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Introduction

The SEDAR 41 data workshop developed time series data for U.S. South Atlantic Gray Triggerfish landings, discards, and abundance. These data were fit with an age structured production model (ASPM) to provide insights into the relationship between removals and population abundance changes. The model code was programmed in AD Model Builder (ADMB), which uses a gradient optimizer for fitting model parameters. Details of the ASPM implementation are listed below and a technical description is included in Appendix 1.

Model Description

Age structured production models have existed since the advent of catch-at-age models in the mid-1980s (Fournier and Archibald 1982, Hilborn 1990, Kimura and Tagart 1982, Ludwig and Walters 1985, Megrey 1989). ASPMs have been used extensively for highly migratory pelagics, where age collection can be difficult, and other stock assessment analyses as well (Cubillos et al. 2002, Geromont and Butterworth 1999, Nishida et al. 2001, Nishida and Rademeyer 2011, Porch 2003, Restrepo 1997, Restrepo and Legault 1998, Ricard and Basson 2002). ASPMs can be viewed as either a simplified version of statistical catch-at-age models or an extension of basic biomass production models (e.g. ASPIC). The primary advantage of ASPM over basic biomass production models is the incorporation of age structured dynamics into the population model (Butterworth and Rademeyer 2008, Punt et al. 2010, Radomski et al. 2005). The simplification from more advanced statistical catch-at-age models is due to the absence of any age or length composition data. Because no age or length data are used in an ASPM, then year class strength is expected to follow a simple two or three parameter production function (i.e. a stock-recruit function;

Butterworth and Rademeyer 2008, Field et al. 2008). With this simplification, ASPMs have a greatly reduced number of parameters compared to a full statistical catch-at-age model. Of course with reduced parameters comes simplifying assumptions (e.g. fixed fleet selectivities).

A parameter analogy can be drawn from ASPM and biomass production models (e.g. ASPIC). In both models fishing mortality (F) values are determined for each year for each fleet. Also, catchability (q) parameters are estimated for each abundance index. For ASPIC models the production function is determined by logistic growth, defined by the K (carrying capacity) and r (intrinsic population growth rate) parameters. In ASPM, using a Beverton-Holt stock recruit function, the corresponding parameters are R_0 (virgin recruitment) and steepness (h - rate of recruitment increase), respectively. Both models have a parameter for estimating the starting condition of the population. In ASPIC this is the B_1/K parameter, while in this ASPM configuration it is modeled with an initial F value parameter (F_{init}).

The advantage of the ASPM over ASPIC is the treatment of the population as age structured. This is considered more realistic and allows for important age specific processes to be incorporated into the population model. Some of the more important age-specific processes include size/weight, maturity, selectivity, and natural mortality.

Data Description

The data inputs used for the ASPM follow the structure outlined in the data workshop report. Specifically, landings and discards were broken into three fleets, commercial handline (cH) (landings only), headboat (HB), and general recreational (GR) (see details below). The ASPM used the single fishery independent MARMAP chevron trap abundance index (1990-2014) developed during the data workshop. No age or length data were used in the ASPM. Life history information was fixed to values recommended by the data workshop.

Landings Data

Landings and discard information were input into the ASPM following the structure outlined in the data workshop report (Table 1). Units of the landings and discards match the units in which the data were collected from harvesters (see Table 1 heading).

Abundance Index

The single fishery independent SERFS chevron trap-video (MCVT) abundance index (1990-2014) from the data workshop report was input into the ASPM. The error values (CVs) were also used in the ASPM (Table 2).

Selectivity

The selectivity configuration was an important part of an ASPM configuration. In this case we used the functions and values obtained from the base Beaufort Assessment Model (BAM) run. This consisted of a single time block (1988-2014) for each of the separate landings fleets, discard fleets, and MCVT index (Table 3). A mixture of logistic (flat-topped) and double logistic (dome-shaped) functions were used to model selectivity for the various landings and discard fleets and MCVT index (Table 3).

Life History

Important life history data used in an ASPM includes age-specific quantities for weight, maturity, proportion female, fecundity, and natural mortality (Table 4). Separate weight-at-age data used for biomass calculations were available for landed catch and the population. The values used for Gray Triggerfish were from either the data workshop report or the BAM base run. Those values are shown in Table 4.

Results

When fit, the ASPM estimates for steepness were hitting an upper bound, suggesting the value is not well defined. Therefore the ASPM results were explored through a range of fixed steepness values. All of these model fits converged properly and no parameter estimates were on bounds. For simplicity, the single abundance index was treated with a likelihood weight of 1 (i.e. no iterative re-weighting was done).

As is commonly the case with stock-recruit derived benchmarks, steepness is an important parameter, such that MSY and derived values are influenced by the set value of this parameter. However, the overall scale of the population estimates for Gray Triggerfish seem to be largely unaffected by values of steepness, as seen in the fairly stable estimates for R_0 and F_{init} across the range of fixed steepness values (Figure 1).

The primary data source being fit in this ASPM was the index time series. The fit to these data are shown in Figure 2. The range of fixed steepness values results in only minor changes to the fit to the MARMAP chevron trap index (Fig. 2).

As mentioned above, when steepness was freely estimated, the result is a value at the upper bound (0.99). A total likelihood profile across values of steepness confirmed that the higher values of steepness were favored by the data fits in the model (Figures 3 and 4). A steepness value at the upper bound does not always confer that the true underlying steepness of the population is high. However, the results over the range of steepness values were qualitatively similar (not over fished and not overfishing). It was only at the extremely low values of steepness that things changed. Time series of biomass (B), spawning biomass (SSB), and fishing mortality (*F*) ratios are shown in Figures 5-7.

Discussion

As pointed out earlier, ASPMs depend on landings and index data. In most fisheries, as is the case for Gray Triggerfish, landings information is usually known fairly well. This leaves the quality of the index as the most important data input for an ASPM. Fortunately, the single index used in this model is from a fishery independent survey, believed to be more reliable than fishery dependent CPUE based abundance indices. The ASPM, like all fisheries models, expects a relationship between landings and indices (i.e. fishing mortality is a driving influence on population fluctuations). If this is not the case, either in actuality or if masked by noisy data, then ASPM will provide poor estimates.

For this application of ASPM, the Gray Triggerfish data raised one concern, the index and landings do not coincide such that increased landings resulted in decreased abundance, nor do decreased landings seem to result in increased abundance values. This can be interpreted in one of two ways, either the population is being harvested at low rates relative to the total population biomass, or the index is somehow not reflecting abundance (e.g. possible hyperstability conditions). If the latter condition exists, then the results from this model may not be useful for management.

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Table 1. Observed time series of landings (L) and discards (D) for commercial lines (cH), headboat (HB), and general recreational (GR). Commercial values are in units of 1000 lb whole weight. Recreational values are in units of 1000 fish. Commercial landings include the weight of dead commercial discards.

Year	cH.L	HB.L	GR.L	HB.D	GR.D
1988	79.832	34.926	113.352	0.165	12.999
1989	96.487	37.367	394.704	0.198	33.973
1990	194.797	71.704	238.471	0.071	11.013
1991	271.694	85.529	313.797	0.016	39.422
1992	263.032	91.733	192.245	0.048	15.78
1993	329.064	107.07	268.924	0.024	10.317
1994	410.679	90.387	171.182	0.024	9.973
1995	488.437	93.367	148.134	0.117	13.301
1996	441.97	89.954	275.301	0.028	16.245
1997	536.451	106.17	290.987	0.088	13.215
1998	424.389	65.857	93.577	0.14	4.928
1999	279.378	37.218	112.743	0.333	7.997
2000	196.777	34.092	107.727	0.42	9.289
2001	215.776	32.978	126.007	0.438	10.384
2002	204.482	57.63	184.651	0.754	14.966
2003	192.111	45.751	170.767	0.294	20.212
2004	252.412	78.073	217.963	0.246	23.61
2005	276.017	63.582	207.746	0.259	22.925
2006	241.487	43.151	169.506	0.363	20.107
2007	323.169	66.403	308.856	0.225	34.17
2008	320.128	44.758	429.321	1.34	23.495
2009	370.283	59.945	497.253	1.294	32.223
2010	453.214	68.807	325.334	2.42	22.782
2011	498.734	53.356	167.214	1.44	9.541
2012	311.367	49.096	264.993	1.581	12.508
2013	339.273	56.487	166.817	1.095	20.26
2014	284.758	53.108	230.238	1.519	21.624

Table 2. Observed index of abundance and CVs from SERFS chevon trap-video (MCVT).

Year	MCVT	MCVT CV
1990	0.28	0.32
1991	1.08	0.24
1992	0.86	0.26
1993	0.8	0.24
1994	1.03	0.23
1995	1.33	0.22
1996	1.58	0.22
1997	1.44	0.22
1998	1.7	0.23
1999	0.75	0.27
2000	0.65	0.28
2001	0.88	0.25
2002	1.5	0.24
2003	0.83	0.31
2004	1.27	0.24
2005	0.77	0.25
2006	0.56	0.27
2007	0.95	0.25
2008	0.89	0.25
2009	0.7	0.26
2010	0.67	0.25
2011	0.87	0.19
2012	1.06	0.18
2013	1.24	0.17
2014	1.29	0.2

Table 3. Selectivity at age for SERFS chevron trap-video index (MCVT), commercial handlines (cH), headboat (HB), and general recreational (GR) landings (L) and discards (D).

Age	MCVT	cH.L	HB.L	GR.L	HB.D	GR.D
1	0.048	0.001	0.015	0.006	1	1
2	0.582	0.015	0.158	0.119	0.206	0.206
3	0.975	0.156	0.758	0.788	0.002	0.002
4	0.999	0.693	1	1	0	0
5	1	0.965	0.771	0.909	0	0
6	1	0.997	0.448	0.721	0	0
7	1	1	0.207	0.474	0	0
8	1	1	0.083	0.255	0	0

Table 4. Life-history characteristics at age, including natural mortality, weight (population and landings), proportion females mature, proportion female, eggs/batch, and batch/yr at age.

Age	Nat. mortality	Wgt Population (lb)	Wgt Landings (lb)	Fem. Maturity	Proportion Female	Eggs/Batch (millions)	Batches/Year
1	0.89	0.88	0.98	0.79	0.5	0.471	0.1
2	0.66	1.59	1.41	0.95	0.5	0.96	2.3
3	0.55	2.26	1.87	0.99	0.5	1.308	3.5
4	0.49	2.85	2.35	1	0.5	1.556	4.6
5	0.46	3.32	2.81	1	0.5	1.732	6.4
6	0.43	3.68	3.27	1	0.5	1.858	7.6
7	0.42	3.96	3.69	1	0.5	1.947	7.6
8	0.4	4.16	4.09	1	0.5	2.011	7.6

Figure 1. ASPM estimates and parameter values for a range of steepness values ranging from 0.25 to 0.95.

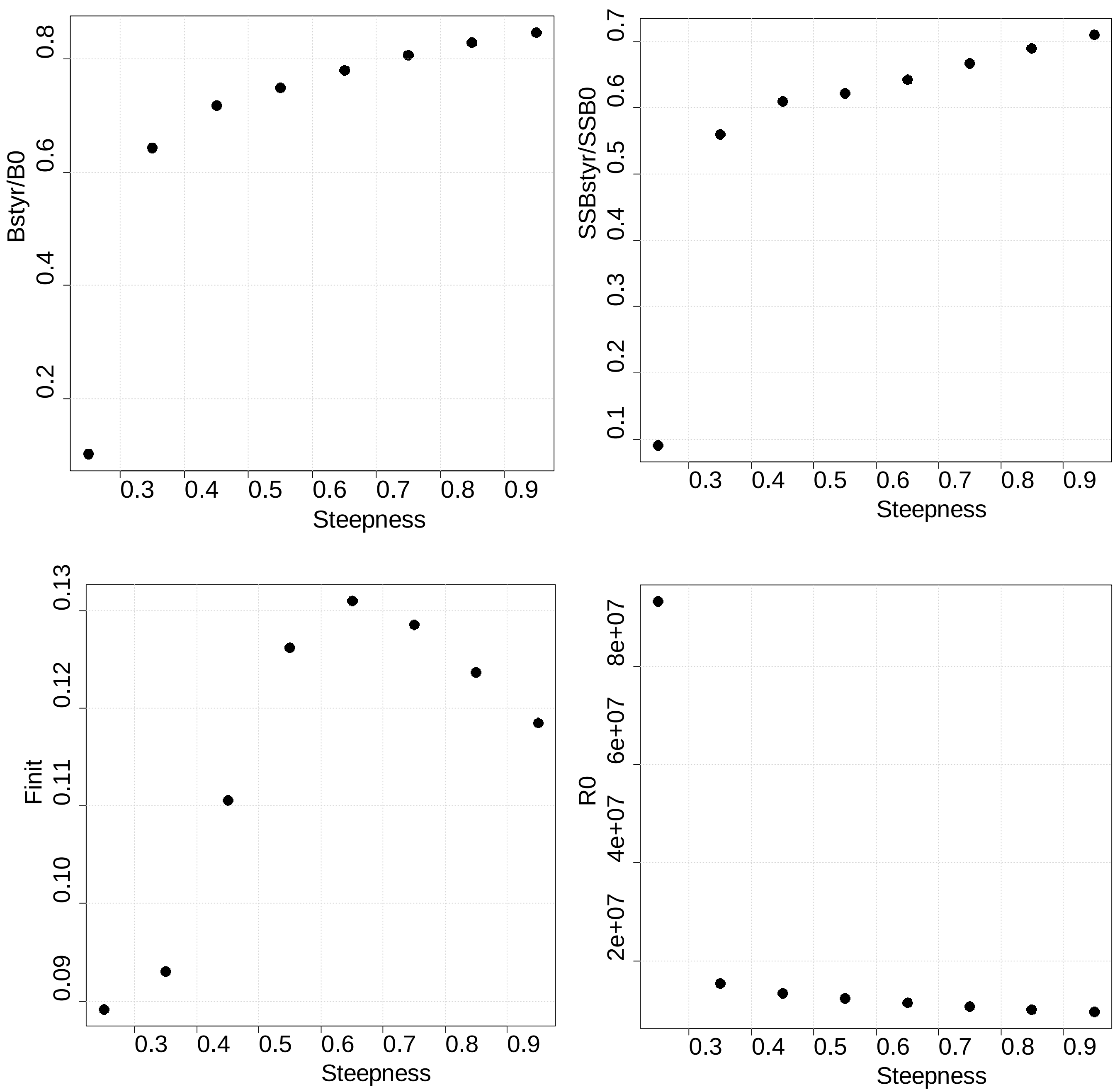


Figure 2. ASPM estimate (colored lines) fits to chevron trap-video CPUE index of abundance (open circles) for a range of steepness values ranging from 0.25 to 0.95.

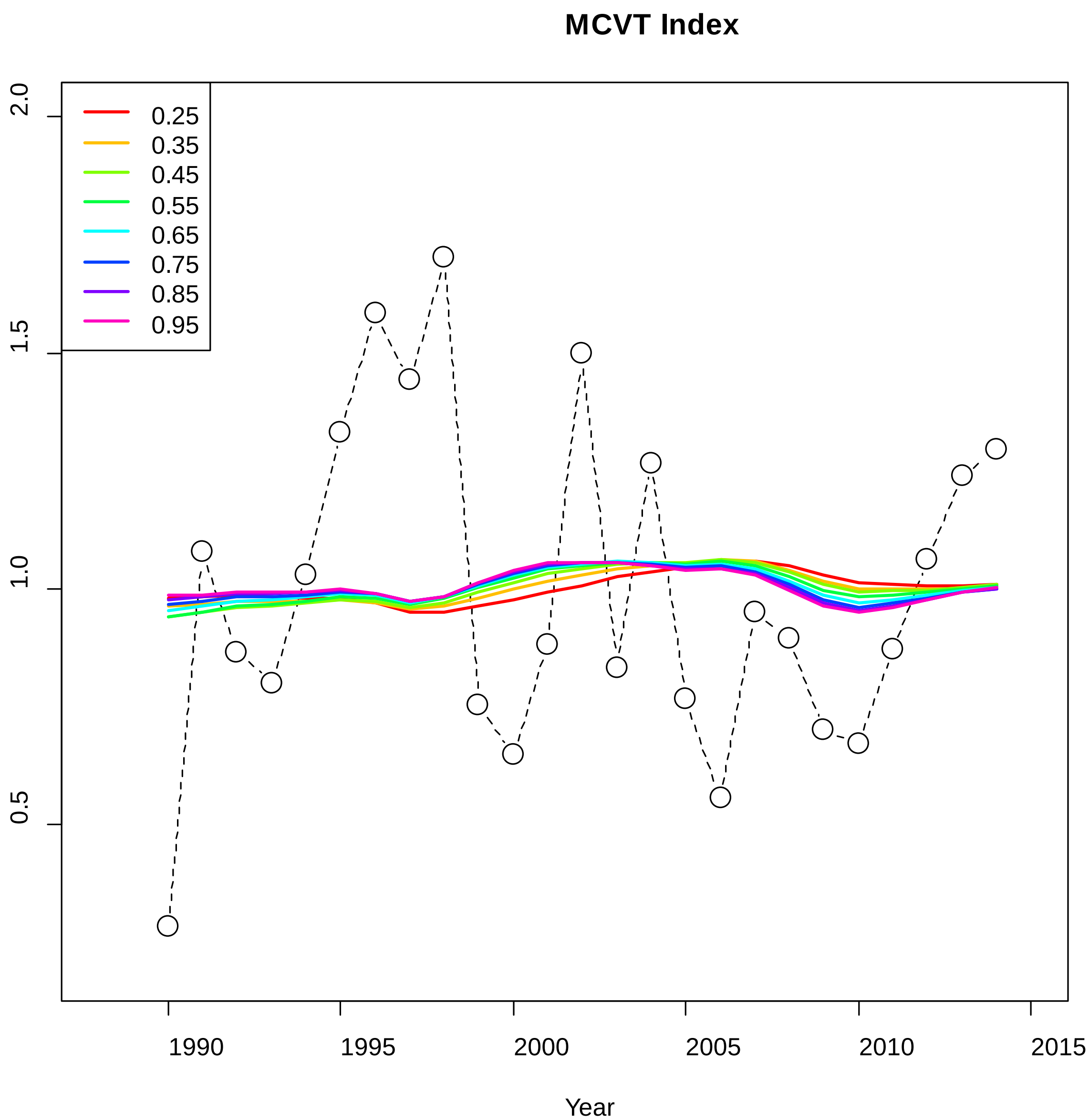


Figure 3. ASPM re-scaled total negative log-likelihood values for a range of steepness values ranging from 0.25 to 0.95.

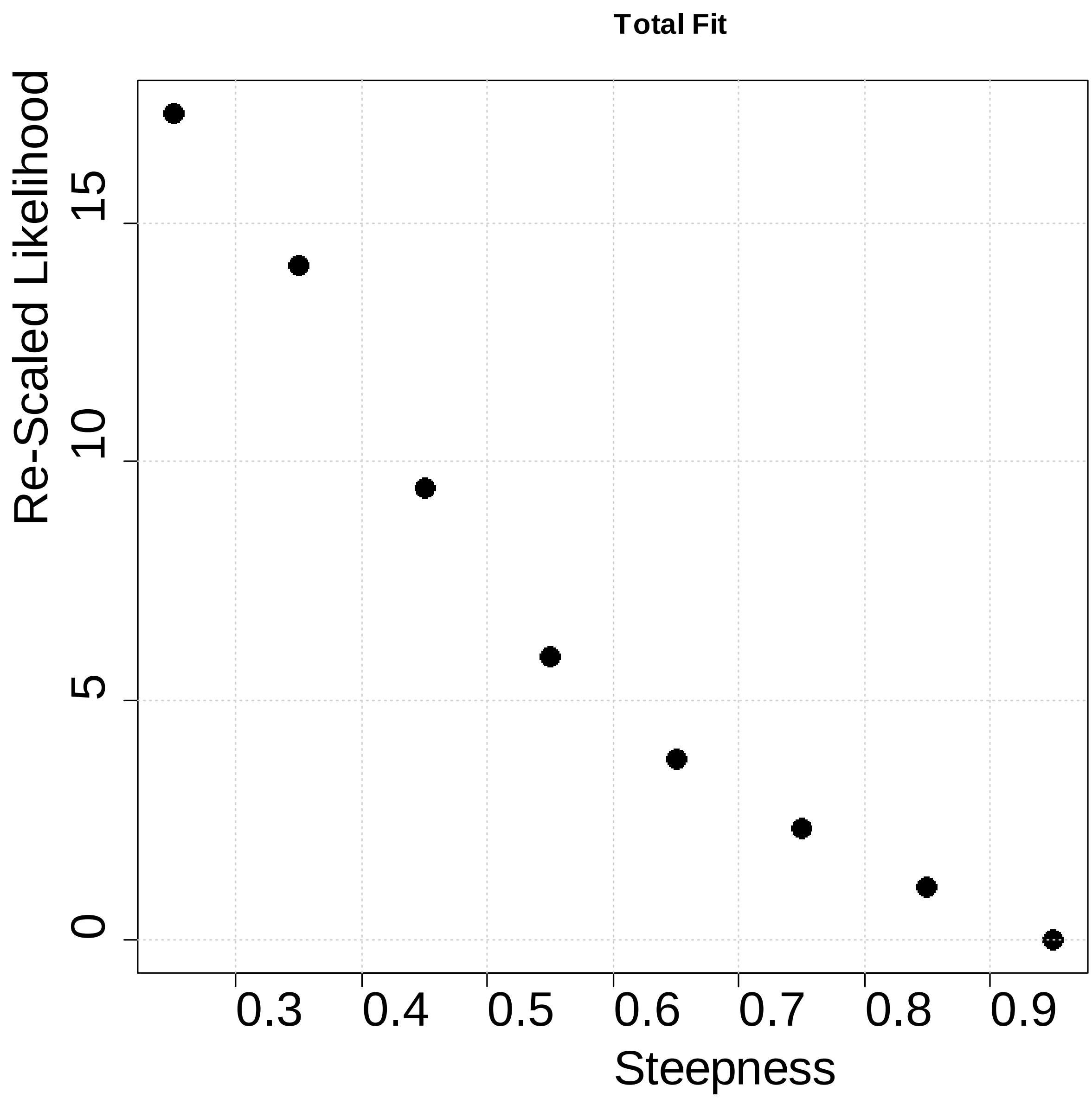


Figure 4. ASPM re-scaled negative log-likelihood values for the abundance index fit for a range of steepness values ranging from 0.25 to 0.95.

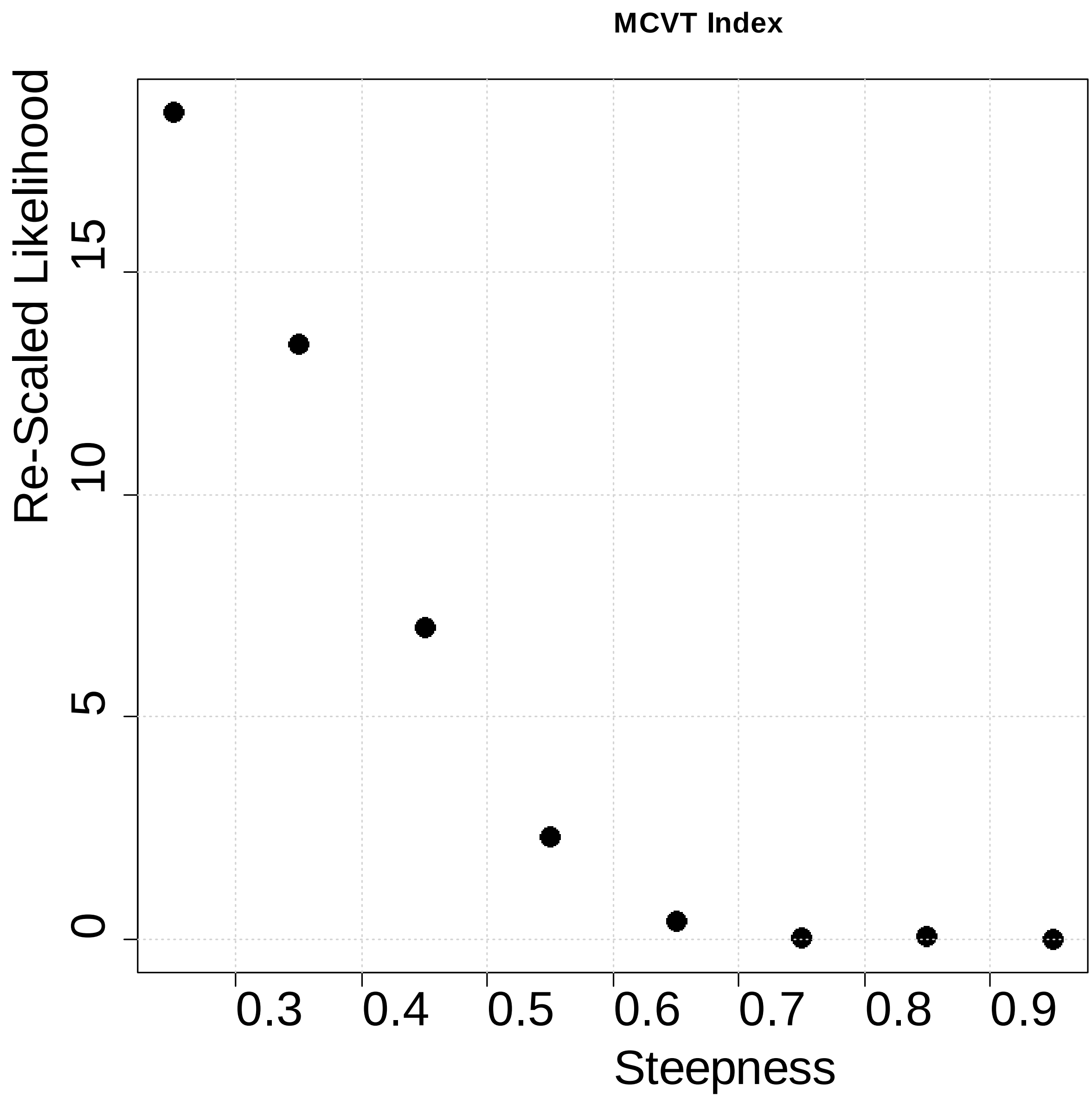


Figure 5. ASPM time series estimates of total biomass (B) relative to virgin biomass (B_0) for a range of steepness values ranging from 0.25 to 0.95. Dashed line at 0.4 represents an approximation of B_{MSY} under typical Beverton-Holt dynamics.

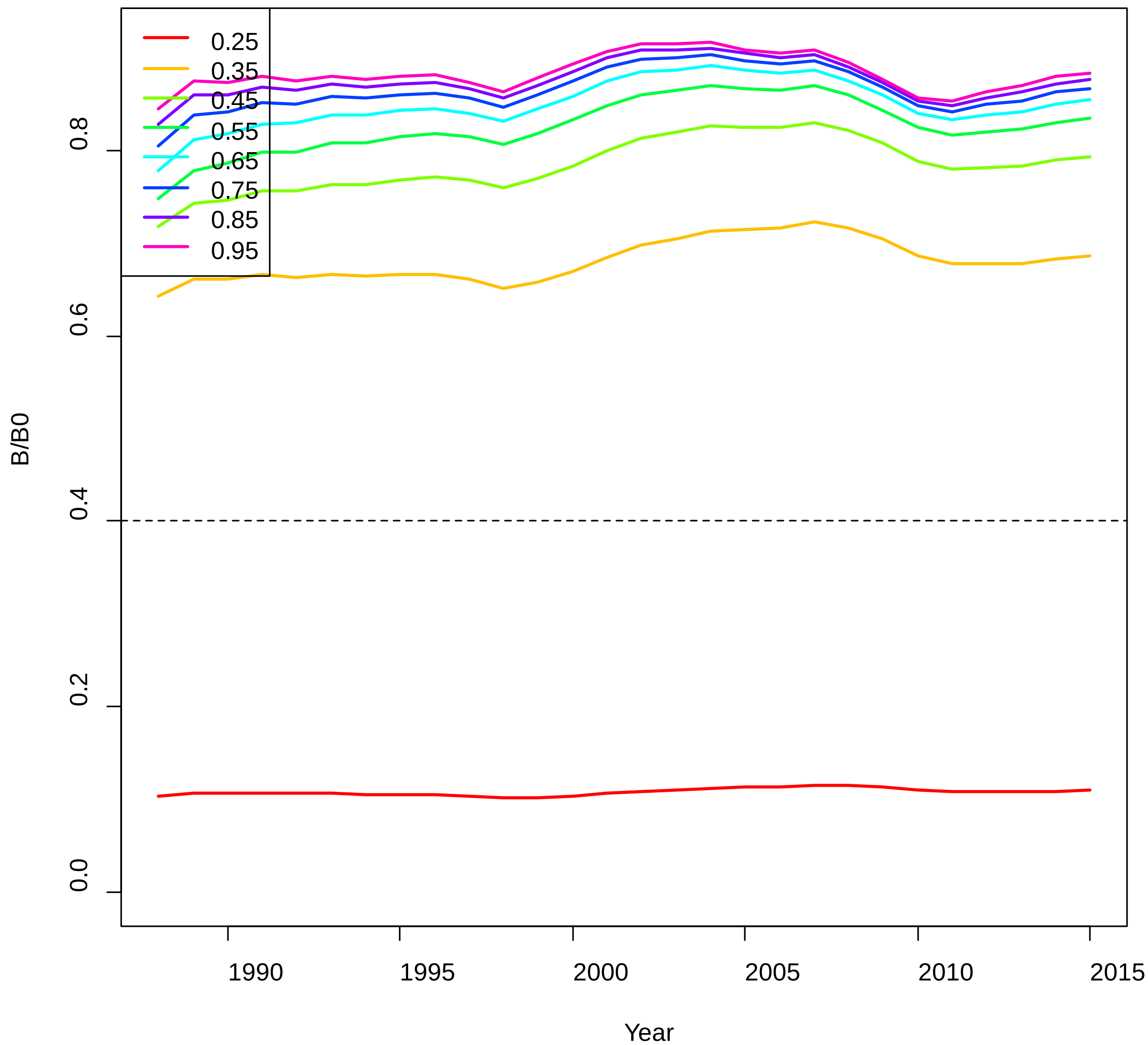


Figure 6. ASPM time series estimates of spawning biomass (SSB) relative to spawning biomass at maximum sustainable yield (SSB_{msy}) for a range of steepness values ranging from 0.25 to 0.95.

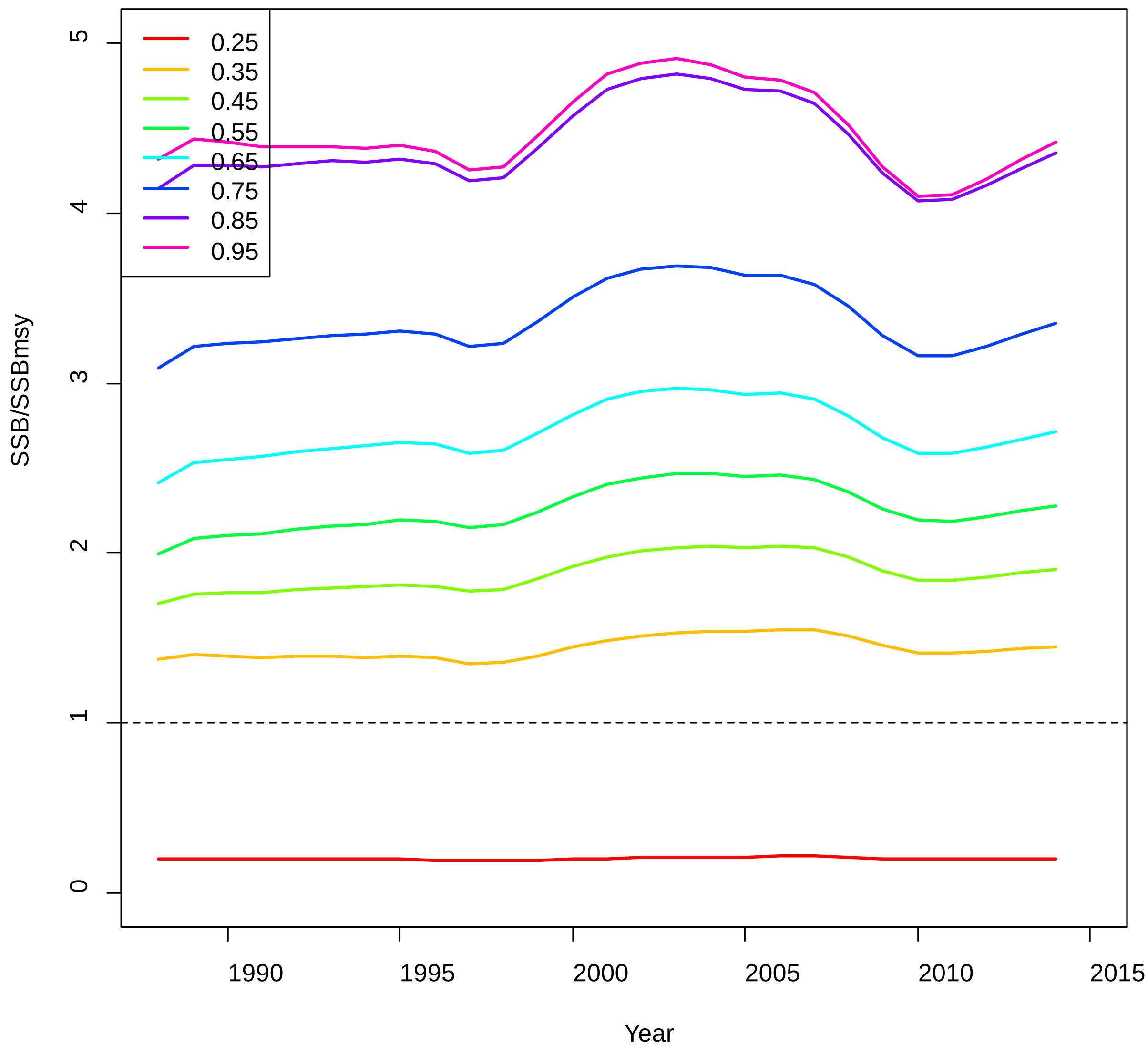
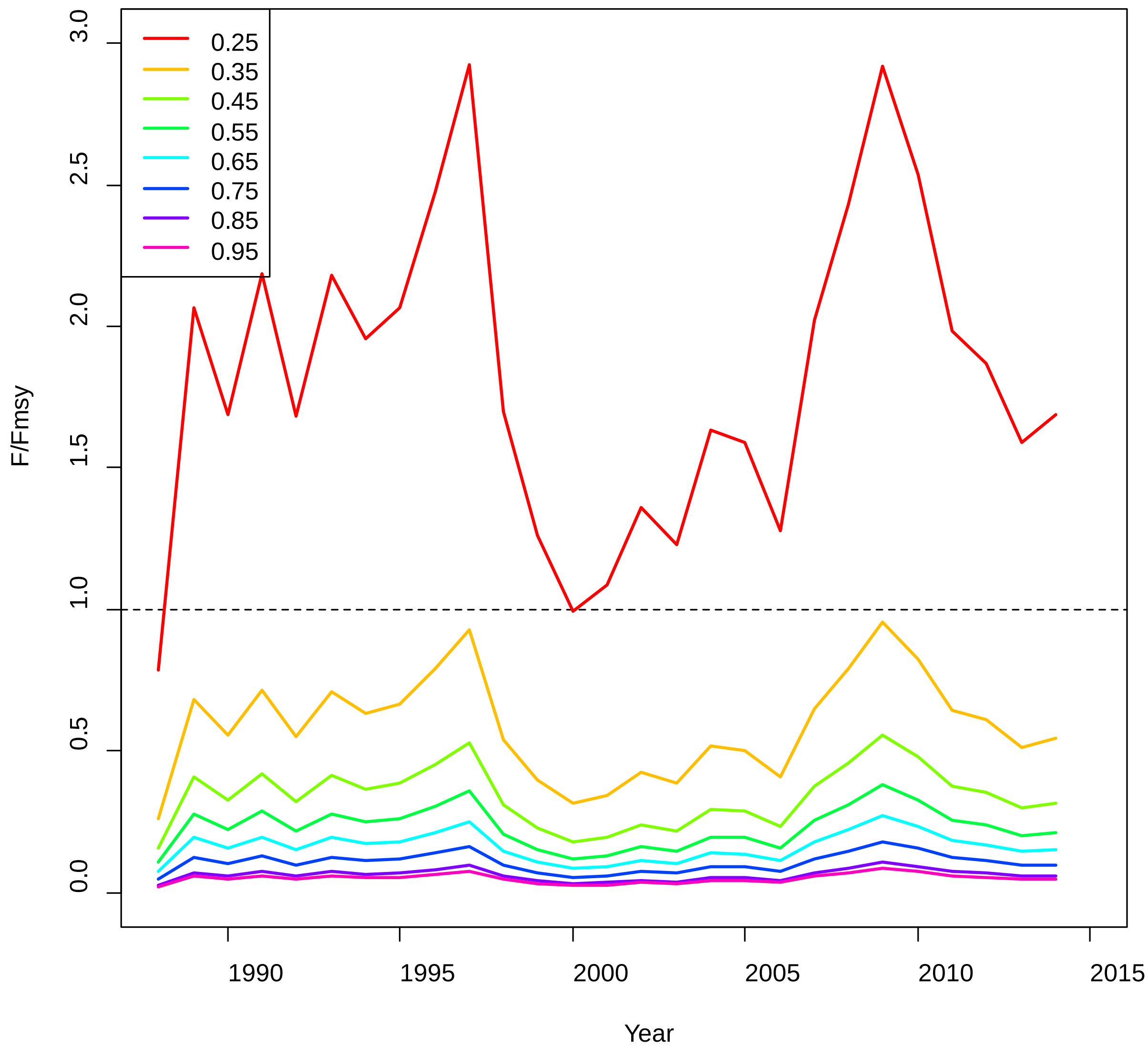


Figure 7. ASPM time series estimates of fishing mortality (*F*) relative to fishing mortality at maximum sustainable yield (F_{msy}) for a range of steepness values ranging from 0.25 to 0.95.



Appendix 1. Age structured production model (ASPM) technical documentation.

Model description

ASPM is fundamentally an age-structured population model with birth and death processes. New biomass is acquired through growth and recruitment, while abundance of existing cohorts experiences exponential decay from fishing and natural mortality. The population is assumed closed to immigration and emigration. The model follows an annual time step for *n* years, $y_1,\ldots, y_n$, and it includes *A* age classes 1-*A*+, where the oldest age class *A*+ allows for the accumulation of fish (i.e., plus group). The youngest age class (recruits) is typically age-1 fish produced by the previous year's spawners, but it could instead be age-0 fish produced by the current year's spawners (and consequently with *A*+1 age classes). Subsequent descriptions assume age-1 is the youngest age class.

Initialization

ASPM computes initial abundance at age, i.e., abundance in the first modeled year as an equilibrium age structure based on natural and initial fishing mortality (F_{init}), where F_{init} is estimated, either freely or with a prior or else fixed at user-specified values.

Life History Information

All the life history information is treated as input into the model. This includes weight at age, which may be derived from length at age, natural mortality, which may be treated as constant or age specific, sex ratio at age, and maturity at age, which may also be a function of length. Fecundity at age may also be included and thus, ASPM is flexible in treating the reproductive output measure, often referred to as spawning stock (*S*).

Recruitment

Expected annual recruitment (R_y) is computed from either the Beverton-Holt or Ricker spawner-recruit model. In ASPM, the Beverton-Holt formulation is,

$$R_{y+1} = \frac{0.8R_0hS_y}{0.2R_0\varphi_0(1-h) + S_h(h-0.2)}$$

where R_0 is virgin recruitment, h is steepness, and φ_0 is the unfished spawners per recruit. The analogous Ricker formulation is,

$$R_{y+1} = \frac{S_y}{\varphi_0} \exp\left(h\left(1 - \frac{S_y}{R_0\varphi_0}\right)\right)$$

Under Beverton-Holt, the expected equilibrium recruitment (R_{eq}) associated with any F is,

$$R_{eq} = \frac{R_0[4h\varphi_F - (1-h)\varphi_0]}{(5h-1)\varphi_F}$$

and under Ricker,

$$R_{eq} = \frac{R_0}{\tau_F}\left(1 + \frac{\log(\tau_F)}{h}\right)$$

where φ_F is spawners per recruit given F, and $\tau_F = \varphi_F/\varphi_0$ is the spawning potential ratio.

In the first year $N_{1,1} = R_{eq}$, based on F_{init}. Computation of R_{eq}, along with the mortality schedule, implies an equilibrium age structure, which would apply to calculations of the initialization (described above) as well as calculations of biological reference points (described below).

Selectivity

In ASPM, selectivity is modeled as a function of age. Selectivity at age ranges on the interval [0,1] and can be modeled for three different types of data: landings, discards, and indices. Because no age or length data is used for ASPM, selectivity has to be fixed. It may be fixed by entering values of selectivity at each age, or through a parametric approach. The parametric approach has the benefit of allowing easy exploration of alternate selectivity assumptions. Parametric functions used in ASPM are similar to those used in other stock assessment models, such as SS (Methot and Wetzel, 2013) or BAM (Williams and Shertzer, 2015).

Fishing

For each fleet being modeled, the ASPM estimates a separate full fishing mortality rate for each year of the time series ($F_{(f,d),y}$), with landings (denoted by subscript *f*) and discards (denoted by subscript *d*) treated as distinct fleets.

Age-specific rates are computed as the product of full F and selectivity (s) at age (i.e., $F_{(f,d),a,y} = s_{(f,d),a,y} F_{(f,d),y}$). Then, the across-fleet annual F_y is represented by apical F, computed as the maximum of F at age summed across fleets,

$$F_{a,y} = \sum_{(f,d)} F_{(f,d),a,y}$$

$$F_y = \max(F_{a,y})$$

Landings and Discards

In ASPM, landings (L) and discards (D) are treated as separate fleets (f or d, respectively). The numbers at age for any of these fleets are predicted using the Baranov catch equation,

$$(L_N \text{ or } D_N)_{(f,d),a,y} = \frac{F_{(f,d),a,y}}{Z_{a,y}} N_{a,y}\left[1 - \exp(-Z_{a,y})\right]$$

where $Z_{a,y} = M_a + F_{a,y}$ [summed across (f,d)] is total mortality at age and $N_{a,y}$ is annual abundance at age. Then, landings or discards at age in weight (L_W or D_W) are calculated multiplying (L_N or D_N) by the fleet specific weight at age. Annual totals are then just the sum across all ages (a) in year (y).

Stock Dynamics

Abundance of recruits ($N_{1,y}$) is described above in the section titled Recruitment. Abundance of each subsequent age at the start of each year is computed assuming exponential decay,

$$N_{a+1,y+1} = N_{a,y} \exp(-Z_{a,y}) \quad \forall_a \in (1 \ldots A-1)$$

$$N_{A,y+1} = N_{A-1,y} \exp(-Z_{A-1,y}) + N_{A,y} \exp(-Z_{A,y})$$

In addition, ASPM can compute abundance later in the year,

$$N'_{a,y} = N_{a,y} \exp(-t_{\text{index}} Z_{a,y})$$

for matching observed indices of abundance. In this calculation, t_{index} represents the fraction of the year over which to apply total mortality, most typically t_{index} = 0.5 for calculating mid-year abundance. Similarly, ASPM computes abundance at the time of peak spawning,

$$N''_{a,y} = N_{a,y} \exp(-t_{\text{spawn}} Z_{a,y})$$

to derive spawning stock. Here, t_{spawn} represents the fraction of the year when peak spawning occurs (e.g., $t_{spawn} = 0.25$ reflects peak spawning at the end of March).

Indices of Abundance

Predicted indices ($U_{u,y}$) for each index (u) are computed from numbers at age, scaled to the relevant portion of the age structure by selectivity (s). A predicted index could additionally be computed in weight, if the observed index is measured in weight.

$$U_{u,y} = \begin{cases} \hat{q}_{u,y} \sum_a s_{u,a} N'_{a,y} & : \text{ if in numbers} \\ \hat{q}_{u,y} \sum_a s_{u,a} w_a N'_{a,y} & : \text{ if in weight} \end{cases}$$

Catchability ($q_{u,y}$) scales indices of abundance to the estimated population at large. For most applications of ASPM, catchability is assumed to be constant across time. Variable catchability could be considered, but the parameters would likely be inestimable, suggesting the better course of action for time varying catchability would be to modify the observed index values to account for the changes in catchability.

Fitting Criteria

Observed landings can be supplied in numbers or in weight for any given fleet. For fitting landings data, ASPM uses the corresponding prediction ($L_{(N \text{ or } W)}$ or $D_{(N \text{ or } W)}$), computed such that units of predictions and observations match. The landings contribution (Λ^L) to the total objective function is

$$\Lambda^L = \sum_f \sum_y \frac{\left[\log\left((L_{f,y} + \epsilon)/(\breve{L}_{f,y} + \epsilon)\right)\right]^2}{2\left(\sigma^L_{f,y}\right)^2}$$

where $\epsilon = 1e^{-5}$ to prevent the optimization procedure from attempting to compute the log of zero (an undefined value), and where $\sigma^L_{f,y}$ are standard deviations in log space. These standard deviations are computed as

$$\sigma^L_{f,y} = \sqrt{\log\left(1 + \left(CV^L_{f,y}/\omega^L_f\right)^2\right)}$$

where $CV^L_{f,y}$ are user-supplied coefficients of variation in arithmetic space and ω^L_f are user-supplied weights. Analogous contributions to the total objective function are computed for discards (Λ^D) and indices of abundance (Λ^U).

Biological Reference Points

Biological reference points (benchmarks) are calculated based on maximum sustainable yield (MSY) estimates from the spawner-recruit model. These benchmarks include MSY, fishing mortality rate at MSY (F_{MSY}), dead discards at MSY (D_{MSY}), and spawning stock at MSY (SSB_{MSY}). The point of maximum yield is identified from the spawner-recruit curve and parameters describing growth, natural mortality, maturity, and selectivity. The value of F_{MSY} is the F that maximizes equilibrium landings (i.e., MSY). The values of D_{MSY} and SSB_{MSY} are those that correspond to F_{MSY}.

The MSY-based benchmarks and proxies are conditional on the fixed selectivity functions. For computation of benchmarks, three composite selectivities are computed from the terminal year of the assessment: 1) selectivity associated with landings, 2) selectivity associated with dead discards, and 3) the sum of the previous two, which describes total fishing mortality and has a peak value of one. The composite selectivities are F-weighted average selectivities across fleets, with F from each fleet estimated as the full F averaged over the last X years of the assessment. Typically, $X = 3$ years.

References

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