# Estimation of mutton snapper total mortality rate from length observations ${ }^{1}$ 

by

Todd Gedamke<br>National Marine Fisheries Service<br>Southeast Fisheries Science Center<br>75 Virginia Beach Drive<br>Miami, FL 33149S

A review of the length frequency data available from the NMFS Trip Interview Program (TIP) database showed that sample sizes were insufficient for a comprehensive analysis in most cases. In both Puerto Rico and the U.S. Virgin Islands, the numbers of sampled yellowfin grouper were insufficient for a length-frequency analysis in all cases (See SEDAR14-DW commercial data report). For mutton snapper, however, a relatively large number of individuals were measured from both the trap and hook-and-line fishery of Puerto Rico. All other combinations of gear types and location proved to contain too few individuals. The following document describes the exploration of these data and attempts at estimating total mortality ( Z ) from a newly derived variant of the BevertonHolt length-based mortality estimator (Beverton and Holt 1956, 1957).

The Beverton-Holt mortality estimator has received widespread use, especially in data-limited situations, because the only required information is the von Bertalanffy growth parameters $K$ and $L_{\infty}$, the so-called length of first capture (smallest size at which animals are fully vulnerable to the fishery and to the sampling gear), $L_{\mathcal{C}}$, and the mean length of the animals ( $\bar{L}$ ) above the length $L_{c}$ :

$$
Z=\frac{K\left(L_{\infty}-\bar{L}\right)}{\bar{L}-L_{c}} .
$$

There are six assumptions behind this method.

1) Asymptotic growth with known parameters $K$ and $L_{\infty}$ which are constant over time.
2) No individual variability in growth.
3) Constant and continuous recruitment over time.

[^0]4) Mortality rate is constant with age for all ages $t>t_{c}$.
5) Mortality rate is constant over time.
6) Population is in equilibrium (i.e., enough time has passed following any change in mortality that mean length now reflects the new mortality level).

The method has been criticized, however because the assumption of equilibrium (6) is very difficult to meet in the real world situations where any change in fishing pressure disrupts the equilibrium stable age distribution. In the case of increased fishing pressure, it simply takes time for the larger and older animals to be removed from the population and the mean length to decrease and reflect the current mortality rate. When fishing pressure is decreased, equilibrium takes even longer to achieve as only time will allow the smaller/ younger animals to grow and the mean length to increase and reflect the current mortality rate (see Table 1 for an example).

Gedamke and Hoenig (2006) developed an extension of the Beverton-Holt lengthbased mortality estimator for use in non-equilibrium situations. This method is attractive because it still only requires minimal data that are commonly available and it does not require the assumption that catch rate is proportional to abundance. It allows for the broader application of a mean length analysis approach by removing an equilibrium assumption that is typically difficult to meet in real world situations. In addition, the transitional form of the model allows mortality estimates to be made within a few years of a change rather than having to wait for the mean lengths to stabilize at their new equilibrium level. In other words, as soon as a decline in mean lengths is detected, this model can be applied and the trajectory of decline can be used to estimate the new Z and how mean lengths will change over time.

The method is described in detail in Gedamke and Hoenig (2006) and will only be described briefly here. Like the Beverton and Holt estimator this extension requires only a series of mean length above a user defined minimum size and the von Bertalanffy growth parameters, so it can be applied in many data poor situations. Gedamke and Hoenig (2006) demonstrated the utility of this approach using both simulated data and an application to data for goosefish caught in the NEFSC fall groundfish survey.

The mean length in a population can be calculated $d$ years after a single permanent change in total mortality from $Z_{1}$ to $Z_{2} \mathrm{yr}^{-1}$ by the following equation:

$$
\bar{L}=L_{\infty}-\frac{Z_{1} Z_{2}\left(L_{\infty}-L_{c}\right)\left\{Z_{1}+K+\left(Z_{2}-Z_{1}\right) \exp \left(-\left(Z_{2}+K\right) d\right)\right\}}{\left(Z_{1}+K\right)\left(Z_{2}+K\right)\left(Z_{1}+\left(Z_{2}-Z_{1}\right) \exp \left(-Z_{2} d\right)\right)} .
$$

This equation can also be generalized to allow for multiple changes in mortality rate over time. A maximum likelihood framework is then used to estimate $\mathrm{Z}_{1}, \mathrm{Z}_{2}$, and the year of change (alternatively d) from the observed mean lengths.

## Application of Gedamke and Hoenig (2006) approach to Mutton Snapper trap fishery in Puerto Rico

The first step in the application of this mean-length approach is to determine the length at which animals become fully vulnerable to the gear, $L_{c}$. A cumulative plot of all individuals captured during the time series was constructed for both the trap and hook-and-line fishery (Figures 1 and 2). The length frequency plot from the trap fishery (Figure 1; n=1275) exhibits an expected pattern of increased vulnerability until approximately $300 \mathrm{~mm}\left(L_{c}\right)$ and then an expected decline in relative abundance of animals above this size due to mortality. Although more individuals were measured ( $\mathrm{n}=1994$ ) in the hook and line fishery (Figure 2), the histogram does not indicate a clear $L_{c}$ with a distribution that is relatively flat and skewed towards larger animals. This in conjunction with inconsistencies in the annual length frequency plots (See SEDAR14-DW commercial data report) resulted in excluding these data from further analysis. As such, the only suitable data set was from the Puerto Rico trap fishery and the remaining analysis refers only to these data.

Sampling was conducted throughout the year and lengths were recorded by interview date. To create the time series, mean lengths were calculated for each of the 326 interview days (Figure 3). To evaluate potential trends in length frequency composition, data were also aggregated for each month (Figure 4) and each year (Figure 5) of the time series. The aggregation of data over the larger time scales appeared to be driven by a few ( $\sim 4-5$ ) samples which had unusually high sample numbers and also large mean lengths (see years 2001 and 2002 in Figure 4, 5).

To avoid potential biases from these few inconsistent data points, the base case analysis was conducted on means from each interview day. Samples sizes were relatively low ranging from 1 to 24 individuals and also very infrequent from 1989 to 1999. The non equilibrium model was constructed with base case parameters of $\mathrm{L}_{\mathrm{inf}}=87 \mathrm{~cm}$ and K $=0.15$. Initially, the model was fit to the data weighted by sample size and a grid search was conducted over all possible years of change in mortality. The likelihood function was maximized when the year of change was 1992 and estimated an initial mortality of $0.48 \mathrm{yr}^{-1}$ and a change to $0.64 \mathrm{yr}^{-1}$ (Figure 6 and 7). Residuals were also plotted and a relatively high number of negative residuals were observed at the end of the time series indicating the influence of a few samples with extremely high recorded mean lengths (Figure 8).

The analysis was then conducted without weighting by sample sizes to avoid the influence of those few samples where a relatively large number of measurements were taken from relatively large fish in one day. The function was maximized when the year of mortality change was 1988 and $\mathrm{Z}_{\text {one }}=0.39 \mathrm{yr}^{-1}$ and $\mathrm{Z}_{\mathrm{two}}=0.80 \mathrm{yr}^{-1}$ (Figures 9 and 10). Note that a specific year of change in fishing mortality is more clearly indicated in this case versus the weighted analysis and the pattern to the residuals is also reduced (Figure

## Sensitivity

A sensitivity of estimates to inputs of growth parameters was also conducted. All combinations of values for K and $\mathrm{L}_{\text {infinity }}$ y from Table 3 in the Life history report were used in the estimation process for the un-weighted model. Estimates for $\mathrm{Z}_{\text {one }}$ ranged from $0.29 \mathrm{yr}^{-1}$ to $0.57 \mathrm{yr}^{-1}$ and estimates for $\mathrm{Z}_{\mathrm{two}}$ ranged from $0.63 \mathrm{yr}^{-1}$ to $1.11 \mathrm{yr}^{-1}$ (Table 2). Although variable the magnitude of change for each combination of $K$ and $L_{\text {infinity }}$ was robust to changes in parameters and estimates were approximately $100 \%$ greater.

Sensitivity of estimates to the selection of the appropriate Lc was also explored. Final estimates were very robust and never varied more than $+/-0.02$ from those presented.

## Potential Biases

- Assumption of Constant recruitment-there has been some indication from other analysis in our group that abundance may have risen in the recent past. Although the stock-recruit relationship may not be well-defined the possibility that there has been some increase in recruitment. This would result in an overestimation of Z as the introduction of a greater number of smaller animals would result in a reduction in mean lengths. Generally, the magnitude of trend in recruitment will reflect the magnitude of bias (i.e. a $10 \%$ trend of increase in recruitment will result in a $10 \%$ overestimation of Z .
- Assumption of knife edge selectivity-in the trap fishery it is likely that some of the largest individuals would not be available to the gear resulting in a dome shaped selectivity curve. This would result in an underestimation of the 'true' mean length of the population and an overestimate of the 'true' Z. More information is necessary to include this type of selectivity curve into the analysis. would exclude larger animals, thereby overestimating Z.
- Selectivity in general-based on the available information there have been no significant changes in the design of trap gear used in Puerto Rico, but an increase in mesh size was mandated. It is possible that this could result in a shift in the length at full vulnerability but the selection of Lc used in our analysis should be high enough for this not to have created a bias. Selectivity information was used from Mahon and Hunte, 2001.


## Conclusions

This type of analysis shows promise for application to the data limited situation that is present in many of the Caribbean fisheries. Unfortunately, data were too limited in most cases to apply the model. In the one case where sufficient samples were taken, the high variability leads to some question as to the validity of the absolute values presented in this analysis. The pattern of change, however, appears to be reliable with an increase in fishing mortality occurring sometime around the late 1980's or early 1990's.

The consistency of sampling protocol over the time series is critical for the success of this method and further information on the selectivity of the gear and protocol for port samplers would be necessary before the estimated total mortality could be given more credibility. For the future, an increase in the number of measured fish over a wider range of vessels/operators will lead to more reliable results.

## References

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Mahon, R. and W. Hunte. 2001 Trap mesh selectivity and the management of reef fishes. Fish and Fisheries 2:356-375.

Table 1. Lag time in years for the mean length of fully vulnerable individuals to approach equilibrium and provide an estimate of mortality within $10 \%$ of the new level following a change in mortality from $\mathrm{Z}_{1}$ to $\mathrm{Z}_{2}$. In this example, growth parameters for goosefish in the southern management region of the Northeast United States were used in the calculations ( $\mathrm{K}=0.1198 \mathrm{yr}^{-1}, \mathrm{~L}_{\infty}=$ $129.2 \mathrm{~cm}, \mathrm{~L}_{\mathrm{c}}=30 \mathrm{~cm}$ ). Note that the natural mortality of goosefish is believed to be around $0.2 \mathrm{yr}^{-1}$ so the first row and first column do not appear possible for goosefish. (From Gedamke and Hoenig, 2006).

|  |  | $\mathrm{Z}_{2}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|  | 0.1 | - | 16 | 14 | 12 | 11 | 10 | 9 | 8 | 7 | 7 |
|  | 0.2 | 23 | - | 9 | 9 | 8 | 8 | 7 | 7 | 6 | 6 |
|  | 0.3 | 26 | 11 | - | 6 | 6 | 6 | 6 | 6 | 6 | 5 |
| $\mathrm{Z}_{1}$ | 0.4 | 27 | 13 | 7 | - | 4 | 5 | 5 | 5 | 5 | 5 |
|  | 0.5 | 28 | 14 | 8 | 4 | - | 3 | 4 | 4 | 4 | 4 |
|  | 0.6 | 28 | 15 | 9 | 6 | 3 | - | 2 | 3 | 4 | 4 |
|  | 0.7 | 29 | 15 | 10 | 7 | 5 | 2 | - | 2 | 3 | 3 |
|  | 0.8 | 29 | 15 | 10 | 7 | 5 | 4 | 2 | - | 2 | 2 |
|  | 0.9 | 29 | 16 | 10 | 8 | 6 | 4 | 3 | 2 | - | 2 |
|  | 1 | 29 | 16 | 10 | 8 | 6 | 5 | 4 | 3 | 2 | - |

Table 2. Sensitivity of $Z$ estimates to inputs of $K$ and $\mathrm{L}_{\infty}$. The range of published values for K and $\mathrm{L}_{\infty}$ were used in the un-weighted estimation procedure for interview date. Note that although estimates are variable the magnitude of change ( $+\sim 100 \%$ ) remains constant.

| Year of Change in <br> Mortality | k | $\mathrm{L}_{\infty}$ | $\mathrm{Z}_{\text {ONE }}$ | $\mathrm{Z}_{\text {TWO }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1988 | 0.13 | 118 | 0.57 | 1.11 |
| 1987 | 0.10 | 118 | 0.43 | 0.85 |
| 1987 | 0.15 | 88 | 0.39 | 0.82 |
| 1987 | 0.20 | 82 | 0.44 | 0.93 |
| 1987 | 0.13 | 80.8 | 0.29 | 0.63 |
| 1988 | 0.25 | 78 | 0.50 | 1.07 |

Figure 1. Cumulative plot of all individuals in the trap fishery of Puerto Rico (19832006). The estimated length at vulnerability to gear $\left(\mathrm{L}_{\mathrm{c}}\right)$ is indicated by the red bar.


Figure 2. Cumulative plot of all individuals in the Hook and line fishery of Puerto Rico (1983-2006).


Figure 3. Mean Length calculated for each interview day. Sample numbers for each interview day have been indicated by both bubble size and number.


Figure 4. Mean Length calculated by month. Sample numbers for each month have been indicated by both bubble size and number.


Figure 5. Mean Length calculated by year. Sample numbers for each year have been indicated by both bubble size and number. Note that means in 2001 and 2002 are extremely high due solely to two samples (see Figure 5).


Figure 6. Results of grid search over all years of change in Z. Likelihood function is weighted by the sample size from each interview day. The function is maximized (indicated by the red dashed box) when the year of mortality change is 1992 and $\mathrm{Z}_{\text {one }}=$ $0.48 \mathrm{yr}^{-1}$ and $\mathrm{Z}_{\mathrm{two}}=0.64 \mathrm{yr}^{-1}$. Note that a year of change of 1993 produces almost identical results of fishing mortality.


Figure 7. Observed and predicted mean lengths for weighted fit of interview day. Year of mortality change is 1992 with $\mathrm{Z}_{\text {one }}=0.48 \mathrm{yr}^{-1}$ and $\mathrm{Z}_{\mathrm{two}}=0.64 \mathrm{yr}^{-1}$.


Figure 8. Residuals of observed and predicted mean lengths for weighted fit by interview day. Note the high number of negative residuals present at the end of the time series.


Figure 9. Results of grid search over all years of change in Z. Likelihood function is not weighted by the sample size. The function is maximized (indicated by the red dashed box) when the year of mortality change is 1988 and $Z_{\text {one }}=0.39 \mathrm{yr}^{-1}$ and $\mathrm{Z}_{\text {two }}=0.80 \mathrm{yr}^{-1}$. Note that a specific year of change in fishing mortality is more clearly indicated in this case versus the weighted analysis.


Figure 10. Observed and predicted mean lengths for non weighted fit by interview day.


Figure 11. Residuals of observed and predicted mean lengths for non weighted fit by interview day. Note that although residuals are a little more balanced than in weighted fit, optimization of function still driven heavily by the relatively high record in 2001.



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