# Growth models for red snapper in U.S. Gulf of Mexico waters estimated from landings with minimum size limit restrictions 

by<br>Guillermo A. Diaz, Clay E. Porch, and Mauricio Ortiz<br>National Marine Fisheries Service<br>Southeast Fisheries Science Center<br>Sustainable Fisheries Division 75 Virginia Beach dr. Miami FL 33149<br>August, 2004<br>Sustainable Fisheries Division Contribution SFD-2004-038

Many samples of age-length pairs collected from fishery landings show a peculiar pattern where young animals appear to grow very quickly and then suddenly slow down. In many cases this is primarily a consequence of minimum size limit regulations; fish smaller than the limit are not landed and therefore do not appear in the sample. This presents no special concerns if the goal is merely to determine the average size at age in the landings. However, when the goal is more ambitious, such as determining the average size at age in the catch, one must use a model-based approach to adjust for the effects of the minimum size limits. This paper presents one such approach.

## METHODS

## Model

The probability that a fish caught at age $a$ will be length $l$ is assumed to be normal distributed with mean $\bar{I}_{a}$ and standard deviation $\sigma_{a}$

$$
\begin{equation*}
\mathrm{p}_{\text {caught }}(| | a, g) \sim \mathrm{N}\left(\bar{l}_{a}, \sigma_{a}\right) \tag{1}
\end{equation*}
$$

When minimum size restrictions are in place, fish less than the minimum size $M$ are discarded and not included in the sampled landings Hence the probability distribution of the samples ought to follow the truncated normal distribution
(2) $\quad \mathrm{p}_{\text {landed }}(l \mid a)=\mathrm{p}_{\text {caught }}(l \mid a) /\left(1-\mathrm{P}_{\text {caught }}(l<M \mid a)\right)$
where $\mathrm{P}_{\text {caught }}$ is the cumulative probability of that a fish caught at age $a$ will be smaller than the size limit. Maximum likelihood estimates of the parameters $\bar{l}_{a}$ and $\sigma_{a}$ of the catch may therefore be obtained from the truncated samples (landings) by minimizing the negative loglikelihood expression

$$
\begin{equation*}
-\sum_{i} \ln \left(\frac{\frac{1}{\sqrt{2 \pi} \sigma_{a}} e^{-0.5^{*}\left(l_{i, a}-\bar{l}_{a}\right)^{2} / \sigma_{a}^{2}}}{1-\int_{-\infty}^{M_{i}} \frac{1}{\sqrt{2 \pi} \sigma_{a}} e^{-0.5^{*}\left(x-\bar{l}_{a}\right)^{2} / \sigma_{a}^{2}} d x}\right) \tag{3}
\end{equation*}
$$

where the subscript $i$ indexes individual observations. The corresponding predictions for the average length-at-age of the landings may be calculated approximately as

$$
\begin{equation*}
\bar{l}_{a, \text { landed }}=\sum_{x} l_{x} p_{\text {landed }}\left(l_{x} \mid a\right) \tag{5}
\end{equation*}
$$

where $x$ indexes a particular discrete length class. As illustrated in Figure $1, \bar{l}_{a, \text { landed }}>\bar{l}_{a}$ for all $M>0$. Note that observations below the minimum size limit assigned to them should be excluded from the analysis and observations collected without minimum size limit restrictions should have an $M$ value of 0 assigned to them.

In the present application the mean length at age in the catch was modeled using the von Bertalanffy growth equation

$$
\begin{equation*}
\bar{l}_{a}=L_{\infty}\left(1-e^{k\left(a-t_{0}\right)}\right) . \tag{4}
\end{equation*}
$$

The standard deviation $\sigma_{a}$ is estimated by a single parameter representing all ages. Preliminary analyses suggested that the standard deviation of length increased linearly with age. This was examined by linearly regressing the standard error of the residuals against the predicted length-at-age (for fish $\leq 8 \mathrm{yr}$ ) and fixing the intercept to zero. The slope of the estimated linear relationship then corresponded to a constant CV model.

## - Data sources

Three different data sets were combined to estimate the parameters of the von Bertalanffy growth model for red snapper
a) Juveniles data set:

The juvenile data set consisted of 27,335 records from independent trawl samples and shrimp trawl bycatch characterization studies (Goodyear 1995). Ages were inferred by length frequency analysis (Goodyear 1995) and ranged from 0.4 to 2 yr .
b) Louisiana State University (LSU) data set:

This data set included 11,620 red snapper age samples collected from 1995 to 2004; 7,353 samples were obtained from commercial handline (Com-HL) landings and 4,267 from recreational handline ( $\mathrm{Rec}-\mathrm{HL}$ ). The estimated fractional age ranged from $1.2-48.7 \mathrm{yr}$ and $0.8-45.0 \mathrm{yr}$ in the Com-HL and Rec-HL samples, respectively. 1,492 samples were collected from AL (all Rec-HL) (East Gulf of Mexico), the rest were collected in TX and LA (West Gulf of Mexico).
c) Panama City Lab data set:

This included 13,508 Com-HL samples (2,454 from East GOM, 6,346 from West GOM, and 4,708 from unknown location), 1,898 Com-LL samples ( 617 Com-LL from East GOM, 924 from West GOM, and 357 from unknown location), and 13,432 Rec-HL samples ( 8,435 from East GOM and 4,897 from West GOM). Age ranged from 0.9-46.8 yr, 1.1-56.7 yr, and 0.2-40.9 in the Com-HL , Com-LL, and Rec-HL samples, respectively. In the case of the recreational fishery, samples with unknown fishing area were assigned to East or West GOM according to the State where the sample was collected.

## RESULTS

## - Length-at-age of landings and catch

Table 1 shows the estimated parameters of the von Bertalanffy growth model for the different fisheries assuming the samples taken from fishery landings were truncated at the minimum size limit. Figures 2-4 show the predicted length-at-age for each fishery. The estimates of $\mathrm{L}_{\infty}$ for the East GOM were higher than those for the West GOM (Table 1), resulting in the prediction of larger fish at age for this area. These differences were greatest for fish older than 10 yr (Fig 2a-4a), largely owing to the low number of large fish in the samples from the East GOM. Table 2 shows that approximately $95 \%$ of all observations corresponded to fish from $0-6$ yr old. Within that age range, no significant differences were observed between East and West GOM (Figures 2b-4b). Thus, a growth equation was estimated for each fishery combining the data from East and West GOM and unknown areas. Figure 5 shows the predicted length-at-age for each of the three fisheries.

Table 1: Estimated growth parameters of the von Bertalanffy growth equation assuming a truncated distribution, $n$-fish corresponds to the number of records from the different fisheries, $n$-juv to the number of records on the juvenile data set, $n$-total to the total number of records used to estimate the growth parameters ( $n$-tot $=n$-fish $+n$-juv).

| DATA | $\mathrm{L}_{\infty}$ | k | $\mathrm{t}_{0}$ | $\sigma$ | $\mathrm{n}-$ fish. | $\mathrm{n}-$ Juv. | n - total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| All combined | 876.9 | 0.22 | 0.37 | 70.38 | 38,200 | 27,335 | 65,535 |
| Com-HL | 911.76 | 0.197 | 0.294 | 63.97 | 20,855 | 27,335 | 48,190 |
| Com-LL | 905.79 | 0.180 | 0.183 | 41.89 | 1,899 | 27,335 | 29,234 |
| Rec-HL | 1031.04 | 0.177 | 0.299 | 58.58 | 17,700 | 27,335 | 45,035 |
| Com-HL East | 1031.93 | 0.162 | 0.218 | 52.36 | 9,806 | 27,335 | 37,141 |
| Com-HL West | 987.43 | 0.162 | 0.177 | 49.96 | 6,346 | 27,335 | 33,681 |
| Com-LL East | 1011.94 | 0.147 | 0.104 | 38.63 | 617 | 27,335 | 27,952 |
| Com-LL West | 936.33 | 0.155 | 0.077 | 35.74 | 924 | 27,335 | 28,259 |
| Rec-HL - East | 1136.23 | 0.150 | 0.241 | 52.46 | 11,310 | 27,335 | 38,646 |
| Rec-HL - West | 1097.96 | 0.157 | 0.246 | 50.19 | 6,389 | 27,335 | 33,724 |

Table 2: Percentage (\%) and cumulative percentage (Cum. \%) of observations at each age range for all data set combined.

| Age range | $\%$ | Cum. \% |
| :---: | :---: | :---: |
| $0-1$ | 21.91 | 21.91 |
| $1-2$ | 19.46 | 41.37 |
| $2-3$ | 17.16 | 58.53 |
| $3-4$ | 20.93 | 79.46 |
| $4-5$ | 11.51 | 90.96 |
| $5-6$ | 4.49 | 95.45 |
| $6-7$ | 2.06 | 97.51 |
| $7-8$ | 0.88 | 98.39 |
| $8-9$ | 0.45 | 98.84 |
| $9-10$ | 0.26 | 99.10 |
| $10-11$ | 0.18 | 99.28 |
| $11-12$ | 0.11 | 99.39 |

No substantial differences were observed between the three curves, so a unique growth curve was estimated combining the data from the 3 fisheries (Table 1, Figure 6).

Figure 7 shows the observed and predicted length-at-age of the landings (under 2 different size limit scenarios) and the predicted length-at-age of the catch. For those ages affected by size limit regulations, the predicted length of the landings will be larger than those of the catch. On the other hand, predicted length of the catch and the landings will be equal for ages unaffected by size limit regulations. clearly, the difference between the predicted length for the catch and landings will increase as the minimum size increases (i.e., larger differences are observed for a 400 mm minimum size than for a 330 mm minimum size). Several different size limits had been in effect during the time period over which the data were collected, so the actual means are somewhere in between these two lines.

## - Coefficient of variation of the von Bertalanffy growth model

Figure 8 shows the plot of the estimated standard error of the residuals at each predicted length and the estimated linear relationship (weighted by number of observations). The linear relationship was significant $(\operatorname{Pr}<0.0001)$ and the estimated value of the CV (slope) was 0.164 (std. err. $=0.0051$ ). This suggests that it may be appropriate to model the standard error as $\sigma_{\mathrm{a}}=\mathrm{CV}^{*} \bar{l}_{a}$, where CV is an estimated parameter. Preliminary attempts to do this caused convergence problems with the EXCEL SOLVER routine used to do the analyses. We do not expect the shape of the final curves to change much with a constant CV model, however it is possible that the predicted lengths values near the origin may increase somewhat and it merits further investigation.

- Comparison of data from the periods 1991-94 and 1995-2002

A comparison of the data used by Goodyear (1995) to estimate a growth function and the data used in the present assessment (excluding juveniles) showed some differences in the size frequency distribution of the youngest animals. A general linear model analysis (GLM) showed that the predicted size-at-age of red snapper in the period 91-94 was smaller than in the period 95-02 (Figure 9). These differences were most notable for the youngest animals (i.e. 1.5 yr old) and became less significant for older animals. Figure 10 shows the observed length-at-age for the periods 91-94 and 95-02. Clearly, samples for the period 91-94 included smaller animals than samples in the period 95-02. These differences in the size distribution at age of the samples are the result of two combined factors: (1) minimum size limits and (2) sample sizes. Samples from the period 91-94 were obtained under size limit restrictions of 330 mm (91-93) and 356 mm for both the recreational and the commercial fisheries. In contrast, samples from the period 95-02 were restricted to a minimum size of 381 mm for the commercial fishery, the recreational fishery had the same size restriction until 1999 and 406 mm from 2000-02. The sample size of the observations in the age range $1.5-4$ were 5,461 and 21,606 for the periods 9194 and $95-02$, respectively. Given the larger sample size corresponding to the period 95-

02 , a wider size frequency distribution at age would be expected. However, given the size limit restrictions imposed after 1994, a larger sample size could only yield larger fish, but not smaller, than the samples from 91-94. Thus, increasing the average size-atage of the samples within the ages affected by the size limit restrictions.

## REFERENCES:

Goodyear, C. P. 1995. Red snapper in U.S. waters of the Gulf of Mexico. Southeast Fisheries Science Center, Coastal Resource Division MIA-95/96-05, 171 p.

Schirripa, M. J. and C. M. Legault. 1999. Status of the red snapper in U.S. waters of the Gulf of Mexico: updated through 1998. Southeast Fisheries Science Center, Sustainable Fisheries Division Contribution: SFD-99/00-75, 44 p., 3 appxs.


Figure 1: Truncated density distribution of length-at-age $a\left(L_{a}\right)$ showing the predicted length-at-age of the catch (point A) and the landings (point B).
a)

b)


Figure 2: Estimated Total length (TL) at age by a von Bertalanffy growth equation assuming a truncated distribution (see text for explanation) for commercial handline (HL) catches from East and West Gulf of Mexico (Note different x-axis scale between graphs a and b).
a)

b)


Figure 3: Estimated Total length (TL) at age by a von Bertalanffy growth equation assuming a truncated distribution (see text for explanation) for commercial longline (LL) catches from East and West Gulf of Mexico (Note different x-axis scale between graphs a and b).
a)

b)


Figure 4: Estimated Total length (TL) at age by a von Bertalanffy growth equation assuming a truncated distribution (see text for explanation) for recreational handline (HL) catches from East and West Gulf of Mexico (Note different x-axis scale between graphs a and b).


Figure 5: Predicted length-at-age for the commercial longline (Com LL), commercial handline (Com HL ) and recreational handline ( Rec HL ) fisheries for the entire Gulf of Mexico.


Figure 6: Observed average length-at-age of the landings and predicted length-at-age of the catch (all data sets combined). The von Bertalanffy growth parameters of the predicted curve are given in Table 1. Vertical lines around the observed average lengths correspond to the standard deviation.


Figure 7: Observed average length-at-age of the landings and associated standard error, predicted length-at-age of the catch (continuous line) and predicted length-at-age of the landings under a 400 mm (squares) and 330 mm (triangles) scenarios.


Figure 8: Observed (Obs) and predicted (Pred) standard error of the residuals at each predicted length. The slope of the line (predicted) correspond to the coefficient of variation (CV) of the estimated growth model.


Figure 9: Estimated length-at-age (by general linear model analysis) for the periods 1991-94 and 1995-2002.


Figure 10: Observed length-at-age in the combined data set (excluding juveniles) for the periods 91-94 and 95-02.

