The Beaufort Assessment Model (BAM) with application to cobia[1]: mathematical description, implementation details, and computer code

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The Beaufort Assessment Model (BAM) with application to cobia[1]: mathematical description, implementation details, and computer code

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1 Note

Previously a working paper from SEDAR 28, this document describes an application of the Beaufort Assessment Model (BAM) to cobia. For application to blueline tilefish, details of the assessment model would be customized to the stock in question. However, the basic structure of the BAM, described below, is expected to remain the same.

2 Overview

The primary model in this assessment was the Beaufort assessment model (BAM), which applies a statistical catch-age formulation. The model was implemented with the AD Model Builder software (Fournier et al. 2012), and its structure and equations are detailed herein. In essence, a statistical catch-age model simulates a population forward in time while including fishing processes (Quinn and Deriso 1999; Shertzer et al. 2008). Quantities to be estimated are systematically varied until characteristics of the simulated population match available data on the real population. Statistical catch-age models share many attributes with ADAPT-style tuned and untuned VPAs.

The method of forward projection has a long history in fishery models. It was introduced by Pella and Tomlinson (1969) for fitting production models and then, among many applications, used by Fournier and Archibald (1982), by Deriso et al. (1985) in their CAGEAN model, and by Methot (1989; 2009) in his Stock Synthesis model. The catch-age model of this assessment is similar in structure to the CAGEAN and Stock Synthesis models. Versions of this assessment model have been used in previous SEDAR assessments in the U.S. South Atlantic, such as red porgy, black seabass, snowy grouper, gag grouper, greater amberjack, vermilion snapper, Spanish mackerel, red grouper, red snapper, and tilefish.

3 Model configuration and equations

Model equations are detailed in Table 3.1. A general description of the assessment model follows.

Stock dynamics In the assessment model, new biomass was acquired through growth and recruitment, while abundance of existing cohorts experienced exponential decay from fishing and natural mortality. The population was assumed closed to immigration and emigration. The model included age classes $1 - 12^{+}$, where the oldest age class 12^{+} allowed for the accumulation of fish (i.e., plus group).

Initialization Initial (1950) abundance at age was computed in the model assuming an equilibrium age structure and fishing mortality rate. The equilibrium age structure was computed for ages $1 - 12^{+}$ based on natural and fishing mortality (F), where F was set equal to the geometric mean fishing mortality from the first three assessment years (1950-1952). This was based on the assumption by the AW panel that the stock was lightly exploited (but less than virgin) prior to the 1950s, particularly during the years following WWII.

Natural mortality rate The natural mortality rate (M) was assumed constant over time, but decreasing with age. The form of M as a function of age was based on Lorenzen (1996). The Lorenzen (1996) approach inversely relates the natural mortality at age to mean weight at age W_a by the power function $\mathrm{M}_a = \alpha W_a^{\beta}$, where α is a scale parameter and β is a shape parameter. Lorenzen (1996) provided point estimates of α and β for oceanic fishes, which were used for this assessment. As in previous SEDAR assessments, the Lorenzen estimates of M_a were rescaled to provide the same fraction of fish surviving from age-1 through the oldest observed age (16 yr) as would occur with constant $M = 0.26$ from the DW. This approach using cumulative mortality is consistent with the findings of Hoenig (1983) and Hewitt and Hoenig (2005).

Growth Mean size at age of the population (fork length, FL) was modeled with the von Bertalanffy equation, and weight at age (whole weight, WW) was modeled as a function of fork length. Parameters of growth and conversions (FL-WW) were estimated by the DW and were treated as input to the assessment model. The von Bertalanffy parameter estimates from the DW were $L_\infty = 1324.4$ mm, $k = 0.27$, and $t_0 = -0.47$ yr. For fitting length composition data, the distribution of size at age was assumed normal with coefficient of variation (CV) estimated by the assessment model. A constant CV, rather than constant standard deviation, was suggested by the size at age data.

Female maturity Females were modeled to be fully mature at age 4 and the proportion mature at ages 1, 2, and 3 were estimated to be 0.0, 0.5, and 0.75 respectively.

Spawning stock Spawning stock was modeled using total mature female biomass measured at the time of peak spawning. For cobia, peak spawning was considered to occur in May. In cases when reliable estimates of fecundity are unavailable, spawning biomass is commonly used as a proxy for population fecundity.

Recruitment Expected recruitment of age-1 fish was predicted from spawning stock using the Beverton–Holt spawner-recruit model. Annual variation in recruitment was assumed to occur with lognormal deviations for the years 1975–2009 only. These deviations were constrained to sum to to 1.0 for the period 1984-2009 when annual age compositions and other data sources providing information on year class strength were available. Estimated recruitment deviations for 1975-1983 were not constrained, and provided a bridge between the data poor period beginning in 1950 and the period when age composition data, which contain information on year class strength, became available (1984) (Methot and Taylor 2011). The ending year of estimated recruitment residuals (2009) is based on the age at full selection and the last year of age composition data.

Landings The model included two time series of combined landings plus discards from 1950-2011: a general recreational fleet and a general commercial fleet. Landings were pooled across all gears in the model. Discards were a small proportion of landings and were combined with landings from the respective fleet after applying discard mortality rates provided by the DW. Commercial and recreational discards were assumed negligible prior to 1983 (the first year of regulation).

The combined landings and discards were modeled with the Baranov catch equation (Baranov 1918) and were fitted in units of weight (1000 lb whole weight, commercial) or numbers of fish (1000 fish, recreational). The DW provided observed commercial landings back to the first assessment year (1950). Observed recreational landings were provided by the DW back to 1981 and hindcasts were provided back to 1955. The hindcasting method was extended an additional five years to the start year of the model (1950).

Fishing Mortality For each time series of removals, the assessment model estimated a separate full fishing mortality rate (F). Age-specific rates were then computed as the product of full F and selectivity at age. Apical F was computed as the maximum of F at age summed across fleets.

Selectivities Selectivity curves applied to landings and CPUE series were estimated using a parametric approach. This approach applies plausible structure on the shape of the curves, and achieves greater parsimony than occurs with unique parameters for each age. Selectivity of landings from the commerical and recreational fleets were modeled as flat-topped, using a two parameter logistic function. Selectivities of the fishery dependent indices (Headboat and South Carolina logbook) were assumed the same as that of the general recreational fleet because all use hook and line gear.

Weak priors were used for estimating slope parameters of both selectivity functions. These priors assumed normal distributions with CV = 0.5 (recreational) or CV=0.25 (commercial) and were intended to provide limited information to help the optimization routine during model execution. Starting values for the slope parameters were based on a method of catch curve analysis that simultaneously estimates selectivity and total mortality, while accounting for age-based variation in natural mortality (Thorson and Prager 2011). Priors help by steering estimation away from parameter space with no response in the likelihood surface. Without these priors, it is possible during the optimization search that a selectivity parameter could become unimportant, for example, if its bounds were set too wide and dependent on values of other parameters. When this happens, the likelihood gradient with respect to the aimless parameter approaches zero even if the parameter is not at its globally best value. Diffuse priors help avoid this situation.

Indices of abundance The model was fit to two indices of relative abundance: the Headboat index (1981-2011) and the South Carolina logbook charterboat index (1998-2011). Predicted indices were conditional on selectivities, which were assumed the same for the two indices given that both use hook and line gear, and were computed from abundance at the midpoint of the year.

Catchability In the BAM, catchability scales indices of relative abundance to estimated population abundance at large. Several options for time-varying catchability were implemented in the BAM following recommendations of the 2009 SEDAR procedural workshop on catchability (SEDAR Procedural Guidance 2009). In particular, the BAM allows for density dependence, linear trends, and random walk, as well as time-invariant catchability. Parameters for these models could be estimated or fixed based on *a priori* considerations. For the base model, the AW assumed time-invariant catchability. For a sensitivity run, however, the AW considered linearly increasing catchability with a slope of 2%, constant after 2003. Choice of the year 2003 was based on recommendations from fishermen regarding when the effects of Global Positioning Systems likely saturated in the southeast U.S. Atlantic (SEDAR 2009). This trend reflects the belief that catchability has generally increased over time as a result of improved technology (SEDAR Procedural Guidance 2009) and as estimated for reef fishes in the Gulf of Mexico (Thorson and Berkson 2010). The value of 2% has been found in other fisheries as well (Zhou et al. 2011). Another sensitivity run applied a random walk approach to estimating catchability. This is notoriously difficult to estimate and often results in the adsorption of noise from the index.

Biological reference points Biological reference points (benchmarks) were calculated based on maximum sustainable yield (MSY) estimates from the Beverton–Holt spawner-recruit model with bias correction (expected values in arithmetic space). Computed benchmarks included MSY, fishing mortality rate at MSY (F_{MSY}), and spawning stock at MSY ($\mathrm{SSB}_{\mathrm{MSY}}$). In this assessment, spawning stock measures total biomass of mature females. These benchmarks are conditional on the estimated selectivity functions and the relative contributions of each fleet's fishing mortality. The selectivity pattern used here was the effort-weighted selectivities at age, with effort from each fishery estimated as the full F averaged over the last three years of the assessment.

Fitting criterion The fitting criterion was a penalized likelihood approach in which observed landings were fit closely, and observed composition data and abundance indices were fit to the degree that they were compatible. Landings and index data were fitted using lognormal likelihoods. Length and age composition data were fitted using robust multinomial likelihoods.

The model includes the capability for each component of the likelihood to be weighted by user-supplied values (for instance, to give more influence to stronger data sources). For data components, these weights were applied by either adjusting CVs (lognormal components) or adjusting effective sample sizes (multinomial components). In this application to cobia, CVs of removals (in arithmetic space) were assumed equal to 0.05, to achieve a close fit to these time series yet allow some imprecision. In practice, the small CVs are a matter of computational convenience, as they help achieve the desired result of close fits to the landings, while avoiding having to solve the Baranov equation iteratively (which is complex when there are multiple fisheries). Weights on other data components (indices, age and length compositions) were adjusted iteratively, starting from initial weights as follows. The CVs of indices were set equal to the values estimated by the DW. Effective sample sizes of the annual length compositions were assumed equal to the annual number of trips sampled. Only number of fish sampled was available for annual age compositions; therefore, effective sample sizes were set to the annual number of fish sampled. Because cobia are caught mostly as one individual fish per trip, the number of fish landed is a probably a good approximation of the number of trips. These initial weights were then adjusted until standard deviations of normalized residuals (SDNRs) were near 1.0 (SEDAR24-RW03, SEDAR25-RW05, Francis 2011). Computed SDNRs accounted for potential correlations in the composition data (TA1.8 in Table A1 of (Francis 2011)). Because only a single pooled age and length composition were available for the commercial fishery, this approach could not be used to derive weights for these data sources. Therefore, weights on commercial age and length compositions were assumed to be the same as those for the comparable recreational data source.

In addition, the compound objective function included several penalties or prior distributions, applied to CV of growth (based on the empirical estimate), the slope of selectivity parameters, and recruitment standard deviation based on Beddington and Cooke (1983) and Mertz and Myers (1996)]. Penalties or priors were applied to maintain parameter estimates near reasonable values, and to prevent the optimization routine from drifting into parameter space with negligible gradient in the likelihood.

Model testing Experiments with a reduced model structure indicated that parameters estimated from the BAM were unbiased and could be recovered from simulated data. Further, the general model structure has been through multiple SEDAR reviews. As an additional measure of quality control, cobia code and input data were examined for accuracy by multiple analysts. This combination of testing and verification procedures suggest that the assessment model is implemented correctly and can provide an accurate assessment of cobia stock dynamics.

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Table 3.1. General definitions, input data, population model, and negative log-likelihood components of the statistical catch-age model applied to cobia. Hat notation ($\widehat{*}$) indicates parameters estimated by the assessment model, and breve notation ($\breve{*}$) indicates estimated quantities whose fit to data forms the objective function.

Quantity	Symbol	Description or definition
General Definitions		
Index of years	y	$y \in \{1950 \ldots 2011\}$
Index of ages	a	$a \in \{1, 2 \ldots A\}$, where $A = 12^+$
Index of length bins	l	$l \in \{1, 2 \ldots 44\}$
Length bins	l'	$l' \in \{200, 370, \ldots, 1970\text{mm}\}$, with midpoint of 30mm bin used to match length compositions. Largest 16 length bins ($FL \geq 1490$ mm) treated as a plus group, but retained for weight calculations.
Index of fisheries	f	$f \in \{1, 2\}$ where 1 = general recreational, 2 = general commercial
Index of CPUE	u	$u \in \{1, 2\}$ where 1 = Headboat, 2 = South Carolina logbook
Input Data		
Observed length compositions	$p^{\lambda}_{(f,u),l,y}$	Proportional contribution of length bin l in year y to fishery f (landings) or index u
Observed age compositions	$p^{\alpha}_{(f,u),a,y}$	Proportional contribution of age class a in year y to fishery f or index u.
Ageing error matrix	$\mathcal{E}$	Estimated from multiple readers ageing the same otoliths.
Length comp. sample sizes	$n^{\lambda}_{(f,u),y}$	Effective number of length samples collected in year y from fishery f or index u
Age comp. sample sizes	$n^{\alpha}_{(f,u),y}$	Effective number of age samples collected in year y from fishery f or index u
Observed landings	$L_{f,y}$	Reported landings in year y from fishery f. Commercial L in 1000 lb whole weight, and recreational L in 1000 fish.
CVs of landings	$c^L_{f,y}$	Assumed 0.05 in arithmetic space
Observed abundance indices	$U_{u,y}$	$u = 1$, Headboat (numbers), $y \in \{1981 \ldots 2011\}$ $u = 2$, South Carolina logbook (numbers), $y \in \{1998 \ldots 2011\}$ Annual values estimated from delta-lognormal GLM. Each time series was scaled to its mean.
CVs of abundance indices	$c^U_{u,y}$	$u = \{1, 2\}$ as above.
Natural mortality rate	M_a	Function of weight at age (w_a): $M_a = \alpha w_a^{\beta}$, with estimates of α and β from Lorenzen (1996). Lorenzen M_a then rescaled based on Hoenig estimate.
Population Model		
Proportion female at age	ρ_a	Considered constant (50:50) across years and ages
Proportion females mature at age	m_a	Increasing with age {0, 0.5, 0.75, 1 ...,1} for ages $1 - 12^+$; assumed constant across years.

Table 3.1. (continued)

Quantity	Symbol	Description or definition
Spawning date	t_{spawn}	Fraction denoting the proportional time of year when spawning occurs. Set to 0.42 for cobia by assuming peak spawning occurs in the end of May.
Mean length at age	l_a	Total length (midyear); $l_a = L_\infty(1 - \exp[-K(a - t_0 + 0.5)])$ where K, L_∞, and t_0 are parameters estimated by the DW
CV of l_a	$\widehat{c}_a^\lambda$	Estimated coefficient of variation of growth, assumed constant across ages
SD of l_a	σ_a^λ	Standard deviation of growth, assumed constant across ages.
Age–length conversion of population	$\psi_{a,l}^u$	$\psi_{a,l}^u = \frac{1}{\sqrt{2\pi}(\sigma_a^\lambda)} \frac{\exp\left[-\left(l_l' - l_a\right)^2\right]}{\left(2(\sigma_a^\lambda)^2\right)}$, the Gaussian density function. Matrix ψ^u is rescaled to sum to one within ages, with the largest size a plus group. This matrix is constant across years.
Age–length conversion of landings	$\psi_{f,a,l,y}^L$	$\psi_{f,a,l,y}^L = \psi_{a,l}^u$
Mean length at age of landings	$\xi_{(f),a,y}^L$	Mean length at age from $\psi_{f,a,y}^L$ for landings.
Individual weight at age of population	w_a	Computed from length at age by $w_a = \theta_1 l_a^{\theta_2}$ where θ_1 and θ_2 are parameters from the DW
Individual weight at age of landings	$w_{(f),a,y}^L$	Computed from length at age by $w_{(f),a,y}^L = \theta_1(\xi_{(f),a,y}^L)^{\theta_2}$
Fishery and index selectivities	$s_{(f,u),a}$	$s_{(f,u),a} = \frac{1}{1+\exp\left[-\widehat{\eta}_{(f,u)}\left(a - \widehat{\alpha}_{(f,u)}\right)\right]}$ where $\widehat{\eta}_{(f,u)}$ and $\widehat{\alpha}_{(f,u)}$ are estimated parameters. Not all parameters were estimated for each fishery or index; some parameters were fixed as described in the text. For instance, the selectivity of the recreational indices (Headboat and South Carolina logbook) were assumed the same as the general recreational fishery.
Fishing mortality rate of landings	$F_{f,a,y}$	$F_{f,a,y} = s_{f,a,y}\widehat{F}_{f,y}$ where $\widehat{F}_{f,y}$ is an estimated fully selected fishing mortality rate by fishery
Total fishing mortality rate	$F_{a,y}$	$F_{a,y} = \sum_f F_{f,a,y}$
Total mortality rate	$Z_{a,y}$	$Z_{a,y} = M_a + F_{a,y}$
Apical F	F_y	$F_y = \max(F_{a,y})$

Table 3.1. (continued)

Quantity	Symbol	Description or definition
Abundance at age	$N_{a,y}$	$N_{1,1950} = \frac{\widehat{R}_0(0.8\varsigma\widehat{h}\phi_{init} - 0.2\phi_0(1-\widehat{h}))}{(\widehat{h}-0.2)\phi_{init}}$ $\widehat{N}_{1+,1950}$ equilibrium conditions expected given assumptions about initial fishing mortality (described below) $N_{1,y+1} = \begin{cases} \frac{0.8\widehat{R}_0\varsigma\widehat{h}S_y}{0.2\phi_0\widehat{R}_0(1-\widehat{h})+(\widehat{h}-0.2)S_y} & \text{for } y < 1975, \\ \frac{0.8\widehat{R}_0\widehat{h}S_y}{0.2\phi_0\widehat{R}_0(1-\widehat{h})+(\widehat{h}-0.2)S_y}\exp(\widehat{R}_{y+1}) & \text{for } y \geq 1975 \end{cases}$ $N_{a+1,y+1} = N_{a,y}\exp(-Z_{a,y}) \quad \forall a \in (0 \ldots A-1)$ $N_{A,y} = N_{A-1,y-1}\frac{\exp(-Z_{A-1,y-1})}{1-\exp(-Z_{A,y-1})}$ $\widehat{R}_0$ (asymptotic maximum recruitment) is an estimated parameter of the spawner-recruit curve, and $\widehat{R}_y$ are estimated annual recruitment deviations in log space for 1975-2009 and zero otherwise. The bias correction is $\varsigma = \exp(\widehat{\sigma_R}^2/2)$, where $\widehat{\sigma_R}^2$ is the estimated variance of recruitment deviations. In the SEDAR-28 baserun, $h = 0.75$ was a fixed parameter. Quantities ϕ_0, ϕ_{init}, and S_y are described below.
Abundance at age (mid-year)	$N'_{a,y}$	Used to match indices of abundance $N'_{a,y} = N_{a,y}\exp(-Z_{a,y}/2)$
Abundance at age at time of spawning	$N''_{a,y}$	Assumed in May to correspond with peak spawning $N''_{a,y} = \exp(-t_{\text{spawn}}Z_{a,y})N_{a,y}$
Unfished abundance at age per recruit at time of spawning	NPR_a	$NPR_0 = 1 \times \exp(-t_{\text{spawn}}M_0)$ $NPR_{a+1} = NPR_a\exp[-(M_a(1-t_{\text{spawn}}) + M_{a+1}t_{\text{spawn}})] \quad \forall a \in (0 \ldots A-1)$ $NPR_A = \frac{NPR_{A-1}\exp[-(M_{A-1}(1-t_{\text{spawn}})+M_At_{\text{spawn}})]}{1-\exp(-M_A)}$
Initial abundance at age per recruit at time of spawning	NPR_a^{init}	Same calculations as for NPR_a, but including fishing mortality (see Z^{init} below).
Unfished spawning biomass per recruit	ϕ_0	$\phi_0 = \sum_{a=0}^{A} NPR_a\rho_a m_a w_a$ In units of mature female weight.
Initial spawning biomass per recruit	ϕ_{init}	$\phi_{init} = \sum_{a=0}^{A} NPR_a^{init}\rho_a m_a w_a$ In units of mature female weight.
Spawning biomass	S_y	$\sum_{a=1}^{A} N''_{a,y}\rho_a m_a w_a$ Spawning biomass is in units of total mature female weight.
Initialization mortality at age	Z_a^{init}	$Z_a^{init} = M_a + s_a^{init}F^{init}$ where F^{init} is an initialization F assumed to be the geometric mean of F from the first three assessment years (1950-1952) and s_a^{init} is the F-weighted average of recreational and commercial selectivity for these three years.
Initial equilibrium abundance at age	N_a^{eq}	Equilibrium age structure given Z_a^{init}
Population biomass	B_y	$B_y = \sum_a N_{a,y}w_a$
Landings at age in numbers	$L'_{f,a,y}$	$L'_{f,a,y} = \frac{F_{f,a,y}}{Z_{a,y}}N_{a,y}[1-\exp(-Z_{a,y})]$

Table 3.1. (continued)

Quantity	**Symbol**	**Description or definition**
Landings at age in whole weight	$L''_{f,a,y}$	$L''_{f,a,y} = w^L_{f,a,y} L'_{f,a,y}$
Index catchability	$q_{u,y}$	$q_{u,1981} = \widehat{q}^0_u f(\text{density})$ $q_{u,y+1} = q_{u,y} f_y(\text{trend}) f_y(\text{random}) f_y(\text{density})$ for $y \geq 1981$ Here, $f_y(\text{density}) = (B'_0)^{\widehat{\psi}} (B'_y)^{-\widehat{\psi}}$, where $\widehat{\psi}$ is a parameter to be estimated, $B'_y = \sum_{a=a'}^{A} B_{a,y}$ is annual biomass above some threshold age a', and B'_0 is virgin biomass for ages a' and greater. In practice, a' should be set high enough to give a reasonable summary of exploitable biomass. The function $f(\text{trend})$ provides a model for linear trend (slope of β_q) in catchability from the start of the index until 2003, where technology effects were thought to saturate (see SEDAR 19 DW report). For example, for an index that starts in 1981, $f_y(\text{trend})$ follows, $f_y(\text{trend}) = \begin{cases} 1.0 & : y = 1981 \\ f_{y-1}(\text{trend}) * (\text{y} - 1981)\beta_\text{q} & : 1981 < y \leq 2003 \\ f_{2003}(\text{trend}) & : 2003 < y \end{cases}$ Finally, $f_y(\text{random}) = \exp(\epsilon_{u,y})$ are lognormal catchability deviations which allow for a random walk in catchability when penalties are placed on the $\epsilon_{u,y}$ (see "Objective Function"). In practice, the catchability function $f_y(\text{trend})$ was used as described for the SEDAR-28 cobia assessment. Density dependence and random walks were not applied in the baserun.
Predicted landings	$\breve{L}_{f,y}$	$\breve{L}_{f,y} = \begin{cases} \sum_a L'_{f,a,y} & : f = 1, \\ \sum_a L''_{f,a,y} & : f = 2 \end{cases}$
Predicted length compositions of fishery independent data	$\breve{p}^\lambda_{u,l,y}$	$\breve{p}^\lambda_{u,l,y} = \frac{\sum_a \psi_{a,l} s_{u,a,y} N'_{a,y}}{\sum_a s_{u,a,y} N'_{a,y}}$
Predicted length compositions of landings	$\breve{p}^\lambda_{f,l,y}$	$\breve{p}^\lambda_{f,l,y} = \frac{\sum_a \psi^L_{f,a,l,y} L'_{f,a,y}}{\sum_a L'_{f,a,y}}$
Predicted age compositions	$\breve{p}^\alpha_{(f,u),a,y}$	$\breve{p}^\alpha_{(f,u),a,y} = \frac{L'_{(f,u),a,y}}{\sum_a L'_{(f,u),a,y}}$
Predicted CPUE	$\breve{U}_{u,y}$	$\breve{U}_{u,y} = \widehat{q}_{u,y} \sum_a N'_{a,y} s_{u,a}$ where $s_{u,a}$ is the selectivity of fishery f in the year corresponding to y.

Table 3.1. (continued)

Quantity	Symbol	Description or definition
Objective Function		
Robust multinomial length compositions	Λ_1	$\Lambda_1 = \sum_{f,u}\sum_{y} 0.5\log(E') - \log\left[\exp\left(-\frac{(p^{\lambda}_{(f,u),l,y} - \breve{p}^{\lambda}_{(f,u),l,y})^2}{2E' \big/ (n^{\lambda}_{(f,u),y}\omega^{\lambda}_{(f,u)})}\right) + x\right]$ where $E' = \left[(1 - p^{\lambda}_{(f,u),l,y})(p^{\lambda}_{(f,u),l,y}) + \frac{0.1}{mbin}\right]$, $mbin$ is the number of length bins, $\omega^{\lambda}_{(f,u)}$ is a preset weight (selected by iterative re-weighting) and x =1e-5 is an arbitrary value to avoid log zero. Bins are 30 mm wide.
Robust multinomial age compositions	Λ_2	$\Lambda_2 = \sum_{f,u}\sum_{y} 0.5\log(E') - \log\left[\exp\left(-\frac{(p^{\alpha}_{(f,u),a,y} - \breve{p}^{\alpha}_{(f,u),a,y})^2}{2E' \big/ (n^{\alpha}_{(f,u),y}\omega^{\alpha}_{(f,u)})}\right) + x\right]$ where $E' = \left[(1 - p^{\alpha}_{(f,u),a,y})(p^{\alpha}_{(f,u),a,y}) + \frac{0.1}{mbin}\right]$, $mbin$ is the number of age bins, $\omega^{\alpha}_{(f,u)}$ is a preset weight (selected by iterative re-weighting) and x =1e-5 is an arbitrary value to avoid log zero.
Lognormal landings	Λ_3	$\Lambda_3 = \sum_{f}\sum_{y} \frac{\left[\log\left((L_{f,y}+x) \big/ (\breve{L}_{f,y}+x)\right)\right]^2}{2(\sigma^{L}_{f,y})^2}$ where x =1e-5 is an arbitrary value to avoid log zero or division by zero. Here, $\sigma^{L}_{f,y} = \sqrt{\log(1 + (c^{L}_{f,y}/\omega^{L}_{f})^2)}$, with $\omega^{L}_{f} = 1$ a preset weight.
Lognormal CPUE	Λ_4	$\Lambda_4 = \sum_{u}\sum_{y} \frac{\left[\log\left((U_{u,y}+x) \big/ (\breve{U}_{u,y}+x)\right)\right]^2}{2(\sigma^{U}_{u,y})^2}$ where x =1e-5 is an arbitrary value to avoid log zero or division by zero. Here, Here, $\sigma^{U}_{u,y} = \sqrt{\log(1 + (c^{U}_{u,y}/\omega^{U}_{u})^2)}$, with ω^{U}_{u} a preset weight.
Lognormal recruitment deviations	Λ_5	$\Lambda_5 = \omega_5\left[\frac{[R_{1975} + (\widehat{\sigma}^2_R \big/ 2)]^2}{2\widehat{\sigma}^2_R} + \sum_{y>1975}^{2009} \frac{[(R_y - \widehat{\varrho}R_{y-1}) + (\widehat{\sigma}^2_R \big/ 2)]^2}{2\widehat{\sigma}^2_R} + n\log(\widehat{\sigma}_R)\right]$ where R_y are recruitment deviations in log space, n is the number of years, $\omega_5 = 1$ is a preset weight, $\widehat{\varrho}$ is the first-order autocorrelation, and $\widehat{\sigma}^2_R$ is the estimated recruitment variance ($\varrho = 0$ in the SEDAR-28 base run).
Additional constraint on early recruitment deviations	Λ_6	$\Lambda_6 = \omega_6\left[\frac{[R_{1975} + (\widehat{\sigma}^2_R \big/ 2)]^2}{2\widehat{\sigma}^2_R} + \sum_{y=1976}^{Y_1} \frac{[(R_y - \widehat{\varrho}R_{y-1}) + (\widehat{\sigma}^2_R \big/ 2)]^2}{2\widehat{\sigma}^2_R} + n\log(\widehat{\sigma}_R)\right]$ where Y_1 is the last year to apply this additional penalty and ω_6 is a preset weight, with ω_6=0.0 for the SEDAR-28 cobia base run.
Additional constraint on final recruitment deviations	Λ_7	$\Lambda_7 = \omega_7\left[\sum_{y=Y_2}^{Y} \frac{[(R_y - \widehat{\varrho}R_{y-1}) + (\widehat{\sigma}^2_R \big/ 2)]^2}{2\widehat{\sigma}^2_R} + n\log(\sigma_R)\right]$ where Y_2 is the first year to apply this additional penalty, Y is the terminal year, and ω_7 is a preset weight, with ω_7=0.0 for the SEDAR-28 cobia base run.
Penalty on random walk on catchability	Λ_8	$\Lambda_8 = \omega_8\sum_{u}\sum_{y} \frac{\epsilon^2_{u,y}}{2(\sigma^q_u)^2}$ where ω_8 is a preset weight and σ^q_u is a control variable input by the user defining the standard deviation of the random walk process. As σ^q_u increases, one essentially estimates each deviation as a free parameter, while values close to zero allow little variation in annual catchability. A random walk on catchability was not used for the SEDAR-28 cobia baserun, thus ω_8=0.0.

Table 3.1. (continued)

Quantity	Symbol	Description or definition
Penalty on initial age structure	Λ_9	$\Lambda_9 = \sum_{a=1}^{A} (\widehat{N}_{a,1984} - N_a^{eq})^2$ where N_a^{eq} is the equilibrium age structure given the initial F, as defined previously. ω_7=0.0 for the SEDAR-28 cobia base run.
Prior distributions and penalties	Λ_{10}	is the sum of penalty terms used to implement prior distributions on several parameters. Normal priors were applied to $\widehat{\eta}_{(f,u)}$. Normal distributions required a value to describe variance. Normal priors assumed CV=0.5 (i.e., diffuse priors) for $\widehat{\eta}_{(1,u)}$ and CV=0.25 for $\widehat{\eta}_{(2,u)}$.
Apical F penalty	Λ_{11}	$\Lambda_{11} = \begin{cases} 0 & :F_{apex} < 3 \\ \omega_{11} \times \exp^{\sqrt{(F_{apex}-1)}} - 1 & :F_{apex} > 3 \end{cases}$ where $\omega_{11} = 0$ for the SEDAR-28 cobia base run.
Total objective function	Λ	$\Lambda = \sum_{i=1}^{11} \Lambda_i$ Objective function minimized by the assessment model