The Beaufort Assessment Model (BAM) with application to black sea bass[1]: mathematical description, implementation details, and computer code

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The Beaufort Assessment Model (BAM) with application to black sea bass[1]: mathematical description, implementation details, and computer code

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1 Note

Previously a working paper from SEDAR 25, this document describes an application of the Beaufort Assessment Model (BAM) to black sea bass. For application to gray triggerfish, details of the assessment model would be customized to the stock in question. However, the basic structure of the BAM, described below, is expected to remain the same.

2 Overview

The primary model in this assessment was the Beaufort assessment model (BAM), which applies a statistical catch-age formulation. The model was implemented with the AD Model Builder software (ADMB Foundation 2011), and its structure and equations are detailed in SEDAR-25-RW-03. In essence, a statistical catch-age model simulates a population forward in time while including fishing processes (Quinn and Deriso 1999; Shertzer et al. 2008). Quantities to be estimated are systematically varied until characteristics of the simulated populations match available data on the real population. Statistical catch-age models share many attributes with ADAPT-style tuned and untuned VPAs.

The method of forward projection has a long history in fishery models. It was introduced by Pella and Tomlinson (1969) for fitting production models and then, among many applications, used by Fournier and Archibald (1982), by Deriso et al. (1985) in their CAGEAN model, and by Methot (1989; 2009) in his Stock Synthesis model. The catch-age model of this assessment is similar in structure to the CAGEAN and Stock Synthesis models. Versions of this assessment model have been used in previous SEDAR assessments of reef fishes in the U.S. South Atlantic, such as red porgy, tilefish, snowy grouper, gag grouper, greater amberjack, vermilion snapper, Spanish mackerel, red grouper, and red snapper, as well as in previous benchmark (SEDAR-02) and update assessments of black sea bass.

3 Model configuration and equations

Model equations are detailed in Table 3.1. A general description of the assessment model follows.

Stock dynamics In the assessment model, new biomass was acquired through growth and recruitment, while abundance of existing cohorts experienced exponential decay from fishing and natural mortality. The population was assumed closed to immigration and emigration. The model included age classes $0 - 11^{+}$, where the oldest age class 11^{+} allowed for the accumulation of fish (i.e., plus group).

Initialization Initial (1978) abundance at age was estimated in the model as follows. First, the equilibrium age structure was computed for ages 1–11 based on natural and fishing mortality (F), where F was set equal to the geometric mean fishing mortality from the first three assessment years (1978-1980) scaled by an estimated multiplier (called $F_{\text{init.ratio}}$). Second, lognormal deviations around that equilibrium age structure were estimated. The deviations were lightly penalized, such that the initial abundance of each age could vary from equilibrium if suggested by early age composition data, but remain estimable if data were uninformative. Given the initial abundance of ages 1–11, initial (1978) abundance of age-0 fish was computed using the same methods as recruits in other years.

A normal prior was applied toward the estimation of $F_{\text{init.ratio}}$. The prior had a mean of 1.0 and a CV of 1.0. The large CV defined the prior to be rather loose.

Natural mortality rate The natural mortality rate (M) was assumed constant over time, but decreasing with age. The form of M as a function of age was based on Lorenzen (1996). The Lorenzen (1996) approach inversely relates the natural mortality at age to mean weight at age W_a by the power function $M_a = \alpha W_a^{\beta}$, where α is a scale parameter and β is a shape parameter. Lorenzen (1996) provided point estimates of α and β for oceanic fishes, which were used for this assessment. As in previous SEDAR assessments, the Lorenzen estimates of M_a were rescaled to provide the same fraction of fish surviving from age-1 through the oldest observed age (11 yr) as would occur with constant $M = 0.38$ from the DW. This approach using cumulative mortality is consistent with the findings of Hoenig (1983) and Hewitt and Hoenig (2005).

Growth Mean size at age of the population (total length, TL) was modeled with the von Bertalanffy equation, and weight at age (whole weight, WW) was modeled as a function of total length. Parameters of growth and conversions (TL-WW) were estimated by the DW and were treated as input to the assessment model. The von Bertalanffy parameter estimates from the DW were $L_\infty = 495.9$, $K = 0.177$, and $t_0 = -0.92$. For fitting length composition data, the distribution of size at age was assumed normal with coefficient of variation (CV) estimated by the assessment model. A constant CV, rather than constant standard deviation, was suggested by the size at age data. For estimating CV of size at age, a normal prior distribution was applied, with mean (0.18) and CV (0.22) provided by the DW.

Sex transition Black sea bass is a protogynous hermaphrodite. Proportion female at age was modeled with a logistic function, estimated by the DW. The age at 50% transition to male was estimated to be 3.83 yr.

Female maturity and fecundity Female maturity was modeled with a logistic function; the age at 50% female maturity was estimated to be ~ 1 yr. Annual egg production by mature females was computed as eggs spawned per batch, a function of body weight, multiplied by the number of batches per year. The number of batches per year was fixed at 31, as recommended by the DW (Danson 2009). Maturity and fecundity parameters were provided by the DW and treated as input to the assessment model.

Spawning stock Spawning stock was modeled as population fecundity of mature females (i.e., total annual egg production) measured at the time of peak spawning. For black sea bass, peak spawning was considered to occur at the end of March.

In cases when reliable estimates of fecundity are unavailable, spawning biomass is commonly used as a proxy for population fecundity. The previous assessment of black sea bass (SEDAR-02) modeled spawning stock as total mature biomass. For protogynous stocks, use of total mature biomass, rather than that of females or males only, has been found to provide more reliable estimates of management quantities over a broad range of conditions (Brooks et al. 2008).

Recruitment Expected recruitment of age-0 fish was predicted from spawning stock using the Beverton–Holt spawner-recruit model. Annual variation in recruitment was assumed to occur with lognormal deviations.

Steepness, h, a key parameter of the Beverton–Holt model, can be difficult to estimate reliably (Conn et al. 2010). Thus, a (beta) prior distribution was applied to steepness. The prior distribution was estimated through meta-analysis on data from 94 stocks of marine demersal fishes (Shertzer and Conn In Press).

The standard deviation of recruitment (σ_R) in log space was estimated using a normal prior distribution, with mean of 0.6 and CV of 0.25, as suggested by the meta-analysis of Mertz and Myers (1996).

Landings The model included time series of landings from five fleets: commercial lines, commercial pots, commercial trawls, headboat, and general recreational. The commercial trawl time series was extended through 1990 (trawling was banned in January, 1989 within federal waters of the SAFMC's jurisdiction).

Landings were modeled with the Baranov catch equation (Baranov 1918) and were fitted in units of weight (1000 lb whole weight). The DW provided observed landings back to the first assessment year (1978) for each fleet except general recreational, because the MRFSS started in 1981. Thus for years 1978–1980, general recreational landings were predicted in the assessment model (but not fitted to data), by applying the geometric mean recreational F from the years 1981–1983.

Discards As with landings, discard mortalities (in units of 1000 fish) were modeled with the Baranov catch equation (Baranov 1918), which required estimates of discard selectivities and release mortality probabilities. Discards were assumed to have gear-specific mortality probabilities, as suggested by the DW (lines, 0.07; pots with 1.5-inch panels, 0.05; and pots with 2-inch panels, 0.01). Annual discard mortalities, as fitted by the model, were computed by multiplying total discards (tabulated in the DW report) by the gear-specific release mortality probability.

For the commercial fleets, discards from handline and pot gears were combined, and were modeled starting in 1984 with implementation of the 8-inch size limit. Commercial discards prior to 1984 were considered negligible and not modeled. Data on commercial discards were available from the DW starting in 1993. Thus for years 1984–1992, commercial discards were predicted in the assessment model (but not fitted to data), by applying the geometric mean commercial discard F from the years 1993–1998 (the 10-inch limit began in 1999).

For headboat and general recreational fleets, discard time series were assumed to begin in 1978, as observations from MRFSS indicated the occurrence of recreational discards prior to implementation of the 8-inch size limit. Headboat discard estimates were separated from MRFSS in 1986, and were combined for 1978–1985. Because MRFSS began in 1981, the 1978–1980 general recreational (plus headboat) discards were predicted in the assessment model (but not fitted to data), by applying the geometric mean recreational discard F from the years 1981–1983.

For fishery discard length composition data collected under a size limit regulation, the normal distribution of size at age was truncated at the size limit, such that length compositions of discards would include only fish of sublegal size. Mean length at age of discards were computed from these truncated distributions, and thus average weight at age of discards would differ from those in the population at large. A portion of commercial discards in 2009–2010 consisted of fish that were of legal size as a result of the closed seasons.

Fishing For each time series of landings and discard mortalities, the assessment model estimated a separate full fishing mortality rate (F). Age-specific rates were then computed as the product of full F and selectivity at age. Apical F was compute as the maximum of F at age summed across fleets.

Selectivities Selectivity curves applied to landings and MARMAP survey gears were estimated using a parametric approach. This approach applies plausible structure on the shape of the curves, and achieves greater parsimony than occurs with unique parameters for each age. Selectivities of landings from all fleets were modeled as flat-topped, using a two-parameter logistic function. Selectivities of fishery-dependent indices were the same as those of the relevant fleet.

Selectivity of each fleet was fixed within each block of size-limit regulations, but was permitted to vary among blocks where possible or reasonable. Commercial fisheries experienced three blocks of size-limit regulations: no limit prior to 1983, 8-inch limit during 1983–1999, and 10-inch limit during 1999–2010. Recreational fisheries experienced four blocks of size-limit regulations, which were the same as those of the commercial fisheries but with a 12-inch size limit implemented in 2007.

Age and length composition data are critical for estimating selectivity parameters, and ideally, a model would have sufficient composition data from each fleet over time to estimate distinct selectivities in each period of regulations. That was not the case here, and thus additional assumptions were applied to define selectivities, as follows. Because no age and very few length composition data were available from commercial trawls, selectivity of this fleet was assumed to mirror that of the commercial pots. With no composition data from commercial fleets prior to regulations, commercial line selectivities in the first and second regulatory blocks were set equal, as were commercial pot selectivities, consistent with the DW recommendation that the 8-inch size limit had little effect on commercial fishing. Length composition data from MRFSS were quite noisy, and thus selectivities of recreational headboat and general recreational fleets mirrored each other.

Selectivities of discards were assumed to be dome-shaped. They were partially estimated, assuming that discards consisted primarily of undersized fish, as implied by observed length compositions of discards. The general approach taken was that age-specific values for ages 0–2 were estimated, age 3 was assumed to have full selection, and selectivity for each age 4^+ was set equal to the age-specific probability of being below the size limit, given the estimated normal distribution of size at age. In this way, the descending limb of discard selectivities would change with modification of the size limit. The exception to the above approach was for commercial discards in years 2009–2010, when a commercial quota was in place. For those years, commercial discard selectivity included fish larger than the 10-inch size limit that would have been released during the closed season. The commercial discard selectivity for these years was computed as the combined selectivities of sublegal-sized fish and landed fish from commercial lines and pots, weighted by the geometric mean (2009-2010) of fleet-specific observed discards or landings.

Diffuse priors were used for estimating parameters of selectivity functions. These priors assumed normal distributions with CV = 1.0 and were intended to provide only weak information to help the optimization routine during model execution. Priors help by steering estimation away from parameter space with no response in the likelihood surface. Without these diffuse priors, it is possible during the optimization search that a selectivity parameter could become unimportant, for example if its bounds were set too wide and depending on values of other parameters. When this happens, the likelihood gradient with respect to the aimless parameter approaches zero even if the parameter is not at its globally best value. Diffuse priors help avoid this situation.

Indices of abundance The model was fit to two fishery-independent indices of relative abundance (MARMAP blackfish/snapper traps 1981–1987; and MARMAP chevron traps 1990–2010) and three fishery-dependent indices (headboat 1979–2010; headboat discards 2005–2010; and commercial lines 1993–2010). Predicted indices were conditional on selectivity of the corresponding fleet or survey and were computed from abundance or biomass (as appropriate) at the midpoint of the year. The headboat discard index, although relatively short in duration, tracks young fish and was included as a measure of recruitment strength at the end of the assessment period. All indices were positively correlated, and in most cases, significantly.

Catchability In the BAM, catchability scales indices of relative abundance to estimated population abundance at large. Several options for time-varying catchability were implemented in the BAM following recommendations of the 2009 SEDAR procedural workshop on catchability (SEDAR Procedural Guidance 2009). In particular, the BAM allows for density dependence, linear trends, and random walk, as well as time-invariant catchability. Parameters for these models could be estimated or fixed based on *a priori* considerations. For the base model, the AW assumed time-invariant catchability, following SEDAR-02. For a sensitivity run, however, the AW considered linearly increasing catchability with a slope of 2%, constant after 2003. Choice of the year 2003 was based on recommendations from fishermen regarding when the effects of Global Positioning Systems likely saturated in the southeast U.S. Atlantic (SEDAR 2009). This trend reflects the belief that catchability has generally increased over time as a result of improved technology (SEDAR Procedural Guidance 2009) and as estimated for reef fishes in the Gulf of Mexico (Thorson and Berkson 2010). The value of 2% has been found in other fisheries as well (Zhou et al. 2011).

Biological reference points Biological reference points (benchmarks) were calculated based on maximum sustainable yield (MSY) estimates from the Beverton–Holt spawner-recruit model with bias correction (expected values in arithmetic space). Computed benchmarks included MSY, fishing mortality rate at MSY (F_{MSY}), and spawning stock at MSY (SSB_{MSY}). In this assessment, spawning stock measures population fecundity of mature females. These benchmarks are conditional on the estimated selectivity functions and the relative contributions of each fleet's fishing mortality. The selectivity pattern used here was the effort-weighted selectivities at age, with effort from each fishery (including discard mortalities) estimated as the full F averaged over the last two years of the assessment. The last two years, rather than three (SEDAR custom), was applied because of the implementation of commercial seasonal closures starting in 2009.

Fitting criterion The fitting criterion was a penalized likelihood approach in which observed landings and discards were fit closely, and observed composition data and abundance indices were fit to the degree that they were compatible. Landings, discards, and index data were fitted using lognormal likelihoods. Length and age composition data were fitted using multinomial likelihoods.

The model includes the capability for each component of the likelihood to be weighted by user-supplied values (for instance, to give more influence to stronger data sources). For data components, these weights were applied by either adjusting CVs (lognormal components) or adjusting effective sample sizes (multinomial components). In this application to black sea bass, CVs of landings and discards (in arithmetic space) were assumed equal to 0.05, to achieve a close fit to these time series yet allow some imprecision. In practice, the small CVs are a matter of computational convenience, as they help achieve the desired result of close fits to the landings, while avoiding having to solve the Baranov equation iteratively (which is complex when there are multiple fisheries). Weights on other data components (indices, age/length compositions) were adjusted iteratively, starting from initial weights as follows. The CVs of indices were set equal to the values estimated by the DW. Effective sample sizes of the multinomial components were assumed equal to the number of trips sampled annually, rather than the number of fish measured, reflecting the belief that the basic sampling unit occurs at the level of trip. These initial weights were then adjusted until standard deviations of normalized residuals were near 1.0 (SEDAR24-RW03, SEDAR25-RW05). Weights on four indices (all but the headboat discard index) were then adjusted upward to a value of 2.5 (SEDAR25-RW05), in accordance with the principle that abundance data should be given primacy (Francis 2011), which would seem particularly true when indices are highly correlated.

In addition, a lognormal likelihood was applied to the spawner-recruit relationship. The compound objective function also included several penalties or prior distributions (e.g., on estimated parameters of selectivity functions). Penalties or priors were applied to maintain parameter estimates near reasonable values, and to prevent the optimization routine from drifting into parameter space with negligible gradient in the likelihood.

Model testing Experiments with a reduced model structure indicated that parameters estimated from the BAM were unbiased and could be recovered from simulated data. Further, the general model structure has been through multiple SEDAR reviews. As an additional measure of quality control, black sea bass code and input data were examined for accuracy by multiple analysts. This combination of testing and verification procedures suggest that the assessment model is implemented correctly and can provide an accurate assessment of black sea bass stock dynamics.

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Table 3.1. General definitions, input data, population model, and negative log-likelihood components of the statistical catch-age model applied to black sea bass. Hat notation ($\widehat{*}$) indicates parameters estimated by the assessment model, and breve notation ($\breve{*}$) indicates estimated quantities whose fit to data forms the objective function.

Quantity	Symbol	Description or definition
General Definitions		
Index of years	y	$y \in \{1978 \ldots 2010\}$
Index of ages	a	$a \in \{0 \ldots A\}$, where $A = 11^+$
Index of size-limit periods	r	$r \in \{1 \ldots 4\}$ where $1 = 1978 - 1983$ (no size limit rec or comm), $2 = 1984 - 1998$ (8-inch limit rec and comm), $3 = 1999 - 2006$ (10-inch limit rec and comm), and $4 = 2007 - 2010$ (12-inch limit rec, 10-inch limit comm); 'rec' = recreational, 'comm' = commercial
Index of length bins	l	$l \in \{1 \ldots 41\}$
Length bins	l'	$l' \in \{100, 110, \ldots, 600\text{mm}\}$, with midpoint of 10mm bin used to match length compositions. Largest 10 length bins (TL > 500 mm) treated as a plus group, but retained for weight calculations.
Index of fisheries	f	$f \in \{1 \ldots 5\}$ where 1 = commercial lines, 2 = commercial pots, 3 = commercial trawls, 4 = recreational headboat, 5 = general recreational
Index of discards	d	$d \in \{1 \ldots 3\}$ where 1 = commercial lines and pots (combined), 2 = headboat, 3 = general recreational
Index of CPUE	u	$u \in \{1 \ldots 5\}$ where 1 = MARMAP blackfish/snapper traps, 2 = MARMAP chevron traps, 3 = commercial lines, 4 = headboat, 5 = headboat discards
Input Data		
Observed length compositions	$p^{\lambda}_{(f,d,u),l,y}$	Proportional contribution of length bin l in year y to fishery f, d (landings or discards) or index u
Observed age compositions	$p^{\alpha}_{(f,u),a,y}$	Proportional contribution of age class a in year y to fishery f or index u
Length comp. sample sizes	$n^{\lambda}_{(f,d,u),y}$	Effective number of length samples collected in year y from fishery f, discards d, or index u
Age comp. sample sizes	$n^{\alpha}_{(f,u),y}$	Effective number of age samples collected in year y from fishery f or index u
Observed landings	$L_{f,y}$	Reported landings in year y from fishery f.
CVs of landings	$c^L_{f,y}$	Assumed 0.05 in arithmetic space
Observed abundance indices	$U_{u,y}$	$u = 1$, commercial lines (weight), $y \in \{1993 \ldots 2010\}$ $u = 2$, headboat (weight), $y \in \{1979 \ldots 2010\}$ $u = 3$, headboat discards (numbers), $y \in \{2005 \ldots 2010\}$ $u = 4$, MARMAP chevron trap (numbers), $y \in \{1990 \ldots 2010\}$ $u = 5$, MARMAP blackfish/snapper trap (numbers), $y \in \{1981 \ldots 1987\}$
CVs of abundance indices	$c^U_{u,y}$	$u = \{1 \ldots 5\}$ as above. Annual values estimated from delta-lognormal GLM. Each time series was scaled to its mean
Observed total discards	$D'_{d,y}$	Discards (1000 fish) in year y from fishery d.

Table 3.1. (continued)

Quantity	Symbol	Description or definition
Discard mortality rate	δ_d	Proportion discards by fishery d that die. The DW recommended $\delta_d = 0.07$ for lines, $\delta_d = 0.05$ for 1.5 inch panel pots, and $\delta_d = 0.01$ for 2 inch panel pots.
Observed discard mortalities	$D_{d,y}$	$D_{d,y} = \delta_d D'_{d,y}$
CVs of dead discards	$c^D_{d,y}$	Assumed 0.05 in arithmetic space
Population Model		
Mean length at age	l_a	Total length (midyear); $l_a = L_\infty(1 - \exp[-K(a - t_0 + 0.5)])$ where K, L_∞, and t_0 are parameters estimated by the DW
CV of l_a	$\hat{c}^\lambda_a$	Estimated coefficient of variation of growth, assumed constant across ages
SD of l_a	σ^λ_a	Standard deviation of growth, $\sigma^\lambda_a = \hat{c}^\lambda_a l_a$
Age–length conversion of population	$\psi^u_{a,l}$	$\psi^u_{a,l} = \frac{1}{\sqrt{2\pi}(\sigma^\lambda_a)} \frac{\exp\left[-\left(l'_l - l_a\right)^2\right]}{\left(2(\sigma^\lambda_a)^2\right)}$, the Gaussian density function. Matrix ψ^u is rescaled to sum to one within ages, with the largest size a plus group. This matrix is constant across years and is used only to match length comps of fishery independent indices.
Age–length conversion of landings	$\psi^L_{f,a,l,y}$	$\psi^L_{f,a,l,y} = \psi^u_{a,l}$
Age–length conversion of discards	$\psi^D_{d,a,l,y}$	$\psi^D_{d,a,l,y} = \begin{cases} \frac{1}{\sqrt{2\pi}(\sigma^\lambda_a)} \frac{\exp\left[-\left(l'_l - l_a\right)^2\right]}{\left(2(\sigma^\lambda_a)^2\right)} & : l_a < l_{\text{limit}} \\ 0 & : \text{otherwise} \end{cases}$ where l_{limit} is the size limit for fishery d in year y (and could be treated as ∞ prior to regulations). Annual matrices $\psi^D_{d,\cdot,\cdot,y}$ are rescaled to sum to one within ages, with the largest size a plus group.
Mean length at age of landings and discards	$\xi^{L,D}_{(f,d),a,y}$	Mean length at age from $\psi^L_{f,a,y}$ for landings or $\psi^D_{d,a,y}$ for discards
Individual weight at age of population	w_a	Computed from length at age by $w_a = \theta_1 l_a^{\theta_2}$ where θ_1 and θ_2 are parameters from the DW
Individual weight at age of landings and discards	$w^{L,D}_{(f,d),a,y}$	Computed from length at age by $w^{L,D}_{(f,d),a,y} = \theta_1(\xi^{L,D}_{(f,d),a,y})^{\theta_2}$
Natural mortality rate	M_a	Function of weight at age (w_a): $M_a = \alpha w_a^\beta$, with estimates of α and β from Lorenzen (1996). Lorenzen M_a then rescaled based on Hoenig estimate.
Proportion male at age	ρ_a	Logistic increase with age; assumed constant across years
Proportion female at age	$1 - \rho_a$	Complement of above
Proportion females mature at age	m_a	Logistic increase with age. All males assumed mature.
Batch fecundity at age	E_a	Eggs spawned per batch, $\log(E_a) = \theta_3 + \theta_4 w_a$ where θ_3 and θ_4 are parameters from the DW
Number annual batches at age	b_a	Number of batches spawned per yr; assumed constant (31) across ages
Annual fecundity at age	$\mathcal{F}_a$	$\mathcal{F}_a = b_a E_a$

Table 3.1. (continued)

Quantity	Symbol	Description or definition
Spawning date	t_{spawn}	Fraction denoting the proportional time of year when spawning occurs. Set to 0.25, assuming peak spawning occurs at end of March
Fishery and index selectivities	$s_{(f,u),a,r}$	$s_{(f,u),a,r} = \frac{1}{1+\exp\left[-\widehat{\eta}_{(f,u),r}\left(a-\widehat{\alpha}_{(f,u),r}\right)\right]}$ where $\widehat{\eta}_{(f,u),r}$ and $\widehat{\alpha}_{(f,u),r}$ are estimated parameters. Not all parameters were estimated for each fishery (or index) and each period of regulations; some parameters were fixed as described in the text. For instance, the selectivity of commercial trawls was assumed equal to that of commercial pots. Commercial line selectivity was set equal between the first and second regulatory periods, as was commercial pot selectivity. Selectivity of headboat and general recreational fleets were set equal.
Discard selectivities	$s'_{d,a,r}$	$s'_{d,0,r} = \text{logit}^{-1}(\widehat{s'_0})$, $s'_{d,1,r} = \text{logit}^{-1}(\widehat{s'_1})$, and $s'_{d,2,r} = \text{logit}^{-1}(\widehat{s'_2})$, where $\widehat{s'_a}$ are estimated parameters; $s'_{d,3,r} = 1.0$; $s'_{d,4^+,r}$ set equal to the age-specific probability of total length below the sector-specific size limit in period r. The exception was commercial discard selectivity during the closure, $s'_{1,a,2009-2010}$, modeled as the weighted average of $s'_{1,a,4}$, $s_{1,a,4}$, and $s_{2,a,4}$ (see text).
Fishing mortality rate of landings	$F_{f,a,y}$	$F_{f,a,y} = s_{f,a,y}\widehat{F}_{f,y}$ where $\widehat{F}_{f,y}$ is an estimated fully selected fishing mortality rate by fishery and $s_{f,a,y} = s_{f,a,r}$ for y in the years represented by r
Fishing mortality rate of discards	$F^D_{d,a,y}$	$F^D_{d,a,y} = s'_{d,a,r}\widehat{F}^D_{d,y}$ where $\widehat{F}^D_{d,y}$ is an estimated fully selected fishing mortality rate of discards by fishery
Total fishing mortality rate	$F_{a,y}$	$F_{a,y} = \sum_f F_{f,a,y} + \sum_d F^D_{d,a,y}$
Total mortality rate	$Z_{a,y}$	$Z_{a,y} = M_a + F_{a,y}$
Apical F	F_y	$F_y = \max(F_{a,y})$
Abundance at age	$N_{a,y}$	$N_{0,1978} = \frac{\widehat{R}_0(0.8\varsigma\widehat{h}\phi_{init}-0.2\phi_0(1-\widehat{h}))}{(\widehat{h}-0.2)\phi_{init}}$ $\widehat{N}_{1+,1978}$ equilibrium conditions expected given assumptions about initial fishing mortality (described below) $N_{0,y} = \frac{0.8\widehat{R}_0\widehat{h}S_y}{0.2\phi_0\widehat{R}_0(1-\widehat{h})+(\widehat{h}-0.2)S_y}\exp(\widehat{R}_y)$ for $y > 1978$ $N_{a+1,y} = N_{a,y}\exp(-Z_{a,y}) \quad \forall a \in (0\ldots A-1)$ $N_{A,y} = N_{A-1,y-1}\frac{\exp(-Z_{A-1,y-1})}{1-\exp(-Z_{A,y-1})}$ Parameters $\widehat{R}_0$ (asymptotic maximum recruitment) and $\widehat{h}$ (steepness) are estimated parameters of the spawner-recruit curve, and $\widehat{R}_y$ are estimated annual recruitment deviations in log space. The bias correction is $\varsigma = \exp(\widehat{\sigma_R}^2/2)$, where $\widehat{\sigma_R}^2$ is the estimated variance of recruitment deviations. Quantities ϕ_0, ϕ_{init}, and S_y are described below.
Abundance at age (mid-year)	$N'_{a,y}$	Used to match indices of abundance $N'_{a,y} = N_{a,y}\exp(-Z_{a,y}/2)$
Abundance at age at time of spawning	$N''_{a,y}$	Assumed late March to correspond with peak spawning $N''_{a,y} = \exp(-t_{\text{spawn}}Z_{a,y})N_{a,y}$

Table 3.1. (continued)

Quantity	**Symbol**	**Description or definition**
Unfished abundance at age per recruit at time of spawning	NPR_a	$NPR_0 = 1 \times \exp(-t_{\text{spawn}} M_0)$ $NPR_{a+1} = NPR_a \exp[-(M_a(1-t_{\text{spawn}}) + M_{a+1} t_{\text{spawn}})] \quad \forall a \in (0 \ldots A-1)$ $NPR_A = \frac{NPR_{A-1} \exp[-(M_{A-1}(1-t_{\text{spawn}})+M_A t_{\text{spawn}})]}{1-\exp(-M_A)}$
Initial abundance at age per recruit at time of spawning	NPR_a^{init}	Same calculations as for NPR_a, but including fishing mortality (see Z^{init} below).
Unfished spawning biomass per recruit	ϕ_0	$\phi_0 = \sum_{a=0}^{A} NPR_a \rho_a m_a \mathcal{F}_a$ In units of population fecundity (number eggs) of mature females.
Initial spawning biomass per recruit	ϕ_{init}	$\phi_{init} = \sum_{a=0}^{A} NPR_a^{init} \rho_a m_a \mathcal{F}_a$ In units of population fecundity (number eggs) of mature females.
Spawning biomass	S_y	$\sum_{a=1}^{A} N''_{a,y} \rho_a m_a \mathcal{F}_a$ in units of fecundity (egg number) of mature females.
Initialization mortality at age	Z_a^{init}	$Z_a^{init} = M_a + F^{init}$ where F^{init} is an estimated initialization F, assumed to be the geometric mean fishing mortality from the first three assessment years (1978-1980) scaled by an estimated multiplier $\widehat{F}_{\text{init.ratio}}$
Initial equilibrium abundance at age	N_a^{eq}	Equilibrium age structure given Z_a^{init}
Population biomass	B_y	$B_y = \sum_a N_{a,y} w_a$
Landing at age in numbers	$L'_{f,a,y}$	$L'_{f,a,y} = \frac{F_{f,a,y}}{Z_{a,y}} N_{a,y}[1-\exp(-Z_{a,y})]$
Landing at age in weight	$L''_{f,a,y}$	$L''_{f,a,y} = w^L_{f,a,y} L'_{f,a,y}$
Discard mortalities at age in numbers	$D'_{d,a,y}$	$D'_{d,a,y} = \frac{F^D_{d,a,y}}{Z_{a,y}} N_{a,y}[1-\exp(-Z_{a,y})]$
Discard mortalities at age in weight	$D''_{d,a,y}$	$D''_{d,a,y} = w^D_{d,a,y} D'_{d,a,y}$

Table 3.1. (continued)

Quantity	Symbol	Description or definition
Index catchability	$q_{u,y}$	$q_{u,1978} = \widehat{q}_u^0 f(\text{density})$ $q_{u,y+1} = q_{u,y} f_y(\text{trend}) f_y(\text{random}) f_y(\text{density})$ for $y \geq 1978$ Here, $f_y(\text{density}) = (B'_0)^{\widehat{\psi}}(B'_y)^{-\widehat{\psi}}$, where $\widehat{\psi}$ is a parameter to be estimated, $B'_y = \sum_{a=a'}^{A} B_{a,y}$ is annual biomass above some threshold age a', and B'_0 is virgin biomass for ages a' and greater. In practice, a' should be set high enough to give a reasonable summary of exploitable biomass. The function $f(\text{trend})$ provides a model for linear trend (slope of β_q) in catchability from the start of the index until 2003, where technology effects were thought to saturate (see SEDAR-19 DW report). For example, for an index that starts in 1978, $f_y(\text{trend})$ follows, $f_y(\text{trend}) = \begin{cases} 1.0 & : y = 1978 \\ f_{y-1}(\text{trend}) * (\text{y} - 1978)\beta_{\text{q}} & : 1978 < y \leq 2003 \\ f_{2003}(\text{trend}) & : 2003 < y \end{cases}$ Finally, $f_y(\text{random}) = \exp(\epsilon_{u,y})$ are lognormal catchability deviations which allow for a random walk in catchability when penalties are placed on the $\epsilon_{u,y}$ (see "Objective Function"). For this assessment, catchability was assumed constant in the base run and the catchability function $f_y(\text{trend})$ was applied as a sensitivity run as described in the SEDAR-25 black sea bass assessment. Density dependence and random walks were not applied in the base run.
Predicted landings	$\breve{L}_{f,y}$	$\breve{L}_{f,y} = \sum_a L''_{f,a,y}$
Predicted discard mortalities	$\breve{D}_{d,y}$	$\breve{D}_{d,y} = \sum_a D'_{d,a,y}$
Predicted length compositions of fishery independent data	$\breve{p}^{\lambda}_{u,l,y}$	$\breve{p}^{\lambda}_{u,l,y} = \frac{\sum_a \psi_{a,l} s_{u,a,y} N'_{a,y}}{\sum_a s_{u,a,y} N'_{a,y}}$
Predicted length compositions of landings	$\breve{p}^{\lambda}_{f,l,y}$	$\breve{p}^{\lambda}_{f,l,y} = \frac{\sum_a \psi^L_{f,a,l,y} L'_{f,a,y}}{\sum_a L'_{f,a,y}}$
Predicted length compositions of discards	$\breve{p}^{\lambda}_{d,l,y}$	$\breve{p}^{\lambda}_{d,l,y} = \frac{\sum_a \psi^D_{d,a,l,y} D'_{d,a,y}}{\sum_a D'_{d,a,y}}$
Predicted age compositions	$\breve{p}^{\alpha}_{(f,u),a,y}$	$\breve{p}^{\alpha}_{(f,u),a,y} = \frac{L'_{(f,u),a,y}}{\sum_a L'_{(f,u),a,y}}$
Predicted CPUE	$\breve{U}_{u,y}$	$\breve{U}_{u,y} = \begin{cases} \widehat{q}_{u,y} \sum_a w^L_{u,a,y} N'_{a,y} s_{u,a,r} & : \quad u = 3,4 \\ \widehat{q}_{u,y} \sum_a N'_{a,y} s_{u,a,r} & : \quad u = 1,2,5 \end{cases}$ where $s_{u,a,r}$ is the selectivity of the relevant fishery in the year corresponding to y.

Table 3.1. (continued)

Quantity	Symbol	Description or definition
Objective Function		
Multinomial length compositions	Λ_1	$\Lambda_1 = -\sum_{f,d,u}\sum_{y}\left[\omega^{\lambda}_{(f,d,u)} n^{\lambda}_{(f,d,u),y} \sum_{l}(p^{\lambda}_{(f,d,u),l,y} + x)\log\left(\frac{(\breve{p}^{\lambda}_{(f,d,u),l,y}+x)}{(p^{\lambda}_{(f,d,u),l,y}+x)}\right)\right]$ where $\omega^{\lambda}_{(f,d,u)}$ is a preset weight (selected by iterative re-weighting) and x =1e-5 is an arbitrary value to avoid log zero. The denominator of the log is a scaling term. Bins are 10 mm wide.
Multinomial age compositions	Λ_2	$\Lambda_2 = -\sum_{f,u}\sum_{y}\left[\omega^{\alpha}_{(f,u)} n^{\alpha}_{(f,u),y} \sum_{a}(p^{\alpha}_{(f,u),a,y} + x)\log\left(\frac{(\breve{p}^{\alpha}_{(f,u),a,y}+x)}{(p^{\alpha}_{(f,u),a,y}+x)}\right)\right]$ where $\omega^{\alpha}_{(f,u)}$ is a preset weight (selected by iterative re-weighting) and x =1e-5 is an arbitrary value to avoid log zero. The denominator of the log is a scaling term.
Lognormal landings	Λ_3	$\Lambda_3 = \sum_{f}\sum_{y}\frac{\left[\log\left((L_{f,y}+x)/(\breve{L}_{f,y}+x)\right)\right]^2}{2(\sigma^L_{f,y})^2}$ where x =1e-5 is an arbitrary value to avoid log zero or division by zero. Here, $\sigma^L_{f,y} = \sqrt{\log(1+(c^L_{f,y}/\omega^L_f)^2)}$, with $\omega^L_f = 1$ a preset weight.
Lognormal discard mortalities	Λ_4	$\Lambda_4 = \sum_{d}\sum_{y}\frac{\left[\log\left((\delta_d D_{d,y}+x)/(\breve{D}_{d,y}+x)\right)\right]^2}{2(\sigma^D_{d,y})^2}$ where x =1e-5 is an arbitrary value to avoid log zero or division by zero. Here, $\sigma^D_{d,y} = \sqrt{\log(1+(c^D_{d,y}/\omega^D_d)^2)}$, with $\omega^D_d = 1$ a preset weight.
Lognormal CPUE	Λ_5	$\Lambda_5 = \sum_{u}\sum_{y}\frac{\left[\log\left((U_{u,y}+x)/(\breve{U}_{u,y}+x)\right)\right]^2}{2(\sigma^U_{u,y})^2}$ where x =1e-5 is an arbitrary value to avoid log zero or division by zero. Here, $\sigma^U_{u,y} = \sqrt{\log(1+(c^U_{u,y}/\omega^U_u)^2)}$, with ω^U_u a preset weight (selected by iterative re-weighting).
Lognormal recruitment deviations	Λ_6	$\Lambda_6 = \omega_6\left[\frac{[R_{1978}+(\widehat{\sigma}^2_R/2)]^2}{2\widehat{\sigma}^2_R} + \sum_{y>1978}\frac{[(R_y-\widehat{\varrho}R_{y-1})+(\widehat{\sigma}^2_R/2)]^2}{2\widehat{\sigma}^2_R} + n\log(\sigma_R)\right]$ where R_y are recruitment deviations in log space, n is the number of years, $\omega_6 = 1$ is a preset weight, $\widehat{\varrho}$ is the first-order autocorrelation, and $\widehat{\sigma}^2_R$ is the estimated recruitment variance ($\varrho = 0$ in the SEDAR-25 base run).
Additional constraint on early recruitment deviation	Λ_7	$\Lambda_7 = \omega_7\left[\frac{[R_{1978}+(\widehat{\sigma}^2_R/2)]^2}{2\widehat{\sigma}^2_R} + \sum_{y=1979}^{Y_1}\frac{[(R_y-\widehat{\varrho}R_{y-1})+(\widehat{\sigma}^2_R/2)]^2}{2\widehat{\sigma}^2_R} + n\log(\sigma_R)\right]$ where Y_1 is the last year to apply this additional penalty and ω_7 is a preset weight, with ω_7=0.0 for the SEDAR-25 black sea bass base run.
Additional constraint on final recruitment deviations	Λ_8	$\Lambda_8 = \omega_8\left[\sum_{y=Y_2}^{Y}\frac{[(R_y-\widehat{\varrho}R_{y-1})+(\widehat{\sigma}^2_R/2)]^2}{2\widehat{\sigma}^2_R} + n\log(\sigma_R)\right]$ where Y_2 is the first year to apply this additional penalty, Y is the terminal year, and ω_8 is a preset weight, with ω_8=0.0 for the SEDAR-25 black sea bass base run.

Table 3.1. (continued)

Quantity	**Symbol**	**Description or definition**
Penalty on random walk on catchability	Λ_9	$\Lambda_9 = \omega_9 \sum_u \sum_y \frac{\epsilon^2_{u,y}}{2(\sigma^q_u)^2}$ where ω_9 is a preset weight and σ^q_u is a control variable input by the user defining the standard deviation of the random walk process. As σ^q_u increases, one essentially estimates each deviation as a free parameter, while values close to zero allow little variation in annual catchability. A random walk on catchability was not used for the SEDAR-25 black sea bass base run, thus ω_9=0.0.
Penalty on initial age structure	Λ_{10}	$\Lambda_{10} = \sum_{a=1}^{A} (\widehat{N}_{a,1978} - N^{eq}_a)^2$ where N^{eq}_a is the equilibrium age structure given the initial F, as defined previously.
Prior distributions	Λ_{11}	Λ_{11} is the sum of penalty terms used to implement prior distributions on several parameters. A beta prior was applied to $\widehat{h}$, and normal priors were applied to $\widehat{\sigma}_R$, $\widehat{c^\lambda_a}$, $\widehat{F}_{\text{init.ratio}}$, $\widehat{\eta}_{(f,u),r}$, $\widehat{\alpha}_{(f,u),r}$, and $\widehat{s'}_{0-2}$. Normal distributions required a value to describe variance. Empirical estimates of standard deviation were available and therefore used for c^λ_a and σ_R. All other normal priors assumed CV=1 (i.e., diffuse priors).
Total objective function	Λ	$\Lambda = \sum_{i=1}^{11} \Lambda_i$ Objective function minimized by the assessment model