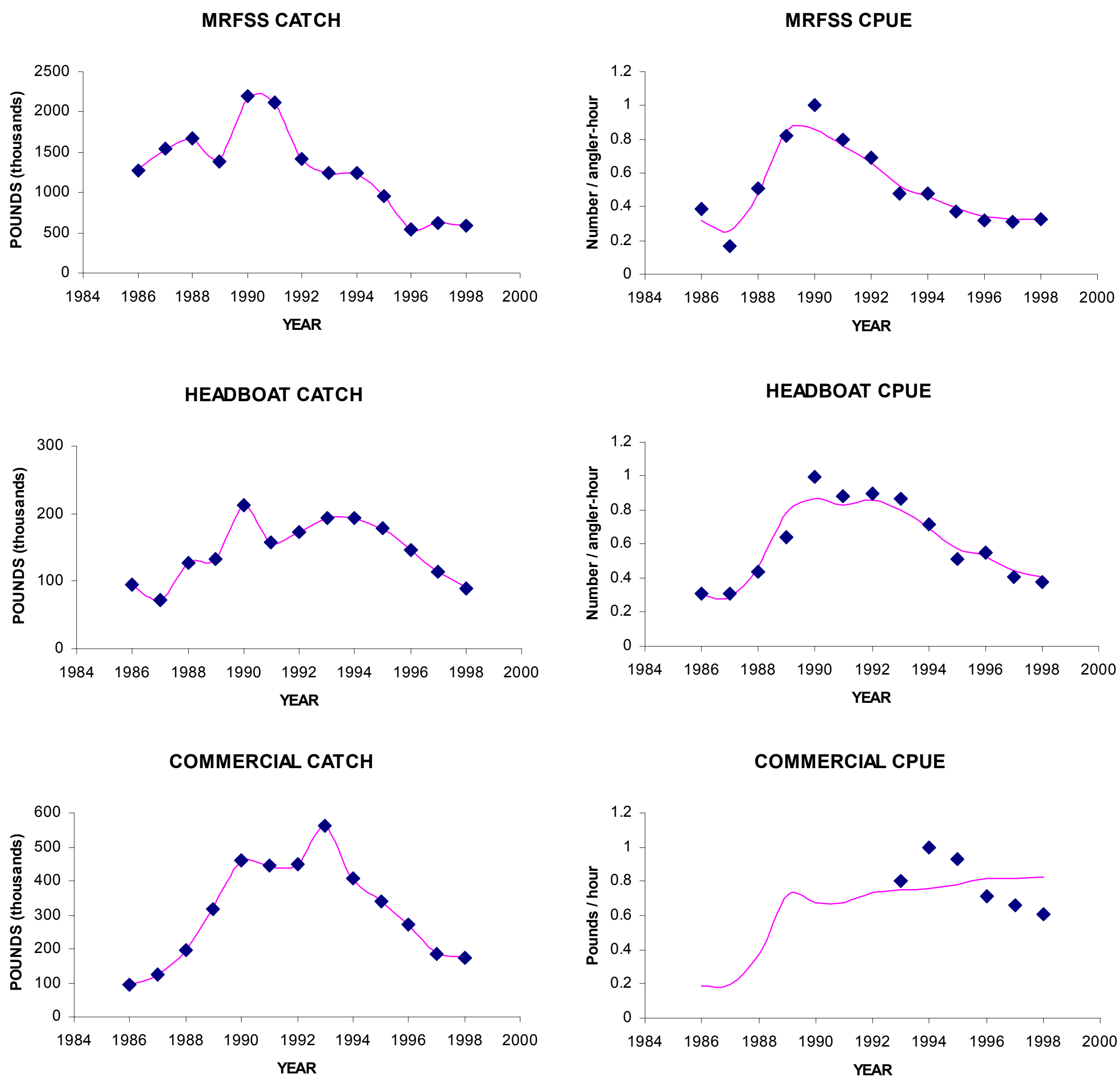

Figure 2f. Model fits (lines) to data (points) from 1986 to 1998 when the variance of the MRFSS CPUE is assumed to be the same as the headboat and commercial CPUE indices ($V_{I,f} = 10$) and large inter-annual variations in the catchability of the MRFSS and headboat fleets are allowed

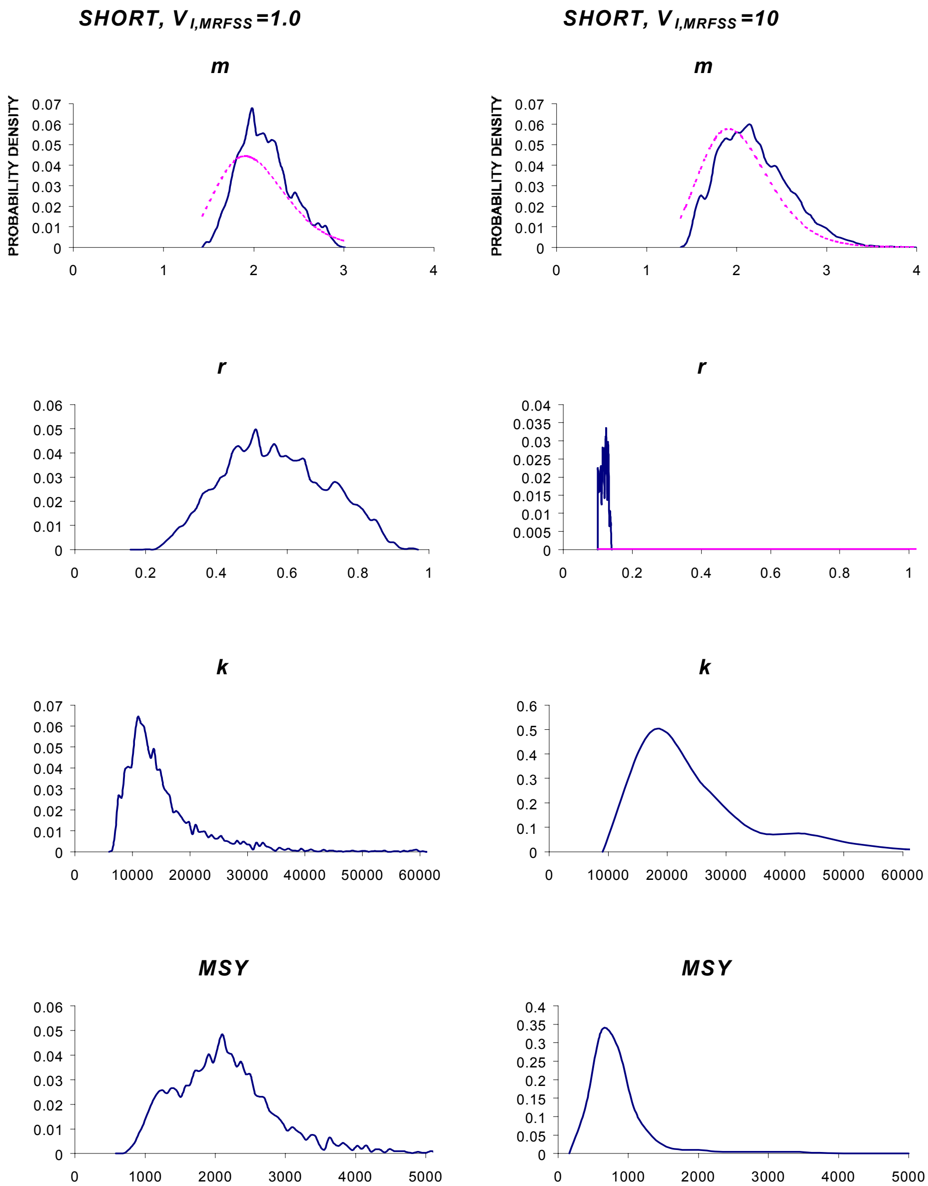

Figure 3a. Bayes posterior distributions for selected parameters estimated from the short time series (1990-1998) using the MCMC algorithm in AD Model Builder (Otter Research Ltd, 2000). The lognormal prior for *m* is indicated by the dotted lines (it differs between panels because the intervals used to generate the probabilities are automated in AD Model Builder and differ between runs). The priors for *r* and *k* are uniform (flat) distributions defined over a broad range with a very low probability for any given value, therefore they do not show up on this scale. Note that the modes of the posteriors for *k* and MSY in the rightmost panels ($V_{I,MRFSS}$=10) do not coincide with the HPD estimates in Tables 4 and 5, suggesting either a poorly behaving solution or perhaps a bug in the ADMB algorithm.

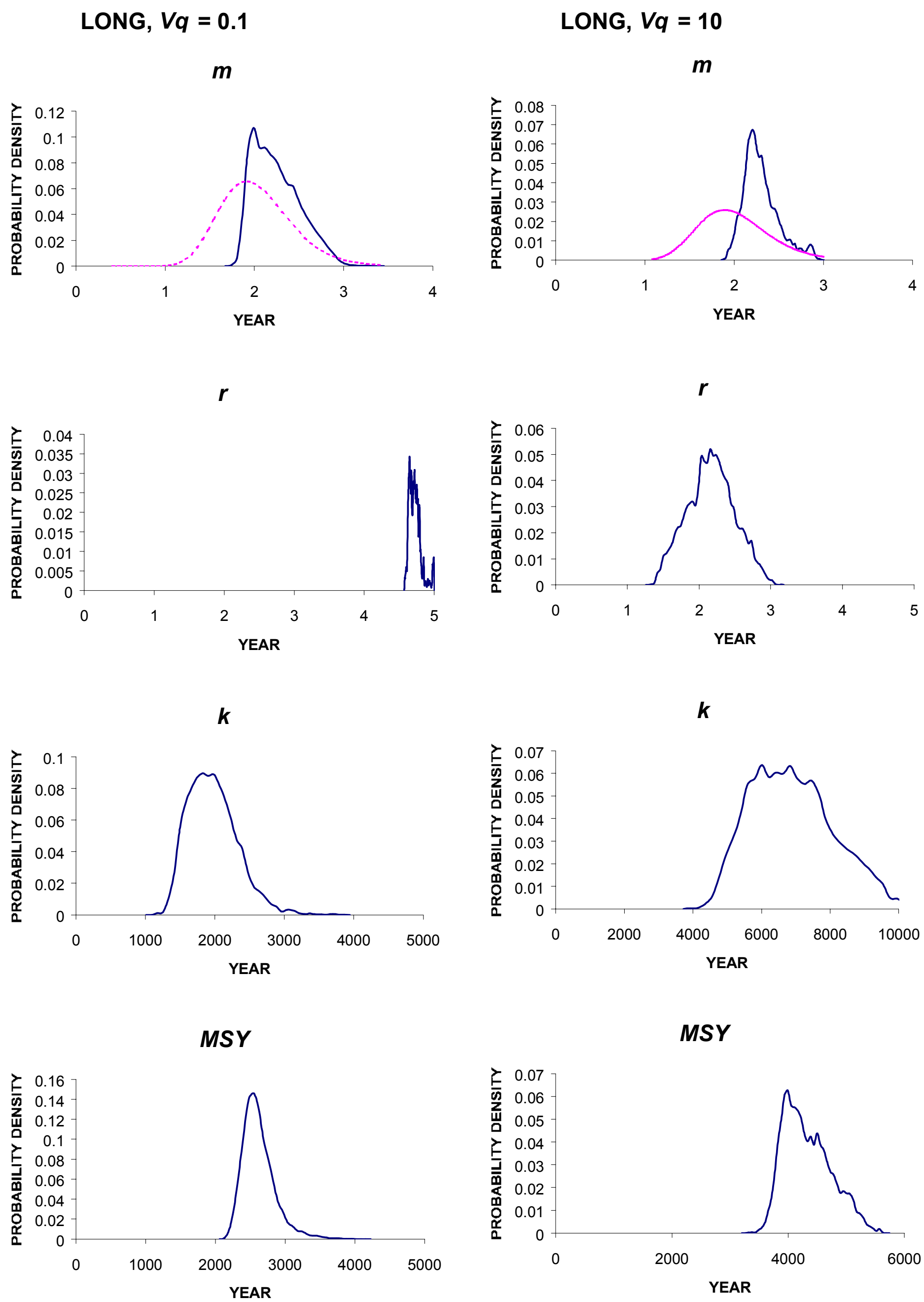

Figure 3b. Bayes posterior distributions for selected parameters estimated from the long time series (1986-1998) with low variance assigned to the MRFSS CPUE series. The lognormal prior for *m* is indicated by the dotted lines (it differs between panels because the intervals used to generate the probabilities are automated in AD Model Builder and differ between runs). The priors for *r* and *k* are uniform (flat) distributions defined over a broad range with a very low probability for any given value, therefore they do not show up on this scale.

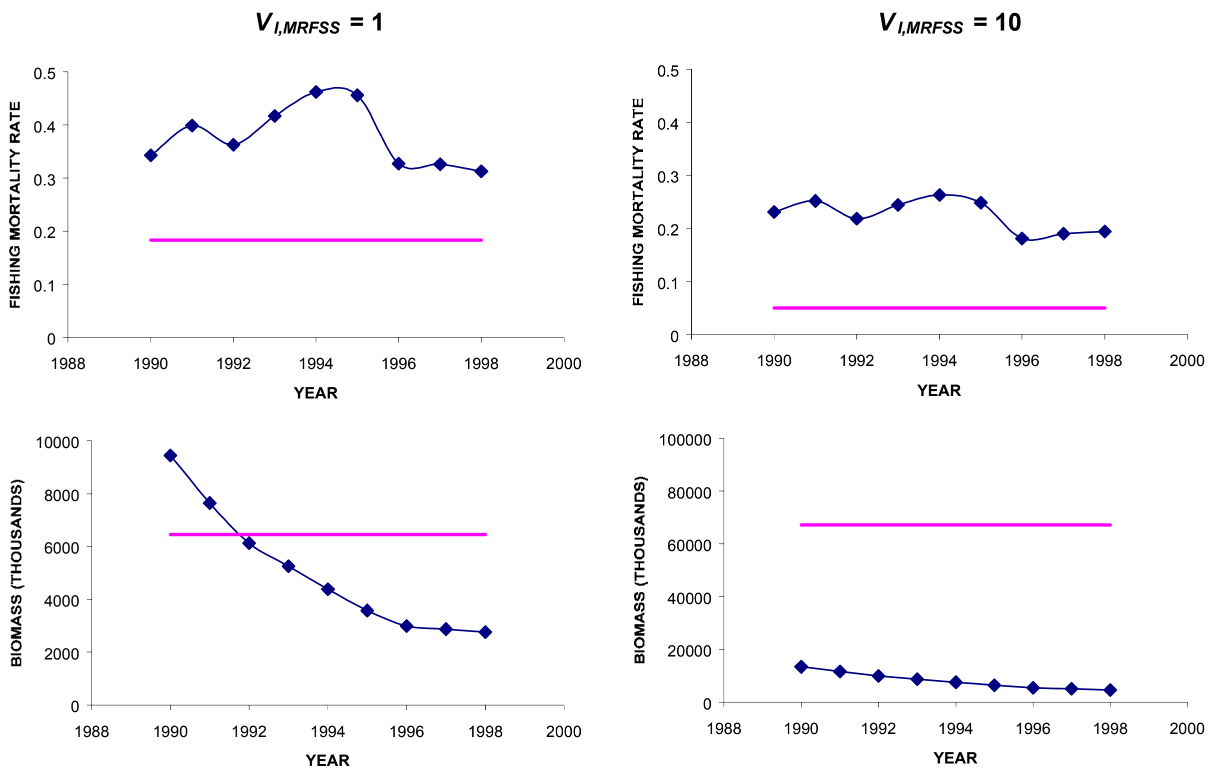

Figure 4a. Fishing mortality rate and biomass estimates for the short (1990-1998) series. The diamonds represent the historical estimates and the solid lines represent F_{MSY} and B_{MSY}.

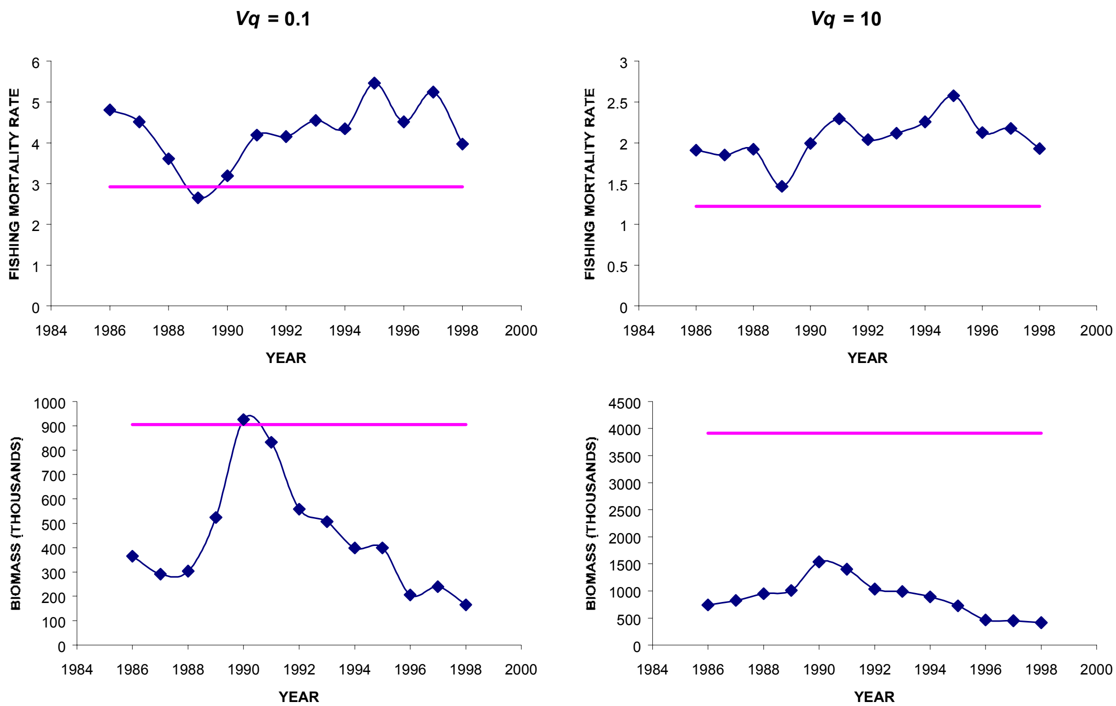

Figure 4b. Fishing mortality rate and biomass estimates for 1986-1998 when the variance of the MRFSS CPUE index was assumed to be the same as that of the MRFSS catch ($V_{I,MRFSS}$=1.0). The diamonds represent the historical estimates and the solid lines represent F_{MSY} and B_{MSY}.

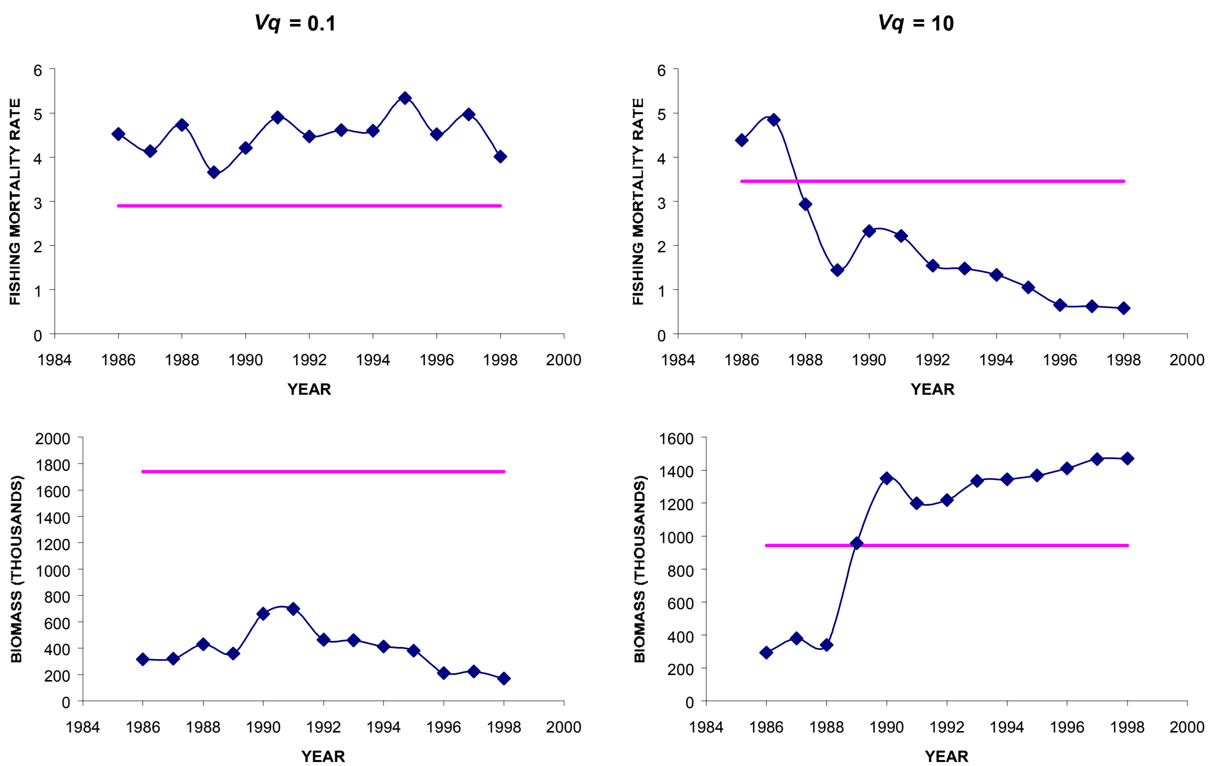

Figure 4c. Fishing mortality rate and biomass estimates for 1986-1998 when the variance of the MRFSS CPUE index was assumed to be the same as for the headboat and commercial CPUE indices ($V_{I,f}$ = 10). The diamonds represent the historical estimates and the solid lines represent F_{MSY} and B_{MSY}.

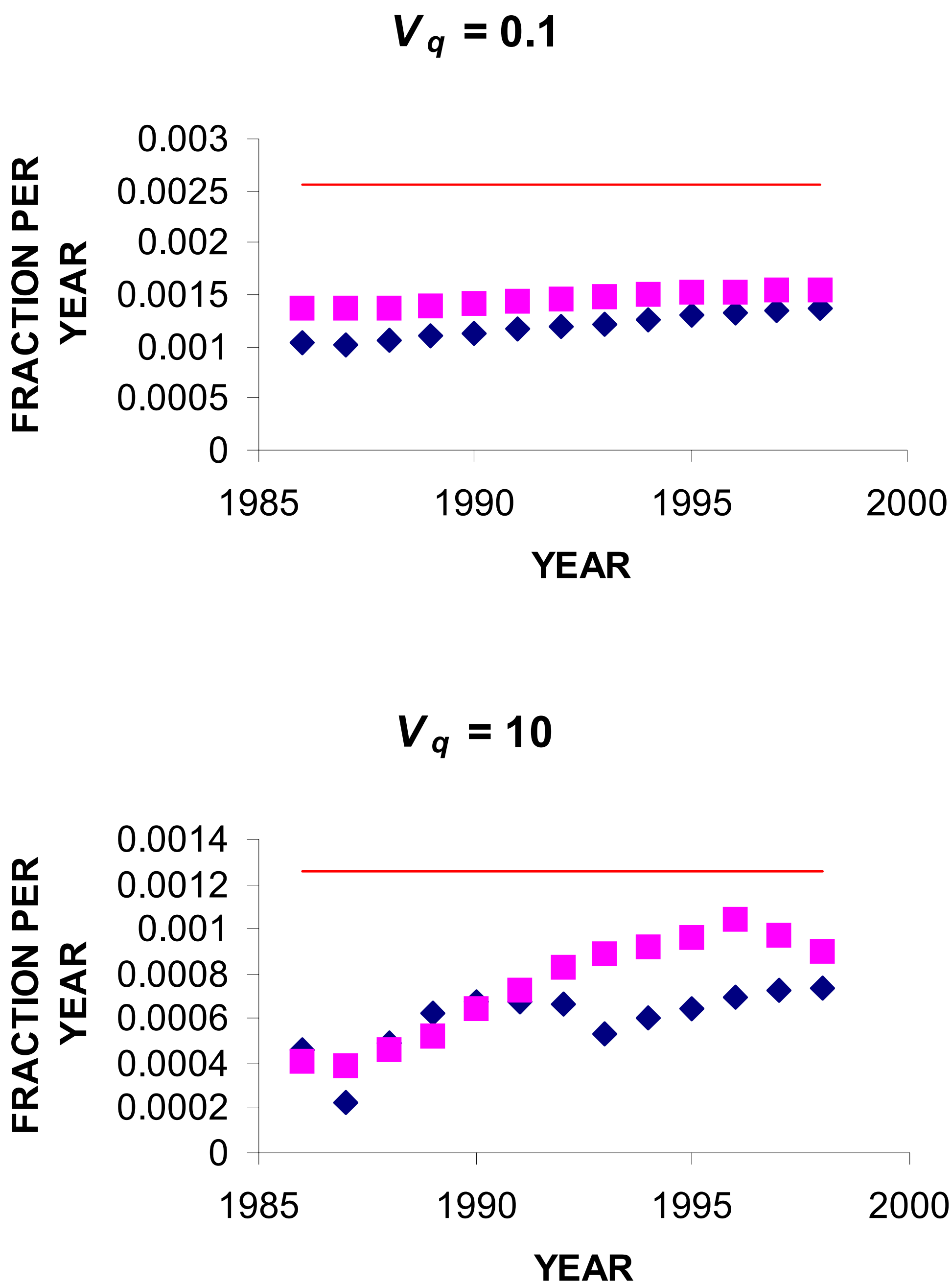

Figure 5a. Estimated catchability coefficients for the MRFSS (squares), Headboat (diamonds), and Commercial (line) fleets when the variance of the MRFSS CPUE index was assumed to be the same as for the corresponding catch ($V_{I,MRFSS}$=1.0).

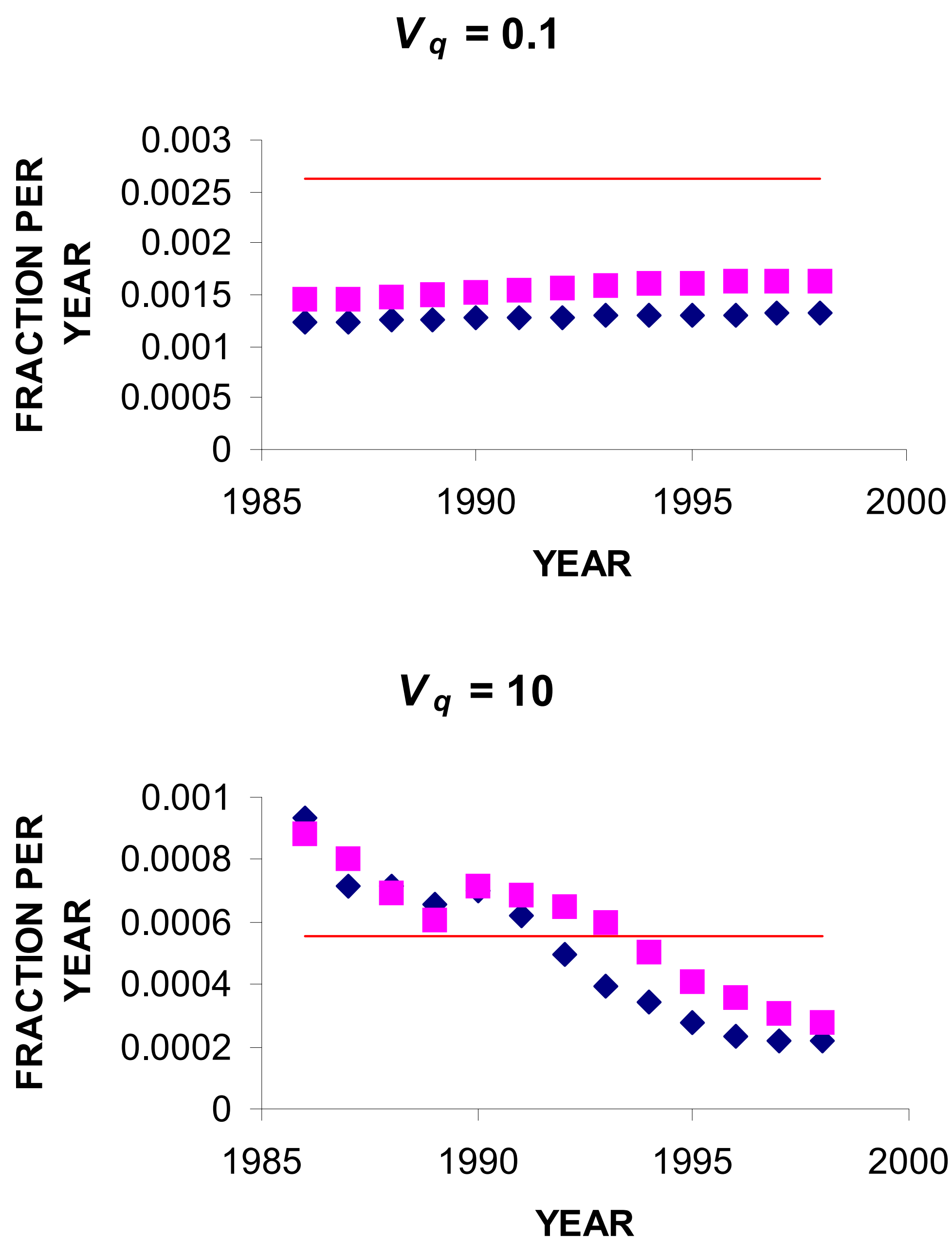

Figure 5b. Estimated catchability coefficients for the MRFSS (squares), headboat (diamonds), and commercial (line) fleets when the variance of the MRFSS CPUE index was assumed to be the same as for the headboat and commercial CPUE indices ($V_{I,f}$= 10).

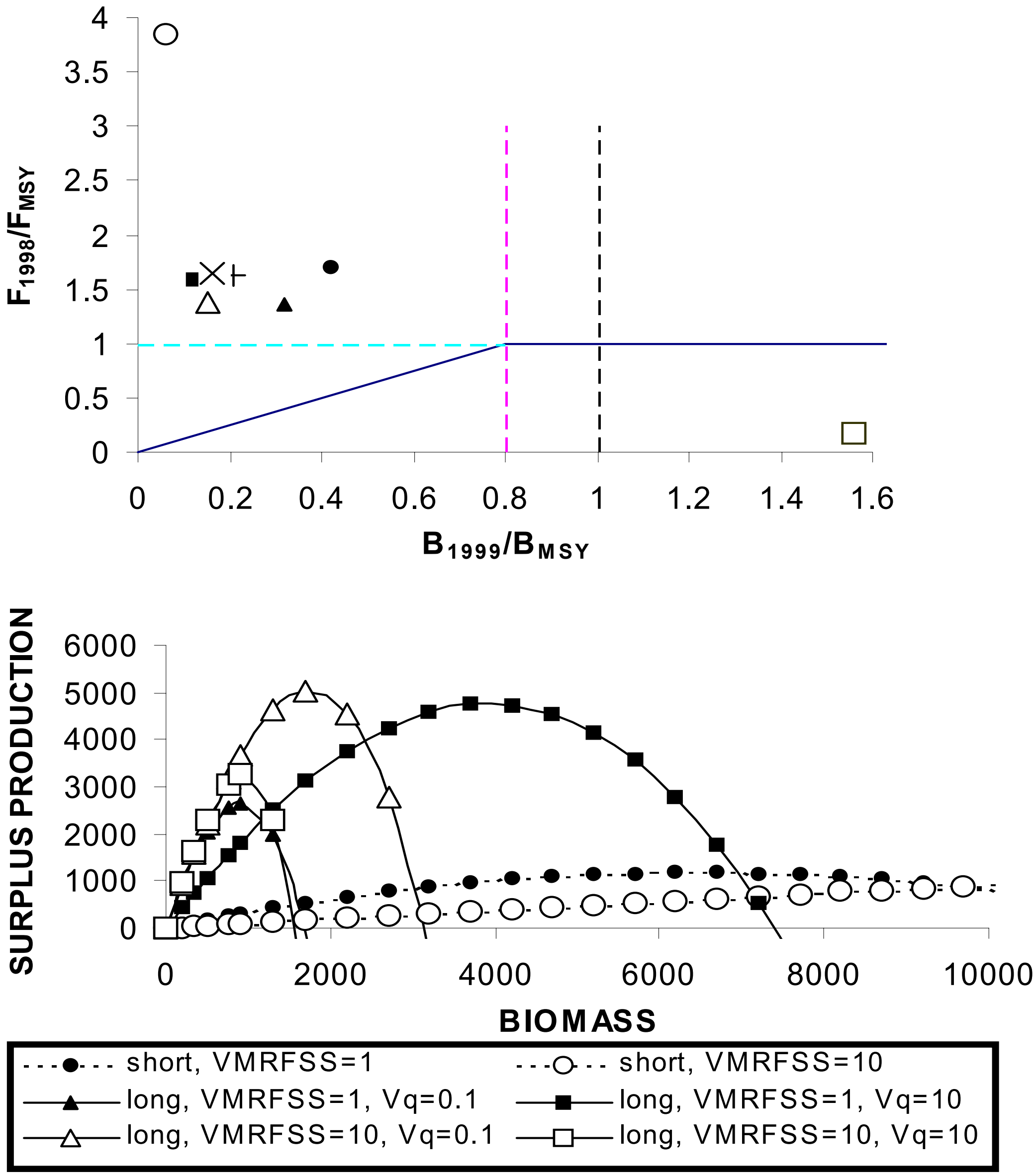

Figure 6. Phase plot of estimated stock status with respect to a hypothetical control rule (top) and surplus production curves (bottom) corresponding to the six models examined. Large hollow symbols represent models with large $V_{I,MRFSS}$ (=10) and small solid symbols represent models with small $V_{I,MRFSS}$ (=1.0). Circles represent the short time series. Triangles and squares represent the long time series with V_q=0.1 and 10.0, respectively. The X and + symbols represent models 1 and 2 of Valle et al. (2001).

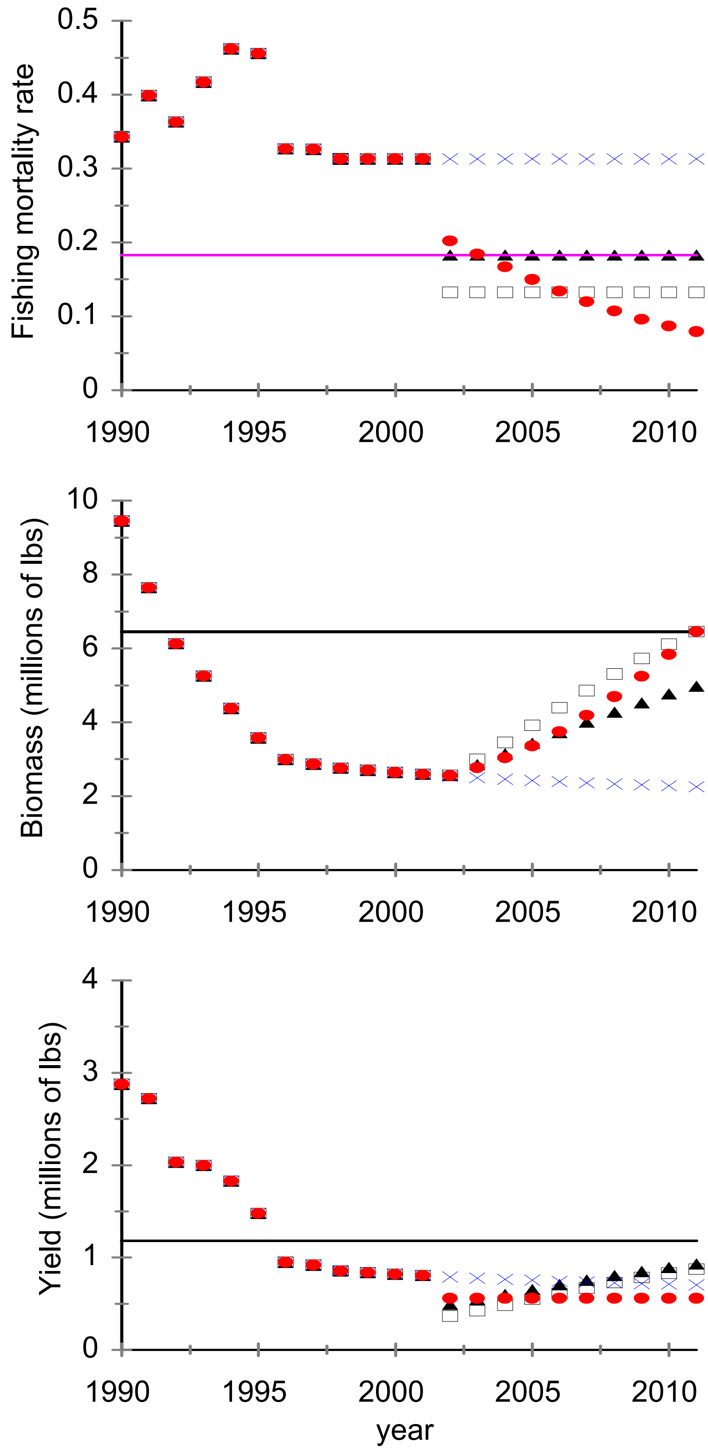

Figure 6. Projections of the short time series model with low MRFSS CPUE variance. The symbols denote fishing at current rates F_{CURR} (X's), fishing at F_{MSY} (triangles), fishing a t a rate that would allow the stock to recover to B_{MSY} within 10 years $F_{RECOVER}$ (squares), and the constant harvest that will allow recovery to B_{MSY} in ten years $Y_{RECOVER}$ (solid circles).