Estimating mutton snapper mortality rates from mean lengths and catch rates in non equilibrium conditions

(SEDAR 14 – RW- 01)

Todd Gedamke and Clay E. Porch

SEDAR26-RD-03

Estimating mutton snapper mortality rates from mean lengths and catch rates in non equilibrium conditions

By

Todd Gedamke and Clay E. Porch
National Marine Fisheries Service
Southeast Fisheries Science Center
75 Virginia Beach Drive
Miami, FL 33149

and

John M. Hoenig
College of William and Mary
Virginia Institute of Marine Science
PO Box 1346
Route 1208 Greate Road
Gloucester Point, Virginia 23062

Sustainable Fisheries Division Contribution SFD-2007-026

In a recent study, Gedamke and Hoenig (2006) derived the transitional form of a mean length mortality estimator for application in non equilibrium conditions. This extension of the Beverton and Holt mortality estimator (1956, 1957) has the same limited data requirements as in the previous formulation and, as such, has the potential for widespread use. The only required information is the von Bertalanffy growth parameters K and L_∞, the so-called length of first capture (smallest size at which animals are fully vulnerable to the fishery and to the sampling gear), L_c, and the mean length of the animals above the length L_c. Unlike the Beverton and Holt mortality estimator (1956, 1957), however, the assumption of equilibrium conditions is not a requirement of the Gedamke and Hoenig (2006) approach. The methodology and an application to goosefish are described in detail in Gedamke and Hoenig (2006) and a summary of the approach and an application to mutton snapper is described in SEDAR14-AW-05.

In this study, we develop an approach to incorporate information from a time-series of catch indices into the mean length estimator to both better detect changes in mortality, and to estimate total mortality rates. The motivation to pursue this research stemmed from the application of the Gedamke and Hoenig (2006) approach to mutton snapper, the high variability of available mean length data, and the resulting uncertainty in total mortality estimates (see SEDAR14-AW-05). Theoretically this proved interesting as catch rates alone can not be used to estimate absolute total mortality rates.

To explain, let us first consider what information catch rates can tell us about total mortality in the simplest scenario when we assume equilibrium conditions, constant recruitment (R), and a constant catchability coefficient (q). The catch index (I) will be a function of the probability of an individual being captured by one unit of effort (q) and the abundance (N) as;

$$I = q \cdot N \quad . \tag{1}$$

Assuming constant recruitment and that total mortality (*Z*) is constant with age, then abundance will be related to total mortality as (Ricker, 1975),

$$N = \frac{R}{Z} \quad . \tag{2}$$

By substitution of eqn (2) into (1) we get

$$I = \frac{q \cdot R}{Z} = \frac{q*}{Z} \tag{3}$$

where $q*$ now represents an unknown scaling parameter that includes both the catchability coefficient (q) and recruitment (R). Thus, unless q* (ie. both q and R) is known, catch rates (I_1 and I_2) from two different mortality levels (Z_1 and Z_2),

$$I_1 = \frac{q*}{Z_1} \quad \text{and} \quad I_2 = \frac{q*}{Z_2} \tag{4}$$

can tell us the relative change in total mortality rates if q^* is assumed to be constant,

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} \qquad . \qquad (5)$$

In real world situations, obtaining reliable estimates of both q and R is extremely rare, therefore estimating absolute values of total mortality from catch rates alone is unlikely. However, since there is information on relative mortality rates we can incorporate this into our length-based non equilibrium mortality estimator.

To accomplish this in a non equilibrium framework we must first recognize that overall abundance, and the corresponding catch index, will not respond instantaneously to changes in total mortality. Equation 3 will only reflect the new mortality rate when enough time has passed for the new equilibrium age structure to be achieved. Using the derivations in Gedamke and Hoenig (2006) we can calculate a relative abundance, in non equilibrium conditions, d years after a permanent change in total mortality from Z_1 to Z_2 yr^{-1}. Let $g = t_c + d$ where t_c is the age at which animals become recruited to the gear. The catch index (I_d) in the population d years after the change in mortality will be equal to,

$I_d = q^* \cdot N_d$ with N_d calculated as,

$$N_d = \int_{t_c}^{g} \exp(-Z_2(t - t_c))dt + \int_{g}^{\infty} \exp(-Z_2 d)\exp(-Z_1(t - g))dt \qquad . \qquad (6)$$

The integral on the left represents fish recruited after the change in mortality – these animals have experienced just the mortality rate Z_2. The integral on the right represents fish that were recruited before the change in mortality – these fish have experienced both the old and the new mortality rates. To illustrate the calculations in equation (6), consider Figure 1 where the response of catch indices to a hypothetical change in total mortality from 0.2 yr^{-1} to 0.4 yr^{-1} is presented. Note that the values on the y-axis are scaled by q* so the absolute values are directly related to recruitment (R) and the catchability coefficient (q).

After integration and simplification equation (6) becomes

$$
\begin{aligned}
N_d &= \frac{1-\exp(-Z_2 d)}{Z_2} + \frac{\exp(-Z_2 d)}{Z_1} \\
&= \frac{Z_1 + (Z_2 - Z_1)\exp(-Z_2 d)}{Z_1 Z_2}
\end{aligned} \quad . \quad (7)
$$

The equation can be modified to incorporate multiple changes in mortality (see generalized equation in appendix B of Gedamke and Hoenig, 2006).

Integrating catch rates and mean lengths

Now that we have laid the groundwork for predicting the relative change in catch rates we can begin to construct a model which incorporates both changes in mean lengths and changes in catch rates. We use the method of maximum likelihood estimation to estimate the year of change and the values of Z_1 and Z_2 that cause predicted mean lengths and catch rates to best match the corresponding time series of data. The probability density function of a normally distributed sample mean $\bar{x}$, when the sample size is m, is

$$f\left(x;\mu,\sigma^2\right)=\frac{\sqrt{m}}{\sqrt{2\pi\sigma^2}}\cdot e^{-\frac{m}{2\sigma^2}\cdot(\bar{x}-\mu)^2} \quad . \qquad (8)$$

The product likelihood function (Λ) for n years of observations (generalized in terms of observed and predicted for application to both mean lengths and indices) is thus,

$$\Lambda=\prod_{y=1}^{n}\frac{m_y}{\sqrt{2\pi\sigma^2}}\cdot e^{-\frac{m_y}{2\sigma^2}\cdot[Observed_y-Predicted_y]^2} \qquad (9)$$

The log-likelihood is proportional to

$$\ln(\Lambda)\propto -n\cdot(\ln\sigma)\ -\frac{1}{2\sigma^2}\cdot\sum_{y=1}^{n}m_y\cdot[Observed_y-Predicted_y]^2 \quad . \qquad (10)$$

To incorporate changes in catch rates and changes in mean lengths we constructed the joint likelihood function as

$$\begin{aligned}\ln(\Lambda)\propto & -n_l\cdot(\ln\sigma_l)-\frac{1}{2\sigma_l^2}\cdot\sum_{y=1}^{n}m_{l,y}\cdot[\bar{L}_y-L_{pred,y}]^2 \\ & -n_i\cdot(\ln\sigma_i)-\frac{1}{2\sigma_i^2}\cdot\sum_{y=1}^{n}m_{i,y}\cdot[I_y-I_{pred,y}]^2\end{aligned} \quad . \qquad (11)$$

The left (or top) part of the function is for the mean lengths where $m_{l,y}$ is the number of fish greater than size L_c measured in year y (or, as is the case for mutton snapper, the number measured on each interview record; a decimal year value is used to correspond to the sampling date), $\bar{L}_y$ is the observed mean length in year y, and $L_{pred,y}$ is the predicted mean length as computed in Gedamke and Hoenig (2006). The right (or bottom) part of the function is for the catch rates where $m_{i,y}$ is the number of observations used to calculate the index in year y, I_y is the index in year y, and $I_{pred,y}$ is the index as predicted by equation (7).

Equation (11) was maximized, and confidence intervals generated for each variable, using the PROC NLP procedure in SAS version 8 (SAS, 1999). Note that including the catch rate data, in addition to the mean length data, required two additional parameters to be estimated: q* in equation (6) and a standard deviation (σ_i) for the catch data. Initially the year in which the mortality change occurred was specified and q*, Z_1, Z_2, and the population standard deviations for both mean length and catch rate data (σ_l, σ_i) were estimated. Once the model proved capable of estimating those 5 parameters from the data, the year of change was also estimated along with the other parameters. Extensive grid searches were conducted over starting estimates for parameters to insure that local maxima were not being identified and the solutions presented represented the best fit of the model to the data.

Application to the mutton snapper pot fishery in Puerto Rico

The application of the Gedamke and Hoenig (2006) approach is described in detail in SEDAR14-AW-05 and only a brief description and pertinent results will be repeated here. Only the pot fishery of Puerto Rico contained enough information to conduct an analysis and those results are presented in Tables 1a and 1b. The results presented from the previous analysis and those of the current study are on the revised data set which was modified by the removal of three inconsistent data points (see Revision of SEDAR14-AW-05 document).

The first step, as described in SEDAR14-AW-05, was to obtain the estimate of length at full vulnerability (L_c) of 30 cm. Mean lengths were then calculated by interview day from all individuals greater than L_c. Standardized catch rates for the Puerto Rico pot fishery were taken from the Addendum to the SEDAR-DW-01 report; which were updated to include 2006 observations. The log likelihood function for mean lengths and catch rates was first constructed assuming one change in mortality and estimates for the unknown parameters were generated (Table 2). Note that unlike the mean length only approach (Table 1), the likelihood function was always weighted by sample size as significant differences in sample sizes in mean lengths and catch rates were present. Mortality was estimated to have increased from 0.47 yr^{-1} to 0.80 yr^{-1} by the end of 1988 (i.e. 1988.9). Confidence intervals were relatively narrow for both the year of change (1987-1991) and for the first level of mortality (0.42 yr^{-1} – 0.53 yr^{-1}), and less so for the second level of mortality (0.63 yr^{-1} – 0.96 yr^{-1}).

An examination of the predicted versus observed values showed a clear pattern to the residuals in the catch indices (Figure 2 and Figure 3) suggesting that the model did not fit the data well. Thus, the likelihood was reformulated to include an additional change in mortality and parameters were estimated again. Note that the penalty for increasing the complexity of the model was the need to estimate two additional parameters: Z_3, and the second year of change. Estimates of the first year of change and the initial mortality rate (first change = 1989, Z_1 = 0.47 yr^{-1}) remained virtually unchanged from those of the simpler model. The initial magnitude of change, however, was estimated to be more substantial with mortality increasing to 1.21 yr^{-1} rather than our earlier estimate of 0.80 yr^{-1} (Table 3). A second change in mortality was detected in 1998 and mortality was estimated to have been reduced to 0.50 yr^{-1}. Confidence intervals for both years of change and the initial mortality rate were relatively narrow, while those of the Z_1 and Z_2 indicate some uncertainty in these estimates (Table 3). An examination of the residuals showed that mean lengths tended to be overestimated after 1998 while the obvious pattern to the residuals of the indices in the one break model was largely removed (Figure 4 and Figure 5). The objective function was reduced by 6.5 which was shown to be significant (P=0.0015) given the two additional parameters.

A comparison of the results from the mean length only and the model with the catch rates integrated into the estimation process is interesting. The mean length only model predicted the same initial increase in mortality rates but when an additional change in mortality was added, the improvement in the objective function was not significant. There simply wasn't enough information in the mean length data to justify the including two additional parameters and even though there appeared to be a decrease around 1999, this estimate could not be deemed reliable (see Table 1). Incorporating the catch indices proved successful even with the penalty of having to estimate two additional parameters (q, and a second standard deviation).

Although the predicted catch indices track the observed values relatively well, the high variability of mean length data results in some uncertainty surrounding the estimates of total mortality since 1990 (Z_2 and Z_3). This is expected when we consider that the clear trend in the catch indices will only provide information on the relative change in mortality and information from the mean lengths is required to estimate the absolute mortality rates. Thus, the variability of the mean length data is reflected in the uncertainty in the absolute estimates of total mortality in the most recent time periods.

There is also some disagreement in the signals from the catch and mean length data that is apparent when we look at the residuals for both data sets. For the catch indices, there is no pattern following 1998 but between 1990 and 1998 the predicted indices are always slightly greater than the observed values. For the mean lengths also, the predicted values are greater than a majority of the observed values. This reflects the conflicting information that is present between the data sets and the resulting estimates are essentially splitting the difference. In essence, the catch indices are indicating a greater relative reduction in total mortality than is supported by the information in the mean length data. This can be seen in a side by side comparison of the previous mean length only analysis and the current results (Figure 6). Although a second change in both analyses suggests a reduction in mortality the magnitude of this change is markedly different. Deciding which estimate is appropriate is extremely difficult given the variability of the mean length data. Alternate weighting schemes may be appropriate if we can quantify the reliability of each data sets. The range of estimates presented here should be used cautiously while slightly more weight can be given to the historical pattern of total mortality. Reliable estimates of total mortality from this approach will only be possible through a more intensive standardized sampling program that includes length frequency data.

References

Beverton, R.J.H. and S.J. Holt. 1956. A review of methods for estimating mortality rates in fish populations, with special reference to sources of bias in catch sampling. Rapports et Proces-verbaux des Reunions, Conseil International Pour L'Exploration de la Mer 140:67-83.

Beverton, R.J.H. and S.J. Holt. 1957. On the dynamics of exploited fish populations. U.K. Ministry of Agriculture, Fisheries, Food, and Fishery Investigations Series II, Vol. XIX.

Gedamke, T. and J.M. Hoenig. 2006. Estimating mortality from mean length data in nonequilibrium situations, with application to the assessment of goosefish. Transactions of the American Fisheries Society 135: 476-487.

Ricker, W.E. 1975. Computation and interpretation of biological statistics of fish populations. Bull. Fish. Res. Board Can. No. 191.

Table 1a. Total mortality rates for the pot fishery of Puerto Rico as estimated from the mean length approach assuming one change in mortality (from revised SEDAR14-AW-05).

Mean Lengths computed by:	Function weighted by Sample Size	Estimated First Mortality Rate (Z_{ONE})	Estimated Second Mortality Rate Z_{TWO}	Estimated Year of Change
Year	**No**	**0.457**	**0.969**	**1992.78**
Year	**Yes**	**0.476**	**0.946**	**1992.65**
Month	**No**	**0.434**	**0.848**	**1987.96**
Month	**Yes**	**0.477**	**0.955**	**1993.65**
Interview Day	**No**	**0.392**	**0.845**	**1988.52**
Interview Day	**Yes**	**0.476**	**0.955**	**1993.60**

Table 1b. Results of the mean length only model that includes two changes in mortality and three different total mortality rates. Note that model fit was not improved significantly (through a likelihood ratio test) by the addition of the two new parameters. However there is some indication in the weighted model that total mortality may have been reduced around 1999 (from revised SEDAR14-AW-05).

Mean Lengths computed by:	Function weighted by Sample Size	Estimated First Mortality Rate (Z_{ONE})	Estimated Second Mortality Rate (Z_{TWO})	Estimated Third Mortality Rate (Z_{THREE})	Estimated First Year of Change	Estimated Second Year of Change
Year	**No**	**0.47**	**0.00**	**0.98**	**1991.02**	**1992.20**
Year	**Yes**	**0.48**	**1.18**	**0.73**	**1993.21**	**1998.84**
Month	**No**	**0.48**	**0.00**	**0.86**	**1985.54**	**1987.11**
Month	**Yes**	**0.48**	**1.17**	**0.75**	**1993.90**	**1999.30**
Interview Day	**No**	**0.39**	**0.70**	**0.89**	**1987.79**	**1994.50**
Interview Day	**Yes**	**0.48**	**1.17**	**0.76**	**1993.84**	**1999.25**

Table 2. Results of model assuming one change in mortality and incorporating both mean lengths and catch data. SD is population standard deviation.

			Confidence	Intervals
Parameter	**Estimate**	**Std Err**	**Lower**	**Upper**
Z_{one}	**0.474**	**0.028**	**0.418**	**0.529**
Z_{two}	**0.798**	**0.084**	**0.633**	**0.963**
Change Year (/100)	**19.889**	**0.010**	**19.869**	**19.909**
q*	**0.839**	**0.138**	**0.569**	**1.108**
Mean Lengths SD	**13.017**	**0.514**	**12.010**	**14.023**
Index SD	**39.115**	**6.902**	**25.587**	**52.642**

Objective function value = -1058.154609

Table 3. Results of model assuming two changes in mortality and incorporating both mean lengths and catch data. SD is population standard deviation.

Parameter	Estimate	Std Err	Confidence Intervals Lower	Upper
Z_{one}	**0.472**	**0.029**	**0.415**	**0.529**
Z_{two}	**1.210**	**0.227**	**0.765**	**1.655**
Z_{three}	**0.501**	**0.084**	**0.336**	**0.665**
First Change Year (/100)	**19.889**	**0.005**	**19.880**	**19.899**
Second Change Year (/100)	**19.982**	**0.007**	**19.968**	**19.997**
q*	**0.825**	**0.112**	**0.606**	**1.045**
Mean Lengths SD	**13.293**	**0.538**	**12.239**	**14.347**
Index SD	**17.947**	**4.364**	**9.394**	**26.500**

Value of Objective Function = -1051.675646

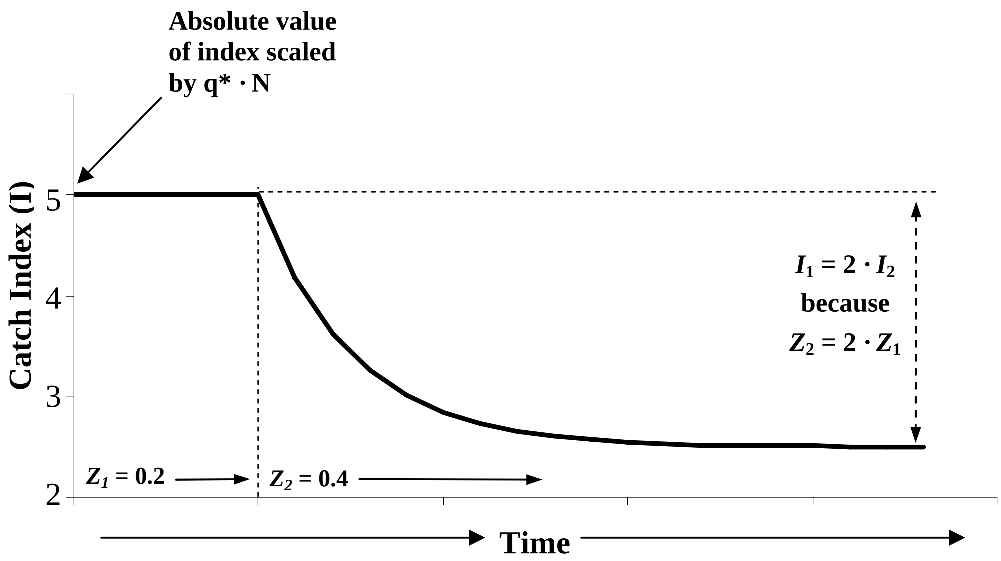


Figure 1. Response of catch indices to a 100% increase in total mortality from 0.2 to 0.4 yr^{-1}. Note that the values on the y-axis are scaled by q* so the absolute values are directly related to recruitment (R) and the catchability coefficient (q).

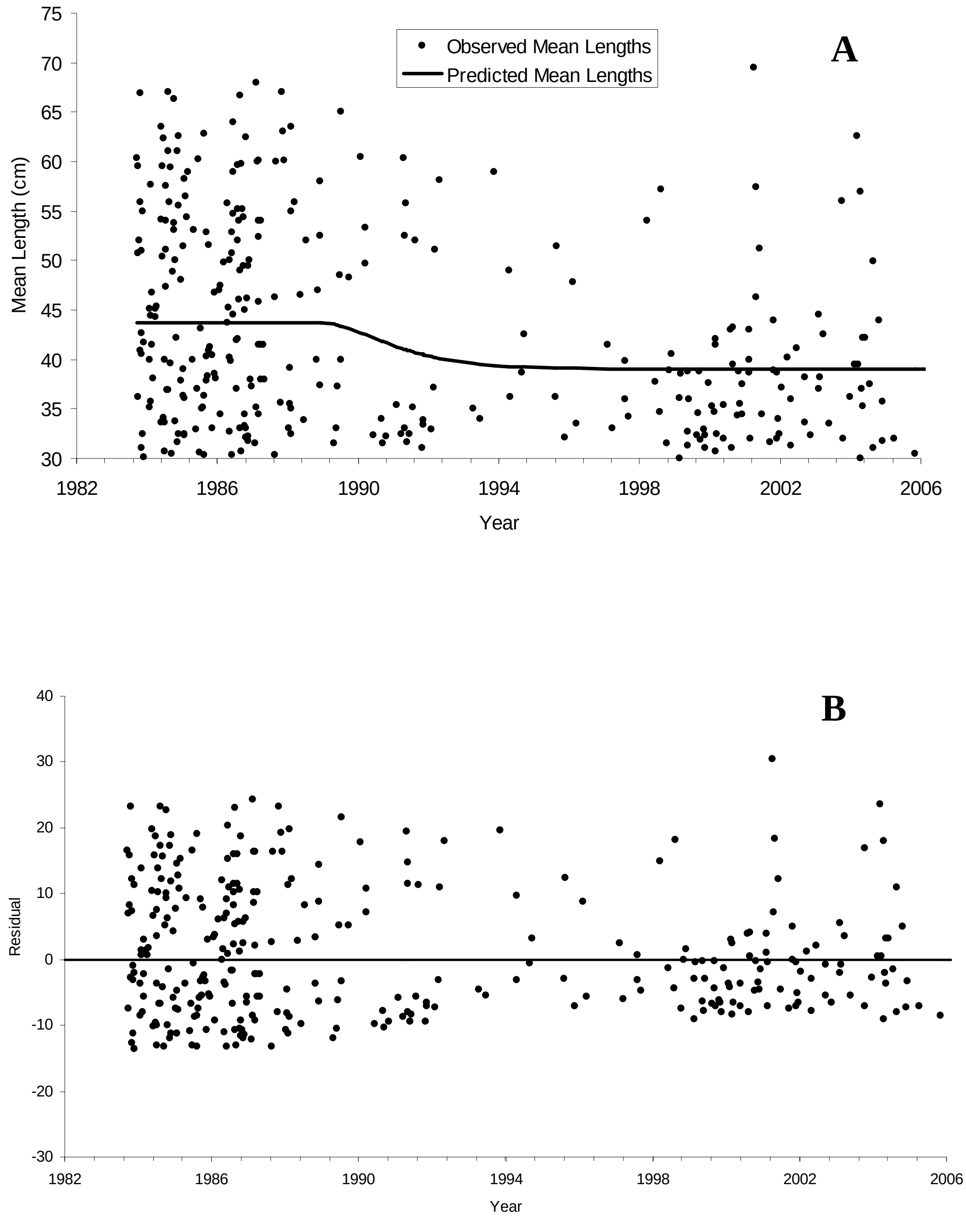


Figure 2. A) Predicted versus observed mean lengths, and residuals (B) for each interview record assuming two changes in mortality and incorporating both mean length and catch data.

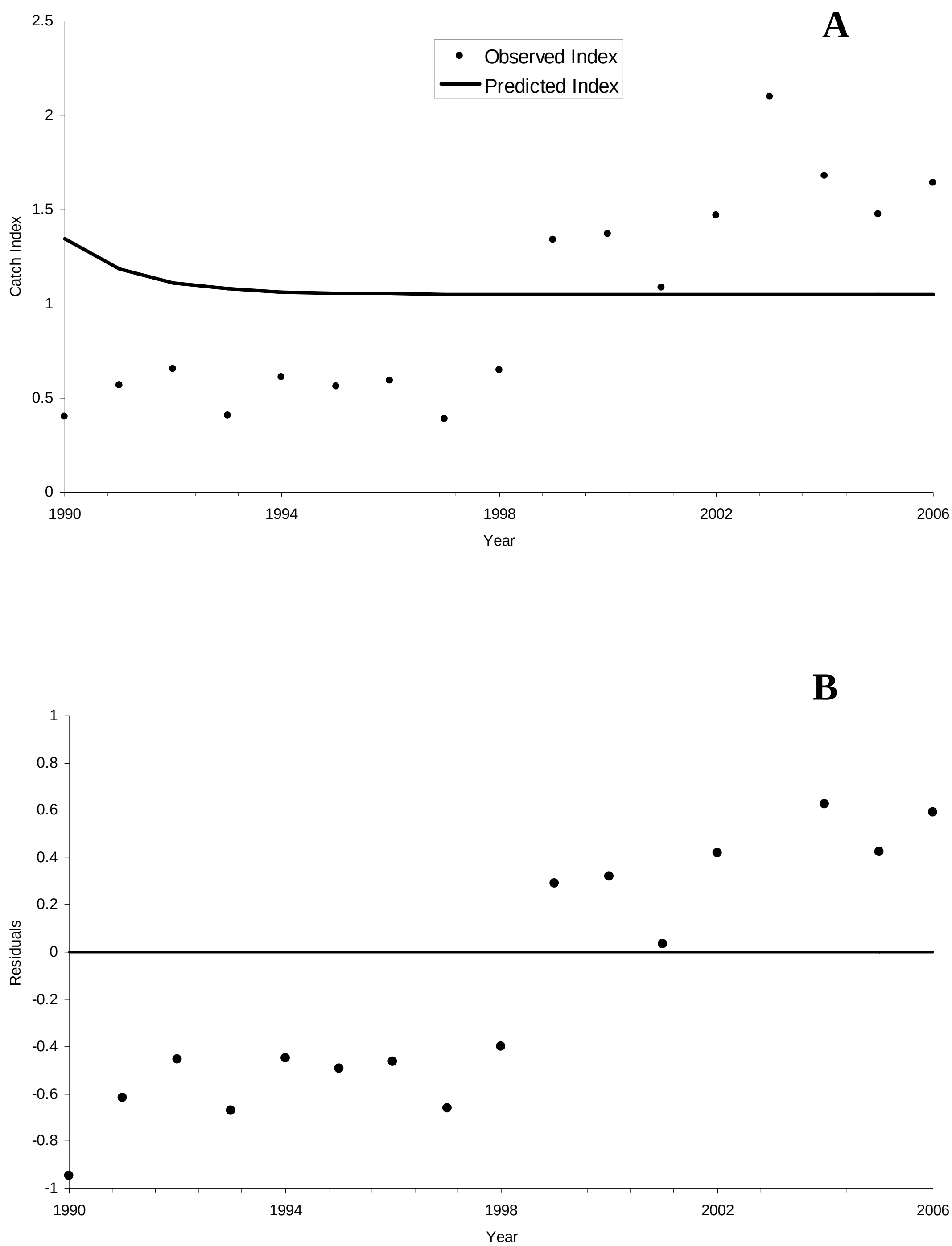


Figure 3. A) Predicted versus observed indices, and residuals (B) for each year assuming one change in mortality and incorporating both mean length and catch data.

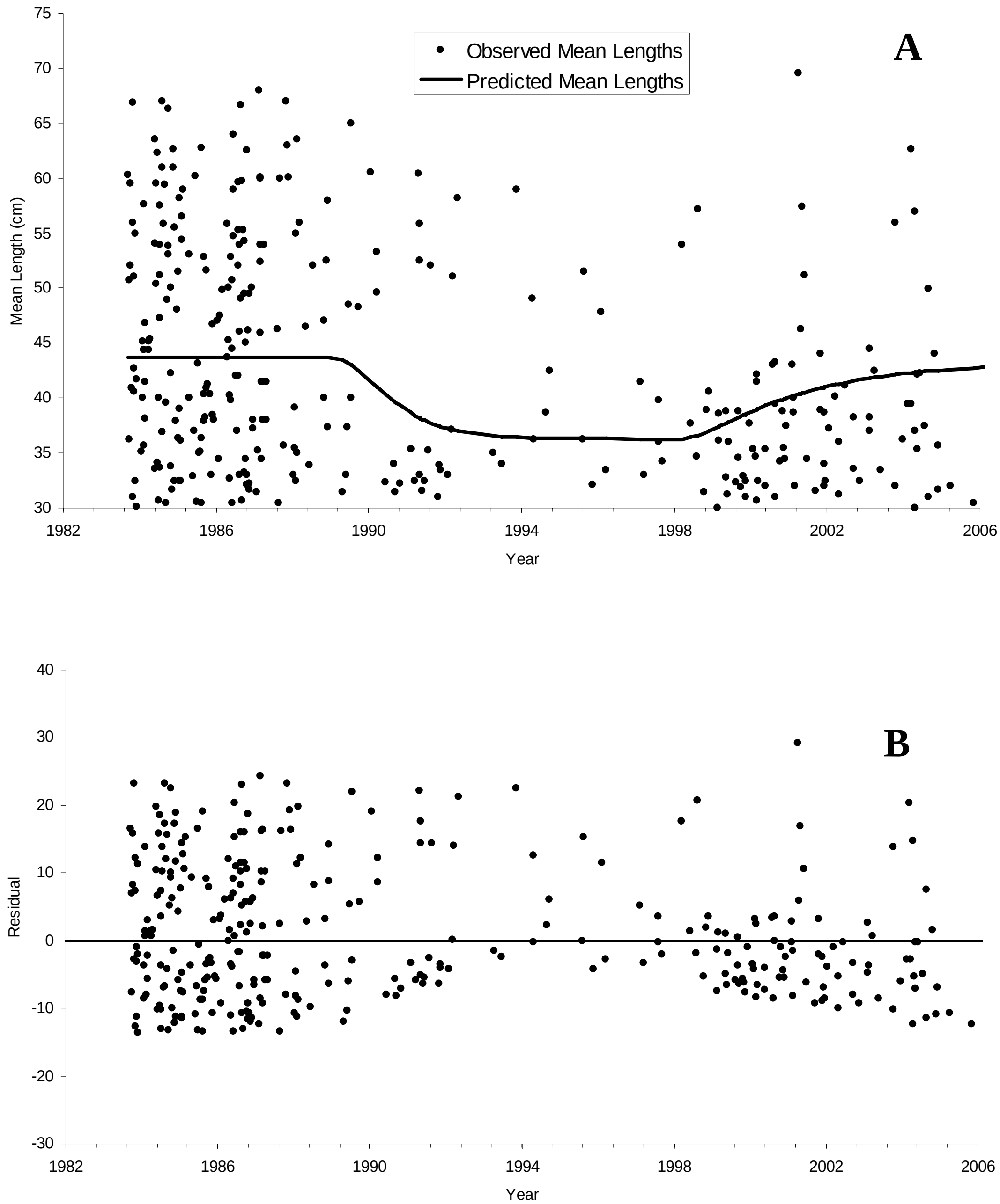
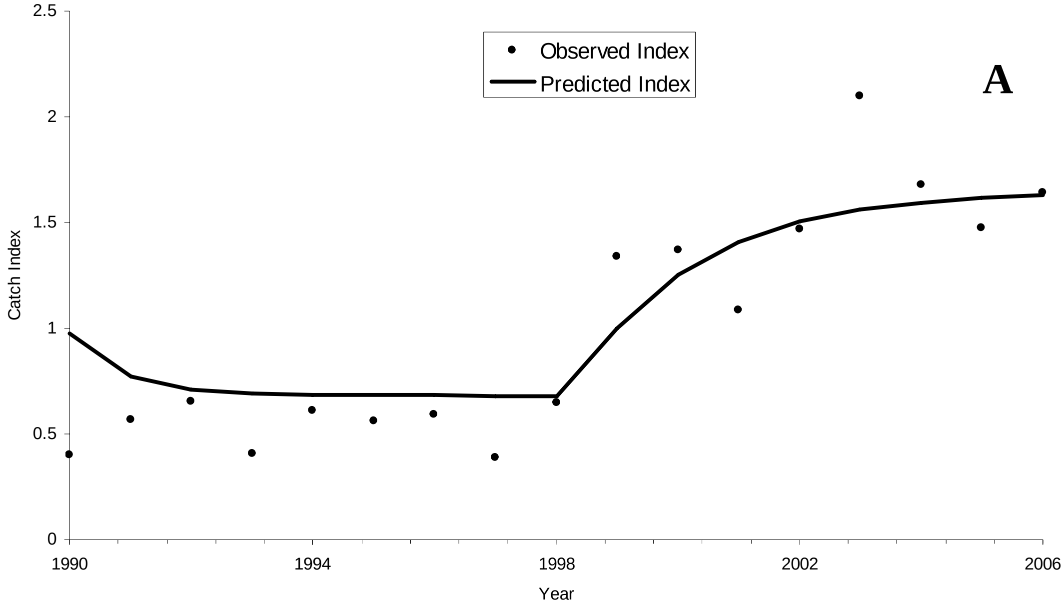


Figure 4. A) Predicted versus observed mean lengths, and residuals (B) for each year assuming two changes in mortality and incorporating both mean length and catch data.



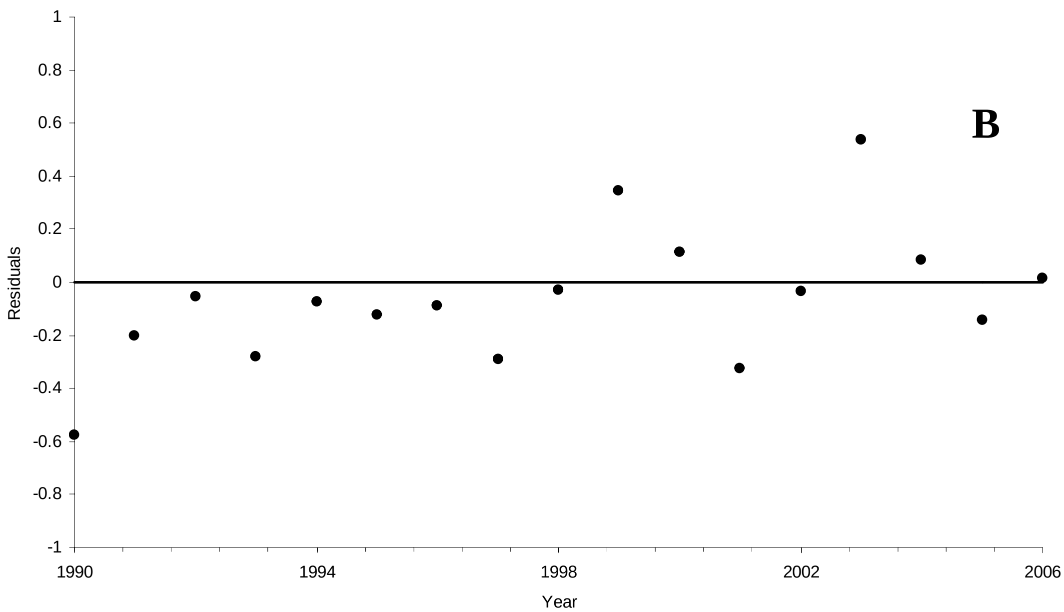


Figure 5. A) Predicted versus observed indices, and residuals (B) for each year assuming two changes in mortality and incorporating both mean length and catch data.

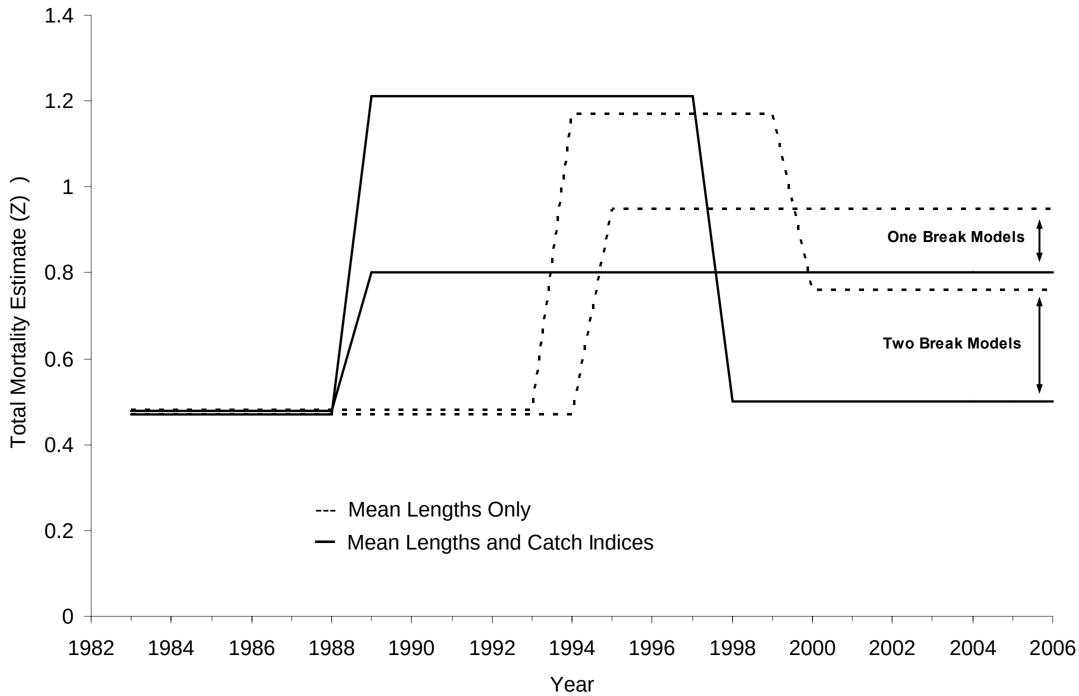


Figure 6. Results of both the mean length only analysis and the analysis which incorporates catch rates into the estimation process. In both cases, results for when the model assumes either one or two changes in mortality are presented.