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**Identifying some approaches to formulating prior probability distributions for natural mortality rates in age zero and age one Gulf of Mexico red snapper**

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## Executive Summary

1. This paper reviews four alternative approaches that could be taken to formulate a prior probability density function for the parameters natural mortality rate for age 0 and age 1 Gulf of Mexico red snapper.
2. Of the approaches, only one appears to offer possibilities. This approach has been applied in other similar circumstances when both expert judgment and observational studies exist that provide a basis to make judgements about the plausibility of factors impacting the values for the parameters of interest and several articles in the peer reviewed literature have already applied this approach. The applications, however, have been to develop priors for the constant of proportionality in relative abundance indices that scale absolute abundance to the values obtained for the abundance indices.
3. A Monte Carlo simulation model is suggested that incorporates results from three different empirical studies that provide approximations of the rate of natural mortality for age 0 and age 1 red snapper and takes into account statistical uncertainty in the empirical estimates and values for the key factors that affect the estimates. A set of key factors that affect the uncertainty and interpretation of the initial estimates of mortality rate provided by these studies are identified and potential ranges of values for them are proposed. Such factors include rates of immigration and emigration of red snapper to and from Dr. Szedlmayer's experimental sites, presumed rates of on site fishing mortality rates, and the ratio of catchabilities for the different surveys of juvenile abundance in regressions of survey catch rates to estimate total mortality rates.
4. Simple mathematical models that describe the potential relationships between the actual rates of natural mortality, the empirically estimated values for total mortality rate and the potential biasing factors are proposed for each of the empirical studies considered in this paper.
5. A Monte Carlo simulation procedure is proposed to compute probability distributions for the rates of natural mortality for age 0 and age 1 fish that integrates results from the three different empirical studies, takes into account key uncertainties and attempts to maintain consistency in assumptions about potential rates of decay in the rate of natural mortality with age and in presumed fishing mortality rates in each of the empirical studies.
6. An alternative protocol is also suggested in which the results from the Monte Carlo analyses are updated sequentially such that the output distribution of the first analysis of Dr. Szedlmayer's field study results becomes the prior probability density function pdf for the analysis of Nichol's regression results for survey catch rates of autumn age 0 and summer age 1 fish. The output distribution from the combination of Szedlmayer's and Nichol's analysis becomes the prior pdf for the analysis of Goodyear's regression results for survey catch rates of summer age 1 fish and autumn age 1 fish. If the output distribution at each stage of the analysis is parameterized as a multi-variate lognormal distribution of the parameters of interest, then these pdfs can be treated as conjugate priors to the next stage of analysis. The final output distribution summarized as a multivariate lognormal pdf for  $M_0$  and  $M_1$  thus could serve as the prior pdf for the stock assessment.

## Introduction

One of the sources of uncertainty in the Gulf of Mexico red snapper (*Lutjanus campechanus*) stock assessment is over the values for natural mortality rate in age 0 and age 1 fish ( $M_0$  and  $M_1$ ). Since the mid 1990's the values applied have been 0.5 and 0.3 yr<sup>-1</sup>, respectively (Goodyear 1995; Schirripa and Legault 1999). There were extensive discussions about the parameters in the April 2004 SEDAR workshop and the workshop report covers this topic in considerable detail (SEDAR7 2004). One of the recommendations in the review report is for a prior probability distribution for these parameters to be formulated to take into account findings from various studies that may shed light on plausible values for  $M_0$  and  $M_1$  for red snapper and take into account uncertainty in values for these parameters. This note identifies and reviews some alternative approaches to formulating prior probability density functions (pdfs) for the rate of natural mortality for red snapper.

There are at least four potential approaches that could be taken to formulate a prior for a parameter in a fish population dynamics model such as the rate of natural mortality. One approach is to formulate a non-informative prior pdf which reflects total ignorance or near total ignorance about the parameter (called hereon a “non-informative prior”) (Box and Tiao 1972). A second approach is to identify subjectively from expert judgement a prior pdf based on biological experts on the species who are most well-qualified to make judgements about values for the parameters of interest (hereon called a “subjective-expert judgement prior”) (Uusitalo 2001; ICES 2004). A third approach is to apply hierarchical probability modelling to datasets from related fish stocks to model the cross stock variability in the parameter and the value for the same parameter in the different populations (Gelman et al. 1995; Liermann and Hilborn 1997; Michielsens and McAllister 2004). The modelling approach permits the computation of a posterior predictive distribution for the parameter in some yet to be sampled population. The posterior predictive distribution then may serve as the prior pdf for the fish stock of interest (hereon called a “hierarchical posterior predictive prior”). A fourth approach is to summarize all empirical information that may be used to formulate plausible values for the parameter and to construct a Monte Carlo simulation model that probabilistically utilizes the various sources of information, probabilistically integrates the information and summarizes the results in an output distribution for the parameter of interest (Punt et al. 1993; McAllister and Ianelli 1997; Boyer et al. 2001). This output distribution then serves as the prior pdf for the parameter in stock assessments (hereon called a “Monte Carlo prior”). The various pros and cons of each of these alternative approaches and their potential merits for the formulation of a prior pdf for  $M_0$  and  $M_1$  for red snapper are reviewed next. Where one of these approaches has apparent merits, some suggestions will be provided on how it might be applied to formulate a prior pdf for  $M_0$  and  $M_1$ .

## Review of different types of priors

### 1. Non-informative prior

Non-informative priors are most appropriate when there is truly little information about the possible values for the parameter of interest and it is expected that there will be sufficient information in the data to which the stock assessment model is to be fitted to be able to estimate precisely the parameter in question (Box and Tiao 1973). This type of prior has often been applied to constants of proportionality in relative abundance indices and variance terms in likelihood functions (Punt et al. 1993; Walters and Ludwig 1994). This type of prior is not commonly applied to natural mortality rate ( $M$ ). This is because the data to which stock assessment models are fitted, e.g., relative abundance index data and catch-age data rarely contain much information in them about  $M$ . If a model is fitted to mark recapture data, the tagging experiment is well-designed and the tag return rate high, then the data may be

informative about the rate of natural mortality. In such a case, a non-informative prior for  $M$  might be appropriate. There exist some mark recapture data for red snapper for age 2 and older but few if any on age zero or age one fish (Schirripa and Legault 1999). Thus a non-informative prior for  $M_0$  and  $M_1$  for red snapper appears to be out of the question.

## 2. Subjective-expert judgement prior

In instances in which fisheries biologists have spent considerable time studying some fish species, they will have developed knowledge of the species and population and others like it to make reasoned judgments about the plausibility of different values for some biological parameter such as the rate of natural mortality (Uusitalo 2001; ICES 2003). This is the case for example among salmon biologists where considerable experience has developed in studying the post smolt mortality rates between hatchery raised and wild anadromous salmonids. Tagging experiments for various Pacific salmon stocks have facilitated the estimation of post smolt mortality rates for these stocks. In such instances, salmon biologists have been in a position to formulate prior probability distributions for post smolt survival rates for wild and hatchery raised salmon stocks for which relatively little data exist (e.g., Baltic salmon) (ICES 2003). Despite the existence of estimates for Pacific salmon, it has been recognized that there remains considerable uncertainty for other salmon stocks, so the priors developed, e.g., for post smolt mortality rates in wild and hatchery-reared Baltic salmon have been quite diffuse, e.g., with prior coefficients of variation larger than 0.5. This implies more than about 10-fold differences between lower and upper 95% prior probability intervals for the parameter. However, it appears that for the rate of natural mortality in juvenile red snapper, few if any such studies exist for other similar populations. Thus, the formulation of a prior for natural mortality rates ( $M$ ) in juvenile red snapper based solely on expert subjective judgement appears to be inappropriate.

Where aging data exist, this permits the formulation of plausible values for  $M$  or the total rate of mortality ( $Z$ ) for mature fish as has already been done for red snapper (Schirripa and Legault 1999). However, such data do not permit judgement about plausible values for  $M$  or  $Z$  for immature fish.

## 3. Hierarchical posterior predictive prior

This approach requires analogous datasets from several different populations that are similar to the one of interest (Gelman et al. 1995). This approach has been applied several times to stock-recruit datasets for groups for similar fish stocks (Liermann and Hilborn 1997; Michielsens and McAllister 2004). The hierarchical models formulated model the parameter estimates for each population but also the mean and variance in the parameter estimates across populations. The set of parameters in the hierarchical model is utilized to compute a posterior predictive distribution for a parameter of interest in a yet unsampled population. This posterior predictive pdf thus becomes the prior pdf for the parameter in the “new” population of interest. This approach is perhaps one of the most empirically-based and sophisticated approaches available to formulate prior pdfs. It requires exchangeability among populations (Gelman et al. 1995) such that the value of the parameter from one of the populations is like a random draw of the parameter from the set of all conceivable populations that have similarities with respect to that parameter. Covariates such as latitude may be utilized to improve the predictive power of this approach. No such datasets for the rate of natural mortality in other similar populations exist for red snapper. Thus, this approach is not available for formulating a prior pdf for  $M_0$  and  $M_1$  for red snapper.

#### 4. Monte Carlo prior

In some instances, indirect observational and experimental information exist that may be drawn upon to develop a Monte Carlo probability model that includes a model of variables contributing to the quantity of interest and empirically or expert-based formulations of prior pdfs for the contributing variables. This approach has been applied several times to develop prior pdfs for constants of proportionality ( $q$ ) in relative abundance indices. One of the first applications developed a prior pdf for  $q$  for the hydro acoustic index of abundance of New Zealand hoki (Punt et al. 1993). Expert judgement was applied to formulate pdfs for several quantities such as target strength, species identification, and bottom back-scatter. These were treated as independent random variables and draws were taken from their distributions. The drawn values were inputted into an equation that related them to  $q$ . The resulting pdf for  $q$  had a coefficient of variation of about 0.7 and this pdf was utilized as an informative prior pdf for  $q$  in the stock assessment. Similar protocols have been applied to formulate priors for  $q$  for research trawl indices of abundance in the Eastern Bering Sea (McAllister and Ianelli 1997) and for hydroacoustic, research trawl and commercial trawl indices of abundance for Namibian orange roughy (Boyer et al. 2001) and spawner abundance indices for north east Atlantic mackerel based on pelagic egg surveys of abundance.

While not entirely analogous, there exist some research datasets for Gulf of Mexico red snapper that could be utilized in a similar manner to formulate a prior pdf for  $M_0$  and  $M_1$ . Approximations of the total mortality rate of age 1 fish ( $Z_1$ ) in the 2<sup>nd</sup> to 3<sup>rd</sup> quarter of the 2<sup>nd</sup> year of life have been obtained by analyzing the relationship between Fall Groundfish indices of age 1 fish and summer SEAMAP indices of age 1 fish (Goodyear 1995; SEDAR7-DW-3). Some recently implemented field studies for example by Szedlmayer (SEDAR7 DW-21) and Rooker et al. (2004) provide direct observations of fish densities of age 0 and age 1 fish on numerous field sites. Szedlmayer provided some estimates of total mortality rates for age 0 fish based on these observations. Yet there remain many sources of uncertainty in the estimates of mortality from all of these studies. The following section reviews the various uncertainties and limitations in these studies relating to  $M_0$  and  $M_1$  and suggests some potential approaches to modelling these uncertainties to compute a prior pdf for  $M_0$  and  $M_1$  that could be utilized in a stock assessment.

#### **Monte Carlo Models to formulate prior pdfs for $M_0$ and $M_1$ .**

##### Using Szedlmayer's results

First I will review the uncertainties and limitations in Szedlmayer's study (SEDAR7 DW-21). Descriptions of this study are provided in (SEDAR7 DW-21) and the SEDAR7 workshop report (SEDAR7 2004). The following two paragraphs are an excerpt from the April SEDAR review report summarizing limitations of Szedlmayer's study.

The new estimate of  $M_0$  may be biased by immigration and emigration of age 0 fish to or from the artificial reef sites. Net immigration from, or net emigration to, the site following the initial July measurement, could bias the estimates of  $M_0$  either too high or too low. Dr. Szedlmayer indicated however, that tagging of some individuals on the sites indicated some emigration and migration but not extensive rates of either [but the key question here is whether one can differentiate between mortality and migration, and it is not clear that a localized tagging experiment can make this distinction]. The habitat upon which the estimates were obtained is only one of several different types of habitat upon which age 0 fish are known to settle. This rate of natural mortality here thus might not be representative of the mean rate of natural mortality of age 0 fish when all habitats

are considered. For example, a study of post-settlement red snapper in Texas showed that they grew faster and has lower mortality rates when living on nutrient-rich inshore mud bottoms than on shell ridges or offshore mud bottom (Landre, unpublished results), which could suggest the shell reef-based estimates could be too high[.] Dispersion of age 1 fish off the reefs would also bias the estimates towards being too high.

Should assessment scientists wish to consider Szedlmayer's estimate further some additional considerations will need to be addressed. For example,  $M_0$  in the stock assessment represents newly settled fish up to the end of the year. This may on average represent fish from a settlement in July to December. Thus  $M_0$  would need to represent on average the rate of natural mortality for the first six months of settlement. The value derived from Szedlmayer's data represents  $M$  for the first year and thus would need to be partitioned between year 0, which encompasses the first six months, and the first half of year 1. Some assumed value would still be required to provide an annual value for  $M$  for the second half of year 1. Because it is generally regarded that the rate of natural mortality in newly settled juvenile fish must decay as the fish grows larger (as indicated in Rooker et al. 2004), then it is likely that the rate of natural mortality for the first six months of settlement would be larger than for the second six months. Thus rather than dividing Szedlmayer's value by two to obtain the value of  $M_0$  for the first six months, some larger fraction of Szedlmayer's value might be assumed for the first six months. For example, if a value 67% of Szedlmayer's value was assumed for the first six months, then a value 33% of that would be assumed for the following six months. A value smaller than 33% Szedlmayer's value would need to be assumed for the latter six months of the first year. This example ... is intended only as an illustration of how  $M_0$  might be partitioned and a decay in  $M$  with size consistently incorporated.

Based on such considerations, it appears that there are several factors that could be employed to utilize Szedlmayer's study results to formulate a prior pdf for  $M_0$ . These include a factor for immigration of age 0 fish onto the site following the density observation in July,  $I_0$ , that would tend to decrease Szedlmayer's estimate of  $Z_0$  from the true annual value. Hence values for  $I_0$  would be less than 1 and larger than 0. A factor for emigration of age 0 fish away from the site following the density observation in July,  $E_0$ , would tend to bias Szedlmayer's estimate of  $Z_0$  too high. Hence values for  $E_0$  would be larger than 1 with an upper limit to be identified. A habitat suitability index,  $H$ , would reflect the relative value for the natural mortality of the shell and concrete artificial reef sites compared to the range of other available habitats. Based on discussions in SEDAR7 (2004), there are some indications that  $H$  may be larger than 1 due to higher survival rates observed elsewhere. However, due to the relative scarcity of sampling across many different types of settlement habitats,  $H$  could conceivably also be less than 1. A parameter,  $\sigma_s$ , would reflect the standard deviation in the estimate of  $Z_0$  and could be set equal to the value obtained from Szedlmayer's study. A factor,  $T$ , would reflect the portion of the annual rate of the estimate of total annual natural mortality rate  $M_0$  that would apply to the first six month of settlement from July until December of the first year. As mentioned in SEDAR7 (2004),  $M_0$  in the first six months of life would be a larger than  $M_0$  in the second six months. Thus  $T$  should be greater than 0.5 and less than 1 with a possible most likely value, for example, at about 0.67.

The factor  $F_0$  reflects the annual fishing mortality rate contributing to the total mortality rate ( $Z_0$ ) estimated by Szedlmayer. The value for  $F_0$  would be the expected average annual value for fishing mortality rate of age 0 fish from the first July to the first June during the course of

the field study. SEDAR7 (2004) suggested that Szedlmayer's estimate could still reflect a small value for  $M_0$  (e.g., 0.5) if fish went off of the artificial reefs at night to forage and many of these fish were captured in shrimp trawls. However, stakes put up around the artificial reefs indicated that no shrimp trawls fished in the vicinity of the reefs because none of the stakes were knocked down on any of the experimental sites. This makes it unlikely that fishing mortality rate could account for much of Szedlmayer's estimate of  $Z_0$ . Thus experimental evidence suggests that  $F_0$  on Szedlmayer's study sites may be very small. A factor for the dispersal of age 1 fish from the site ( $E_1$ ) before the end of the first year would tend to inflate the estimate of  $M_0$ . A factor for immigration of age 1 to the site before the end of the first year ( $I_1$ ) would decrease the estimate of  $M_0$ . A factor  $A_0$  would lower the observed rate of total mortality rate for age 0 fish that had settled earlier than July due to the known decay in  $M$  with age. Thus, the potential values for  $A_0$  would be less than 1.

The resulting model relating Szedlmayer's estimate of total annual mortality rate of age 0 fish on his site to the average value for  $M_0$  for red snapper in the Gulf of Mexico for the months until December in the first year is as follows:

$$(1) \quad \hat{Z}_0 = ((M_0 \times H) + F_0) \times I_0 \times E_0 \times A_0 \times I_1 \times E_1 \times \exp(\varepsilon)$$

where  $\varepsilon \sim Normal(0, \sigma_s^2)$

Rearranging to solve for  $M_0$ , and taking into account  $T$ , the fraction of  $M_0$  in the first six months, the natural mortality rate up until the end of December in the initial year of life for the average site, this becomes

$$(2) \quad M_{0(1-6)} = \frac{T}{H} \times \left( \frac{\hat{Z}_0}{I_0 \times E_0 \times A_0 \times E_1 \times I_1 \times \exp(\varepsilon)} - F_0 \right)$$

The value for  $\hat{Z}_0$  applied in equation 2 is fixed at Szedlmayer's estimate from his study (2.3 yr<sup>-1</sup>). A value for  $M_0$  in the second six months (the first January to the first June) can be obtained by:

$$(3) \quad M_{0(7-12)} = (1 - T)M_0 = (1 - T) \frac{M_{0(1-6)}}{T}$$

A model and empirical basis for the instantaneous natural mortality in the second half of the second calendar year for age 13 months to 18 months is however still needed. As mentioned above, the value for  $M_{1(13-18)}$  from the second July to the second December in the fish's second year of life should be no larger than  $M_{0(7-12)}$ . It is suggested that the regression estimates of total mortality rate using the summer SEAMAP catch rates and autumn groundfish survey catch rates could provide an empirically based probabilistic approximation for  $M_{1(13-18)}$  from July to December (see below).

Providing that probability pdfs can be constructed for each of the input parameters on the right hand side of equation 2 ( $T, H, I_0, E_0, A_0, E_1, I_1$  and  $F_0$ ), this model will permit the computation of a pdf for  $M_0$  in the first six months and a value for  $M_1$  in months seven to twelve,  $M_{0(7-12)}$  or  $M_{1(7-12)}$  if we take the subscript before the parenthesis to indicate calendar year age.

A study by Rooper et al. (2004) provided an estimate of 0.12 d<sup>-1</sup> for days 20-30 after settlement. It is expected that the rate of mortality decays with age and the rate of decay remains highly uncertain, especially in the earliest phases within the first year of life. Thus

the usefulness of this estimate in formulating a prior pdf for  $M_0$  in the first six months is dubious. This value could potentially be applied to indicate an upper bound for  $M_0$  in the first six months. The dubious expansion of the value to the first half year provides a very high estimate of  $M_0$  of 21.9 for these first six months. This is an extremely high estimate and may give higher plausibility to high estimates of  $M_0$  rather than low estimates. The formulation of a quantitative method to include this particular estimate in the construction of a prior pdf for  $M_0$  however remains unclear to me.

#### Using results from analyses of survey catch rates of age 0 and age 1 fish

A similar Monte Carlo protocol to that described for Szedlmayer's results could be applied to utilize results of analyses of catch rates between successive surveys of age 0 and age 1 fish to compute a probability distribution for natural mortality rate in the second half of the first calendar year  $M_{1(13-18)}$  (Goodyear 1995; SEDAR7 DW-2). For example, Goodyear (1995) provided an approximation of  $Z_1$  by regressing Autumn groundfish survey catch rates of age 1 fish on summer SEAMAP survey catch rates of age 1 fish. SEDAR7 DW2 provided approximations of  $Z_0$  between autumn survey and summer surveys by regressing the summer survey catch rate of age 1 on the autumn survey catch rate of age 0 fish. This latter analysis could be utilized to make further inferences about plausible values for natural mortality rate in the first half of the second calendar year,  $M_{1(7-12)}$ .

Goodyear's (1995) regression has provided an approximation of the average total mortality rate of age 1 fish for the mid-second trimester to the mid-third trimester. The following protocol relates the annual rate of natural mortality between months seven (first January) to eighteen (second December) to the regression estimate of the total mortality rate between the two surveys. The unmodified regression estimate of total mortality rate assumes that the catchability of the two surveys is the same. A factor,  $R_q$ , can be formulated to reflect the potential value for the ratio of catchability of the autumn survey to the summer survey. The estimated regression slope was 0.473 and suggests a value for  $Z$  for this trimester of 0.75. If the rate of natural mortality were the same from month seven to month eighteen, then this could be expanded to an annual estimate of  $2.25 \text{ yr}^{-1}$ . However, since the value is derived from observations between the mid second and mid third trimesters, the estimate could be expected to be lower than the annual average value for  $M_1$ . The expected ratio of the estimate of the average annual rate of natural mortality  $M_1$  derived from this trimester period to the true average value for natural mortality over the year could be given the factor  $V$ . Because the instantaneous rate of natural mortality is expected to decay with age in the first few years of life and the estimate is derived in the later part of the period of interest, the value for  $V$  could be expected to be less than 1 and the lower bound would need to be agreed. Over and above, the factor  $V$ , the total annual mortality rate must be decreased by a factor of 1/3 to match the fraction of the year represented by this regression estimate. The presumed average total fishing mortality rate over this period over the time series is represented by  $F_{F1,S1}$ . Also, due to variability in the catch rate data there is error in the estimate of the slope,  $\varepsilon_{b,S1,F1}$ . The variance in this error, is delimited by  $\sigma_{b,S1,F1}^2$ .

The estimated slope, based on the regression of autumn groundfish age 1 on summer SEAMAP age 1 catch rates can be related to the annual rate of natural mortality as follows:

$$(4) \quad \hat{b}_{F1,S1} = \exp\left(-\left(\frac{(\bar{M}_{1,1} \times V)}{3} + F_{F1,S1}\right)\right) \times R_{q,S1,F1} + \varepsilon_{b,S1,F1}$$

where  $\varepsilon_{b,S1,F1} \sim \text{Normal}(0, \sigma_{b,S1,F1}^2)$



Given this model for the estimate of the slope,  $\hat{b}_{F1,S1}$ , the value for the average natural mortality rate can be modelled using:

$$(5) \quad \bar{M}_{1,1} = -\frac{3}{V} \left( \log \left( \frac{\hat{b}_{F1,S1} - \varepsilon_{b,F1,S1}}{R_{q,F1,S1}} \right) + F_{F1,S1} \right)$$

subject to  $\hat{b}_{F1,S1} - \varepsilon_{b,F1,S1} > 0$  and the term inside of the widest bracket being less than zero.

The value applied for  $\hat{b}_{F1,S1}$  in equation 5 is fixed at the regression estimate (e.g., 0.473). Providing that probability pdfs can be constructed for each of the input parameters on the right hand side of the equation ( $V$ ,  $R_{q,F1,S1}$ , and  $F_{F1,S1}$ ), this model will permit the computation of a pdf for  $\bar{M}_{1,1}$ . Further equations to model a value for natural mortality in the second half of the second calendar year ( $M_{I(13-18)}$ ) are provided further below.

Similar issues regarding the interpretation of the mortality estimates provided by regressions of summer survey catch rates of age 1 fish on autumn catch rates of age 0 fish apply in the formulation of a probability distribution for natural mortality rate in the first half of the second calendar year ( $M_{I(7-12)}$ ). Presumably the period covered by these surveys is from November of the first calendar year until June of the second calendar year, about eight months. As an approximation of instantaneous total annual mortality for fish from January to December in their first and second years, this estimate could be expected to be too high. The factor  $W$  thus represents the ratio of the total annual natural mortality rate obtainable in the regression to the actual average total annual natural mortality rate from January to December in the first and second year of life. Because the instantaneous rate of natural mortality is expected to decay with age in the first few years of life and the estimate is derived in the earlier part of the period of interest, the value for  $W$  could be expected to be more than 1 and the upper bound would need to be agreed. Over and above the factor  $W$ , the total annual mortality rate must be decreased by a factor of 2/3 to match the fraction of the year represented by this regression estimate. The presumed average total fishing mortality rate over this period over the time series is represented by  $F_{S1,F0}$ .

The estimated slope, based on the regression of summer SEAMAP age 1 catch rates on autumn groundfish age 0 catch rates can be modelled as follows:

$$(6) \quad \hat{b}_{S1,F0} = \exp \left( - \left( \frac{2}{3} \times \bar{M}_{1,2} \times W + F_{S1,F0} \right) \right) \times R_{q,S1,F0} + \varepsilon_{b,S1,F0}$$

where  $\varepsilon_{b,S1,F0} \sim Normal(0, \sigma_{b,S1,F0}^2)$

Given this model for the estimate of the slope,  $\hat{b}_{S1,F0}$ , the value for the average natural mortality rate for the second calendar year can be modelled also using:

$$(7) \quad \bar{M}_{1,2} = -\frac{3}{2 \times W} \left( \log \left( \frac{\hat{b}_{S1,F0} - \varepsilon_{b,S1,F0}}{R_{q,S1,F0}} \right) + F_{S1,F0} \right)$$

subject to  $\hat{b}_{S1,F0} - \varepsilon_{b,S1,F0} > 0$  and the term inside of the widest bracket being less than zero.

Thus equations 5 and 7 provide two related protocols to compute distributions for the rate of natural mortality from months seven to eighteen,  $\bar{M}_1$ . Equations 5 and 7 also utilize data from the same surveys and hence are not truly independent sources of information about  $\bar{M}_1$ . If it was desired to take this dependence into account, it could be possible to bootstrap the annual survey observations such that the years of datapoints drawn are selected at random with replacement, and the slopes for the two regressions then estimated. The values for the other random variables could then be drawn. The presumed values for the fishing mortality rates in both equations might also be seen to be positively correlated. Thus, a bivariate distribution for these two fishing mortality rate values could be specified with a positive correlation. There would be no need to simulate values for the regression error term because the bootstrapping would take this uncertainty into account. To maintain consistency in the assumption about the decay in the rate of natural mortality with age, it may also be appropriate to assume a negative correlation between  $V$  and  $W$ , since if one is high, it is likely that the other is low and vice versa.

One possible approach to obtaining a pdf for the rate of natural mortality in the second draw that would use the results from equations 5 and 7 would be as follows. For each draw that results in a computed value for  $\bar{M}_{1,1}$  and  $\bar{M}_{1,2}$  from equations 5 and 7, respectively, the average of the two values could be computed and saved. The histogram of saved values for each averaged  $\bar{M}_1$  could then be used as the prior pdf of  $\bar{M}_1$ . This however ignores the results from equation 3 for  $M_0$ .

#### Combining results from Equations 2,3, 5 and 7 to formulate a joint prior pdf for $M_1$ and $M_0$ .

Because the instantaneous rate of natural mortality is expected to decay within the first few years of life, the value for  $\bar{M}_1$  can be expected to be smaller than the value for  $M_0$ . If the method using Szedlmayer's results (Equation 2) is to be applied also to compute a prior for  $M_0$ , this method actually provides joint value for  $M_0$  (first July to first December) and  $M_1$  from the first January to the first June (age 7 months to 12 months).

The methods utilizing the catch rate data (Equations 5 and 7) could then provide probabilistic estimates of the value for  $M_1$  for age 13 months to 18 months  $\bar{M}_{1(13-18)}$  and also ages 7 to 12 months ( $\bar{M}_{1(7-12)}$ ), respectively. The latter value could be combined with the value for  $\bar{M}_{1(13-18)}$  provided in equation 3 to obtain an improved approximation for  $\bar{M}_{1(13-18)}$ . A simulation trial result for  $\bar{M}_{1(7-12)}$  however would be rejected, if it was larger than the concurrently simulated value for  $M_{0(1-7)}$ .

The computation of the half year mortality rate between ages 13 and 18 months,  $\bar{M}_{1(13-18)}$  could be obtained as follows. In each simulation of  $\bar{M}_{1,1}$  from Equation 5, the value  $V$  is drawn. The value  $V$  pertains to month eight in the second calendar year and application of it to  $\bar{M}_{1,1}$  could provide an estimate of the instantaneous natural mortality in the eighth month. However, we would like an approximation for the average instantaneous mortality rate in the second half of the second calendar year. This should thus pertain to month nine. Using a linear approximation of decay in  $M$ , the per month slope of the decay is given by:

$$(8) \quad m_{1,1} = \bar{M}_{1,1} \times \frac{(V-1)}{2}$$

The average instantaneous mortality rate in the second half of the calendar year can be approximated by the interpolated value for month nine:

$$(9) \quad \bar{M}_{1,1(13-18)} = \bar{M}_{1,1} + m_{1,1} \times 3$$

where the slope  $m_{1,1}$  is derived from equation 8. The half year mortality rate for ages 13 to 18 is approximated by halving the value in equation (9):

$$(10) \quad M_{1,1(13-18)} = 0.5\bar{M}_{1,1(13-18)}$$

The half year mortality for ages seven months to twelve months can be approximated by using results from equation 7 in an analogous manner to the use of results from equation 5. For each value computed for  $\bar{M}_{1,2}$  from Equation 7, the value  $W$  is drawn. The value  $W$  pertains to month two in the second calendar year and application of it to  $\bar{M}_{1,2}$  could provide an estimate of the instantaneous natural mortality in the eighth month. However, we would like an approximation for the average instantaneous mortality rate in the first half of the second calendar year. This should thus pertain to month three. Using a linear approximation of decay in  $M$ , the per month slope of the decay is given by:

$$(11) \quad m_{1,2} = \bar{M}_{1,2} \times \frac{(W-1)}{4}$$

The average instantaneous mortality rate in the first half of the calendar year can be approximated by the interpolated value for month three in the second calendar year:

$$(12) \quad \bar{M}_{1,2(7-12)} = \bar{M}_{1,2} + m_{1,2} \times 3$$

where the slope,  $m_{1,2}$ , is derived from equation 11.

The half year mortality rate for ages 7 to 12 months could be obtained by:

$$(13) \quad M_{1,2(7-12)} = 0.5\bar{M}_{1,2(7-12)}$$

Following the approximation of this quantity in equation 3 ( $M_{0(7-12)}$ ), equation 13 offers a second approximation of the half year mortality rate for ages 7 to 12 months ( $M_{1,2(7-12)}$ ). To obtain a single approximation for this quantity the results from equations 3 and 13 could be averaged to provide a synthesized approximation:

$$(14) \quad M_{1(7-12)} = 0.5(M_{0(7-12)} + M_{1,2(7-12)})$$

The annual natural mortality rate for the second year (ages 7 to 18 months) could be obtained by summing the results from equations 10 and 13.

$$(15) \quad M_1 = M_{1(7-12)} + M_{1,1(13-18)}$$

To achieve consistency in the assumptions about presumed fishing mortality rates, the values assumed for  $F_0$  and  $F_i$  in equations 2, 5 and 7 could be modelled to have a positive correlation. The value for  $T$  in the simulation of  $M_0$  could be assumed to be negatively correlated with  $V$  and positively correlated with  $W$  so that consistent assumptions are made about the potential decay in the rate of natural mortality with age. Thus a trivariate distribution between  $T$ ,  $V$ , and  $W$  could be assumed with the negative and positive correlations between the variables incorporated. Thus, once a value for  $M_0$  is drawn, a value for  $M_1$  in months seven to twelve is also provided with the use of the parameter  $T$ . Values for  $\bar{M}_{1,1}$  and

$\bar{M}_{1,2}$  could be simulated using equations 5 and 7 and utilizing the correlations between the various input parameters. If the value for  $\bar{M}_{1,1}$  or  $\bar{M}_{1,2}$  happens to be larger than that for  $M_{0(1-6)}$ , then these values for  $\bar{M}_{1,1}$  and  $\bar{M}_{1,2}$  could be discarded and another pair of values for  $\bar{M}_{1,1}$  and  $\bar{M}_{1,2}$  could be drawn until their values were lower than that for  $M_{0(1-6)}$ . The values for  $M_1$  for ages seven months to 12 months would then be added to the value for  $M_1$  for ages 13 to 18 months in equation 15. Thus, a joint distribution for  $M_0$  and  $M_1$  could be provided in which all of the values for  $M_1$  are lower than the paired values for  $M_0$ .

## Results

To illustrate how the first protocol might be applied, values for input parameters to equations 2, 5, and 7 were arbitrarily specified (Table 1) and a Monte Carlo simulation was run. The results are shown in Table 2 and Figures 1 to 8. With the many contributing sources of uncertainty incorporated the output distributions for  $M_0$  and  $M_1$  (Figures 3 and 8, Table 1) are very wide, as expected. Due to the multiplicative error factors, the distributions are skewed and appear to be lognormal. The output distributions for  $M_0$  and  $M_1$ , despite the arbitrary specification of input distributions, are centred over plausible values, however. It must be noted that the results displayed are strictly for the purposes of illustration and are not intended for use in stock assessment.

## Discussion

The most difficult part of this protocol would be to specify distributions for the uncertain parameters contributing to equations 2, 5 and 7. For example, it would be difficult to specify fishing mortality rate variables that are plausible but that ignore the empirical estimates in previous assessments. If values for  $F_0$  and  $F_1$  were derived from historic stock assessments, then incorporation of the resulting prior pdfs into the assessment would be effectively using the data twice. Thus, it would be best to specify relatively flat distributions for  $F_0$  and  $F_1$  that spanned a wide range of plausible values. Moreover, the values chosen for  $F_0$  and  $F_1$  would require that the values for  $M_0$  and  $M_1$  computed were positive.

One possible extension of the methodology would be one in which the results from the Monte Carlo analyses are updated sequentially such that the output distribution of the first analysis of Dr. Szedlmayer's field study results becomes the prior probability density function pdf for the analysis of Nichol's regression results for survey catch rates of autumn age 0 and summer age 1 fish. The output distribution from the combination of Szedlmayer's and Nichol's analysis becomes the prior pdf for the analysis of Goodyear's regression results for survey catch rates of summer age 1 fish and autumn age 1 fish. If the output distribution at each stage of the analysis is parameterized as a multi-variate lognormal distribution of the parameters of interest, then these pdfs can be treated as conjugate priors to the next stage of analysis. The final output distribution summarized as a multivariate lognormal pdf for  $M_0$  and  $M_1$  thus could serve as the prior pdf for the stock assessment.

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Table 1. Illustrative specification of pdfs for factors in equations 2, 5 and 7.

Model	Parameter	Brief description	Specifications
Equation 2 for $M_{0(1-6)}$	$Z_0$	Empirical estimate of total mortality rate from months 1 to 12	Set equal to Szedlmayer's estimate of $2.3 \text{ yr}^{-1}$
	H	Effect of habitat on this site specific estimate of natural mortality	$H \sim \text{Normal}(\text{mean} = 1.1, \text{standard deviation} = 0.2)$
	$F_0$	Presumed total annual rate of fishing mortality on age 0 in Szedlmayer's study	$F_0 \sim \text{Normal}(\text{mean} = 0.6 \text{ yr}^{-1}, \text{standard deviation} = 0.2 \text{ yr}^{-1})$ (Note could be much lower)
	$I_0$	Effect of immigration of age 0 fish after July on total mortality estimate	$I_0 \sim \text{Normal}(\text{mean} = 1.1, \text{standard deviation} = 0.2)$
	$E_0$	Effect of emigration of age 0 fish after July on total mortality estimate	$E_0 \sim \text{Normal}(\text{mean} = 0.9, \text{standard deviation} = 0.2)$
	$I_1$	Effect of immigration of age 1 fish before June on total mortality estimate	$I_1 \sim \text{Normal}(\text{mean} = 1.1, \text{standard deviation} = 0.2)$
	$E_1$	Effect of emigration of age 1 fish on before June total mortality estimate	$E_1 \sim \text{Normal}(\text{mean} = 0.9, \text{standard deviation} = 0.2)$
	$A_0$	The effect of settlement of age 0 fish before July on mortality estimate	$A_0 \sim \text{Normal}(\text{mean} = 0.95, \text{standard deviation} = 0.1)$
	$e_0$	The estimation error in Szedlmayer's estimate of total mortality rate	$e_0 \sim \text{Normal}(\text{mean} = 0, \text{standard deviation} = 0.4)$
	T	The fraction of total annual natural mortality presumed to occur in from age 0 to 6 months	$T \sim \text{Normal}(\text{mean} = 0.7, \text{standard deviation} = 0.1)$

Equation 5 for $M_{1(13-18)}$ .	$b_{12}$	The estimate of the slope in the regression of autumn survey age 1 fish on summer survey age 1 fish	$b_{12}$ is set equal to the regression estimate of 0.473 from Goodyear (1995).
	V	Ratio of instantaneous rate of natural mortality in month 8 of first calendar year to the average rate for the entire first year.	$V \sim \text{Uniform}(0.6, 0.9)$
	$F_{12}$	The fishing mortality rate in the second half of the first calendar year	$F_{12} \sim \text{Normal}(\text{mean} = 0.6, \text{standard deviation} = 0.2)$
	$R_{12}$	The ratio of catchability coefficients for the autumn survey and the summer survey for age 1 fish	$R_{12} \sim \text{Normal}(\text{mean} = 1, \text{standard deviation} = 0.2)$
	$e_{12}$	The error deviation in the estimate of the slope $b_{11}$ and the true slope.	$e_{12} \sim \text{Normal}(\text{mean} = 0, \text{standard deviation} = 0.1)$
Equation 5 for $M_{1(13-18)}$ .	$b_{11}$	The estimate of the slope in the regression of summer survey age 1 fish on autumn survey age 0 fish	$b_{11}$ is set equal to a possible regression estimate of 0.3 from Nichols (SEDAR7-3) (actual values for estimates not available at time of drafting of this paper).
	WV	Ratio of instantaneous rate of natural mortality in month 2 of first calendar year to the average rate for the entire first year.	$W \sim \text{Uniform}(1, 1, 1.4)$
	$F_{11}$	The fishing mortality rate from ages four months to 12 months	$F_{11} \sim \text{Normal}(\text{mean} = 0.6, \text{standard deviation} = 0.2)$
	$R_{11}$	The ratio of catchability coefficients for the summer survey age 1 and autumn survey for age 0 fish	$R_{11} \sim \text{Normal}(\text{mean} = 1, \text{standard deviation} = 0.2)$
	$e_{11}$	The error deviation in the estimate of the slope $b_{11}$ and the true slope.	$e_{11} \sim \text{Normal}(\text{mean} = 0, \text{standard deviation} = 0.1)$

Table 2. Summary statistics for some illustrative computations of proposed procedures to compute prior pdfs for  $M_0$  and  $M_1$ . Note that these results are purely illustrative and not intended for use in stock assessment.

**Statistics**

		$M_1$ from Goodyear regression	$M_1$ from Nichols regression	$M_0$ 1st 6 months Szedlmayer	$M_1$ 1st 6 months Szedlmayer	$M_1$ 2nd 6 months Goodyear ( $M_{12}$ )	$M_1$ 1st 6 months Nichols	$M_1$ 1st 6 months combined from Szedlmayer and Nichols ( $M_{11}$ )	$M_1$ final estimate from adding $M_{11}$ and $M_{12}$
N	Valid	647	981	989	989	647	981	970	970
	Missing	353	19	11	11	353	19	30	30
Mean		1.3921	1.0290	1.7814	.7577	.9980	1.1098	.9278	1.6734
Std. Error of Mean		.04017	.01746	.05143	.02515	.02249	.02019	.01569	.03782
Median		1.2158	.9506	1.3074	.5421	1.0064	1.0114	.8406	1.3864
Mode		.00(a)	.00(a)	.00(a)	.00(a)	-.26(a)	.00(a)	.05(a)	.10(a)
Std. Deviation		1.02170	.54684	1.61733	.79082	.57208	.63230	.48870	1.17789
Variance		1.04387	.29903	2.61576	.62540	.32728	.39981	.23882	1.38741
Skewness		1.276	1.305	2.941	3.539	.356	1.651	1.957	3.075
Std. Error of Skewness		.096	.078	.078	.078	.096	.078	.079	.079
Kurtosis		3.169	4.467	15.285	21.230	-.043	6.729	8.882	18.173
Std. Error of Kurtosis		.192	.156	.155	.155	.192	.156	.157	.157
Range		7.90	4.94	16.50	8.86	3.32	6.09	5.16	13.97
Minimum		.00	.00	.00	.00	-.26	.00	.05	.10
Maximum		7.90	4.94	16.50	8.86	3.05	6.09	5.21	14.07
Percentiles	10	.2490	.4186	.4278	.1564	.2357	.4286	.4263	.6646
	25	.5988	.6582	.7682	.2857	.5446	.6862	.5938	.9456
	50	1.2158	.9506	1.3074	.5421	1.0064	1.0114	.8406	1.3864
	75	1.9167	1.3224	2.2637	.9453	1.3843	1.4259	1.1486	2.0138
	90	2.7449	1.7572	3.6477	1.5693	1.6981	1.8922	1.4922	2.9304

a Multiple modes exist. The smallest value is shown



Figure 1. Frequency distribution of results for the rate of natural mortality for age 1 fish based on Goodyear (1995) regression results (Equation 5) and inputs in Table 1.

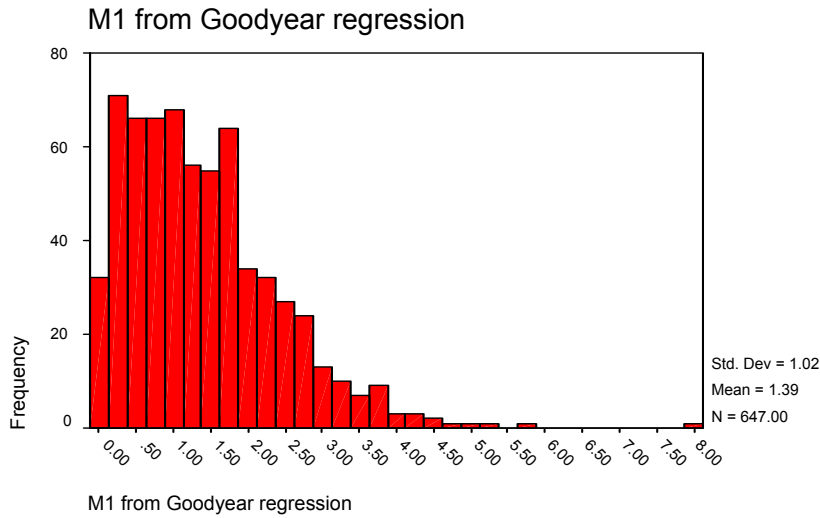


Figure 2. Frequency distribution of results for the rate of natural mortality for age 1 fish based on presumed results from Nichol's (1995) regressions (Equation 7) and inputs in Table 1.

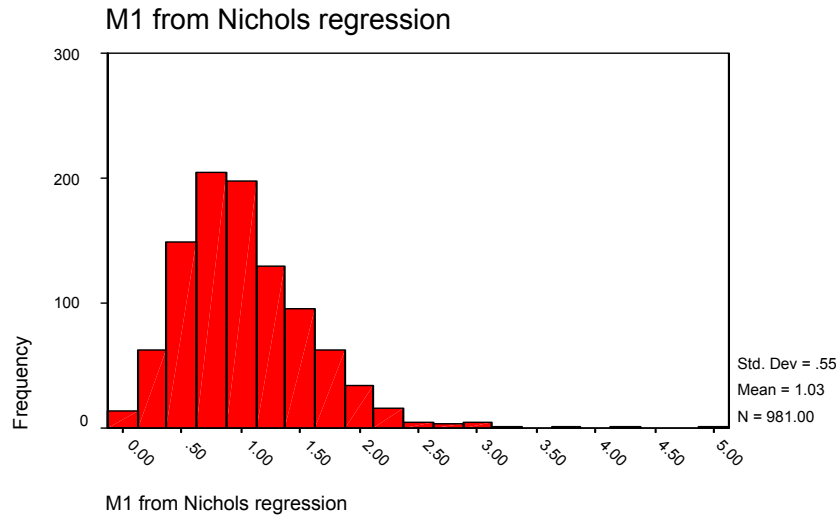


Figure 3. Frequency distribution of results for the rate of natural mortality age 0 to six months based on Szedlmayer's results (Equation 2) and inputs in Table 1.

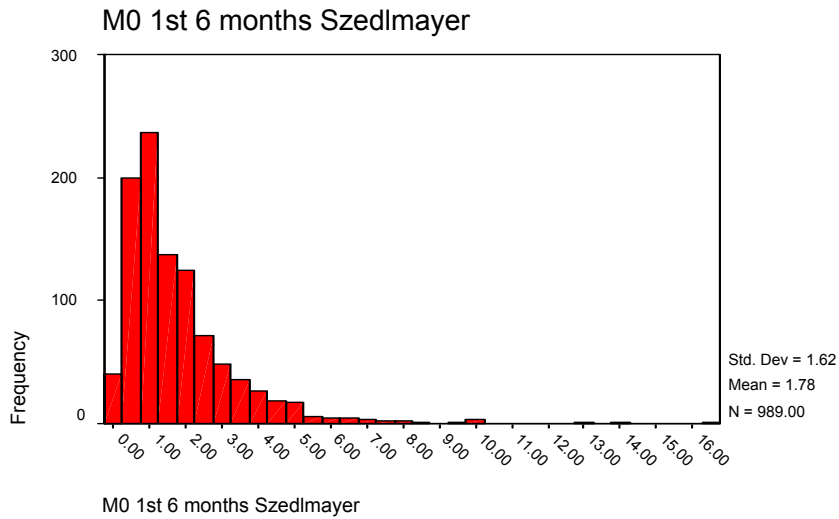


Figure 4. Frequency distribution of results for the rate of natural mortality age 7 to 12 months based on Szedlmayer's results (Equation 2) and inputs in Table 1

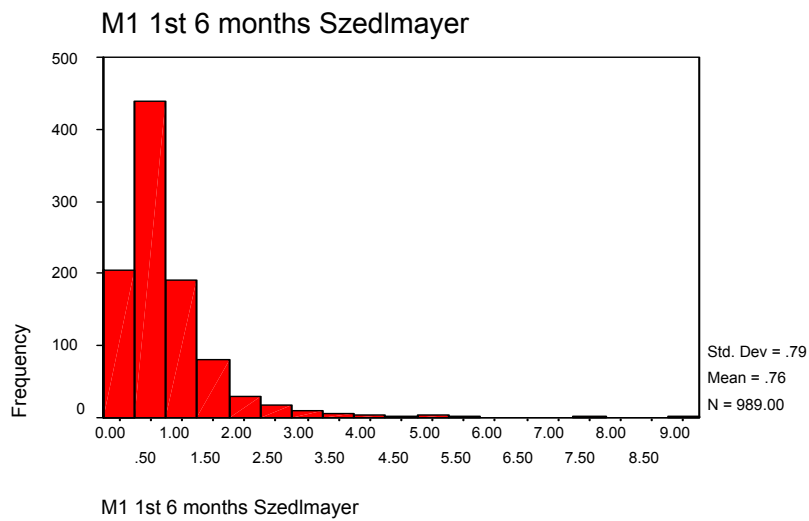


Figure 5. Frequency distribution of results for the rate of natural mortality age 13 to 18 months based on Goodyear (1995) regression results (Equation 2) and inputs in Table 1

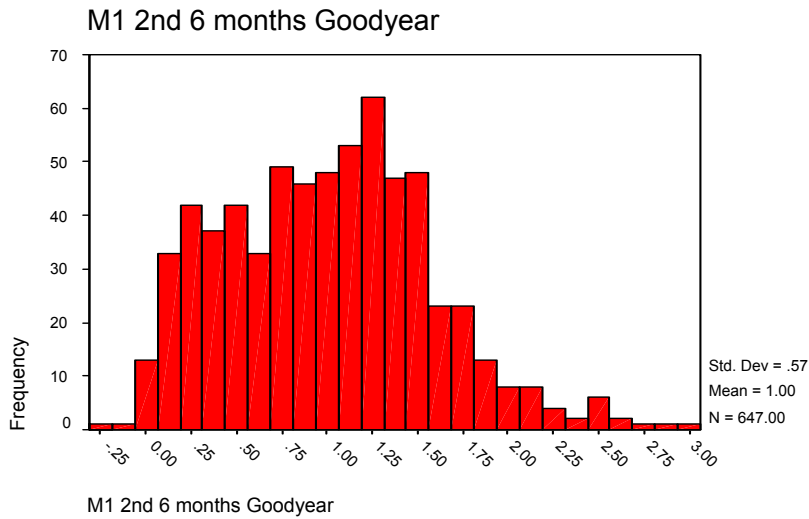


Figure 6. Frequency distribution of results for the rate of natural mortality age 7 to 12 months based on Nichol's regression results (Equation 5) and inputs in Table 1

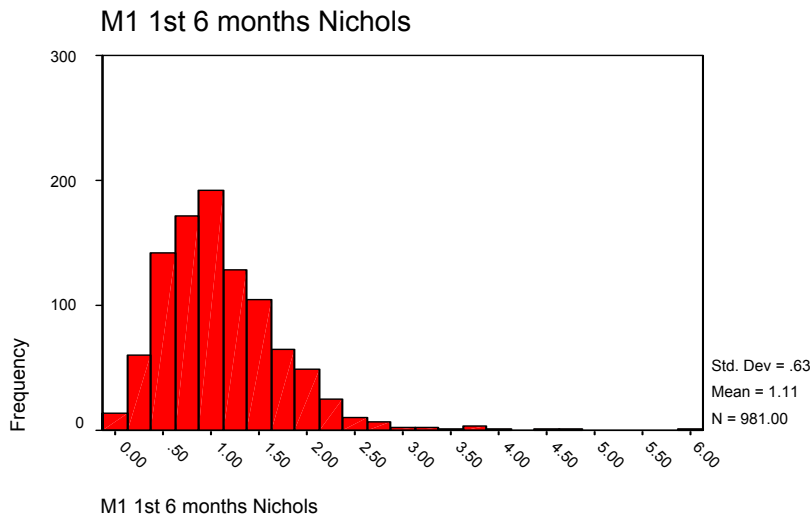


Figure 7. Frequency distribution of results for the rate of natural mortality age 7 to 12 months based on Szedlmayer's and Nichols' results (Equations 2 and 5) combined and inputs in Table 1

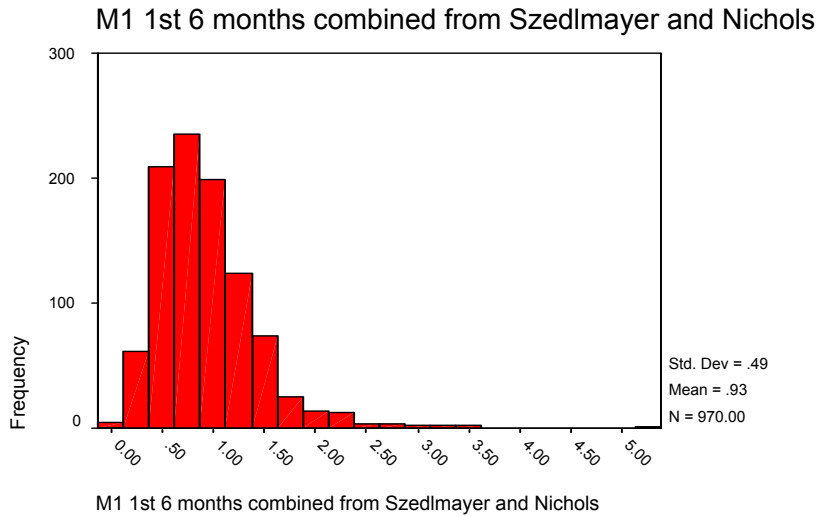


Figure 8. Frequency distribution of results for the rate of natural mortality age 7 to 18 months based on (Equations 2, 5 and 7) and inputs in Table 1

