

MRFSS/MRIP Calibration Workshop

Ad-hoc Working Group Report

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One outcome of the MRFSS/MRIP Calibration Workshop was the formation of an ad-hoc working group charged with the following: 1) Establish a priority list in each region for which species assessments should be updated to incorporate the new MRIP-derived catch estimates; and, 2) Provide a technical approach (or approaches) to hind-casting and forecasting catch estimates, including examples. The ad-hoc working group included representatives from the NEFSC, SEFSC, GSMFC, ASMFC, and S&T Headquarters.

Species Prioritization

At the workshop participants discussed how priorities for conducting updated and benchmark assessments might be changed based on the results of re-estimation of 2004 to 2011 recreational catches for managed species. Although benchmark and updated assessment schedules are already set for 2012 and 2013, decisions have to be made on how to prioritize future assessments that will use the new MRIP numbers. The ad-hoc committee was asked to develop a metric that could be used to rank species based on the potential impact the switch from MRFSS to MRIP estimates could have on assessment outcomes. The metric was based on criteria related to the magnitude and significance of differences between MRFSS and MRIP catch estimates and the relative importance of the recreational catch time series in the overall assessment model. It was noted during the workshop that many other criteria, unrelated to the re-estimation of MRFSS numbers, will likely also affect scheduling species for updated and benchmark assessments (e.g., socio-economic importance, stock status, and political considerations). Nevertheless, workshop participants did see value in having an objective and understandable set of recreational data metrics that could be used as part of the stock assessment prioritization process.

Six criteria were used to rank species:

1. Total MRIP A and B1 in numbers
2. Mean percent difference between MRFSS and MRIP AB1 numbers calculated as:

$$100 * \frac{1}{n} \sum_{i=1}^n \frac{(MRFSS AB1_i - MRIP AB1_i)}{MRFSS AB1_i}$$

3. Mean percent difference between MRFSS and MRIP B2 numbers calculated as:

$$100 * \frac{1}{n} \sum_{i=1}^n \frac{(MRFSS B2_i - MRIP B2_i)}{MRFSS B2_i}$$

4. Fraction of discards to total catch

$$100 * \frac{1}{n} \sum_{i=1}^n \frac{MRFSS B2_i}{(MRFSS AB1_i + MRFSS B2_i)}$$

5. Multiple R² (Pearson correlation squared) between the annual MRIP AB1 and MRFSS AB1 values calculated from a linear regression of one versus the other or, equivalently:

$$\text{corr}(MRFSS AB1_{i=1:n}, MRIP AB1_{i=1:n})^2$$

6. Percent of total landings attributed to the recreational sector

The six criteria were chosen to represent a combination of factors that would be important in prioritization of species. First the total A plus B1 numbers give an idea of the magnitude of the recreational fishing mortality associated with landings. Next the percent difference between both AB1 and B2 (released alive) numbers provide an idea of the average difference between MRFSS and MRIP estimates; while noting that the average can be low if positive and negative differences cancel each other out. The fraction of discards provides a measure of the importance of discards which can be quite influential in many assessments. The correlation between the annual AB1 numbers provides an estimate of how well the estimates track each other, noting that the estimates could differ in magnitude but might still have the same trend. Finally, the percent of landings attributed to the recreational sector provide an idea of how influential the recreational landings may be in the assessment model, compared to commercial landings, and how sensitive the results may be to changes in recreational inputs.

For each of the six criterion species were initially assigned categorical ranks ranging from one through the total number of species. For example, 16 species were compared for Northeast region with one representing the lowest priority species for that criterion and 16 the highest priority. Ranks were then scaled back to a 10 point scale to provide relative ranks which could be compared across regions as follows:

$$\text{Rank 10-point scale} = 10 * \text{Initial Rank/Number of Species}$$

The overall priority rank score was calculated as the average of the categorical ranks across the six criteria. Tables 1, 2 and 3 give rankings for the Northeast, South Atlantic and Gulf of Mexico species, respectively. It should be noted that regional separations were based upon MRIP subregions (Northeast

= 4 & 5, South Atlantic = 6, and Gulf of Mexico = 7) which do not necessarily reflect the regional partitions used in all stock assessments.

Table 1. Metrics and rankings for Northeast species prioritization based on projected impact of changes in recreational time series data on stock assessments.

Northeast Region	MRIP AB1 (Number of Fish) Sum 2004-2011		Mean % Difference AB1 Catch		Mean % Difference B2 Catch		Relative Importance of Discards (B2 catch)		R2 Correlation Coefficient MRFSS and MRIP AB1		Avg % Recreational Landings (2004 - 2011)		Overall Priority Rank (higher values indicate greater priority)
	Value (1,000s)	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	
tautog	6,508	4.4	0.083	5.6	0.085	6.9	0.092	7.5	0.883	7.5	91%	10.0	7.0
scup	28,205	7.5	-0.157	9.4	-0.136	9.4	0.076	3.8	0.818	6.9	32%	4.4	6.9
spot	69,387	8.8	0.096	6.9	0.042	5.0	0.043	0.6	0.982	9.4	43%	5.6	6.0
spotted seatrout	104,875	10.0	-0.022	2.5	-0.024	3.1	0.080	4.4	0.770	5.0	87%	8.8	5.6
striped bass	18,350	5.6	-0.060	4.4	0.011	0.6	0.108	8.8	0.802	6.3	80%	8.1	5.6
weakfish	4,268	3.8	0.089	6.3	-0.014	1.9	0.090	6.9	0.991	10.0	41%	5.0	5.6
bluefish	52,848	8.1	0.020	1.9	0.011	1.3	0.081	5.0	0.956	8.1	71%	7.5	5.3
red drum	26,154	6.9	0.012	1.3	-0.041	4.4	0.089	6.3	0.748	3.8	89%	9.4	5.3
atlantic cod	2,908	3.1	0.242	10.0	0.313	10.0	0.086	5.6	0.516	0.6	18%	2.5	5.3
summer flounder	482	1.3	0.048	3.8	0.098	7.5	0.119	9.4	0.732	3.1	45%	6.3	5.2
atlantic croaker	82,482	9.4	-0.036	3.1	-0.048	5.6	0.074	3.1	0.796	5.6	26%	3.1	5.0
spiny dogfish	156	0.6	0.107	7.5	0.103	8.1	0.122	10.0	0.588	1.3	3%	0.6	4.7
pollock	1,348	1.9	0.121	8.1	0.064	6.3	0.054	1.3	0.968	8.8	8%	1.9	4.7
black sea bass	14,738	5.0	0.008	0.6	0.036	3.8	0.105	8.1	0.595	1.9	51%	6.9	4.4
winter flounder	1,736	2.5	0.148	8.8	0.129	8.8	0.055	1.9	0.611	2.5	5%	1.3	4.3
spanish mackerel	20,804	6.3	0.077	5.0	0.020	2.5	0.061	2.5	0.757	4.4	30%	3.8	4.1

Table 2. Metrics and rankings for South Atlantic species prioritization based on projected impact of changes in recreational time series data on stock assessments.

South Atlantic Region	MRIP AB1 (Number of Fish) Sum 2004-2011		Mean % Difference AB1 Catch		Mean % Difference B2 Catch		Relative Importance of Discards (B2 catch)		R2 Correlation Coefficient MRFSS and MRIP AB1		Avg % Recreational Landings (2004 - 2011)		Overall Priority Rank (higher values indicate greater priority)
	Value (1,000s)	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	
red snapper	313	3.6	0.185	8.6	0.123	6.8	0.102	9.5	0.978	8.6	74%	7.7	7.5
gray snapper	2,781	7.3	0.164	8.2	0.071	3.6	0.097	7.7	0.986	9.1	71%	6.8	7.1
mutton snapper	940	5.0	0.055	4.1	0.127	7.3	0.073	6.8	0.971	8.2	78%	8.2	6.6
black sea bass	4,023	8.2	0.083	5.0	0.074	4.1	0.104	10.0	0.958	7.7	36%	2.3	6.2
sheepshead	4,599	8.6	0.119	6.4	0.082	4.5	0.055	3.6	0.851	4.5	81%	8.6	6.1
wahoo	340	4.1	-0.088	5.5	-0.320	9.5	0.008	0.5	0.947	6.4	95%	9.1	5.8
blue runner	5,581	9.1	0.049	3.2	0.070	3.2	0.065	5.5	0.894	5.5	72%	7.3	5.6
red porgy	297	3.2	-0.288	9.1	-0.525	10.0	0.055	4.1	0.840	4.1	37%	2.7	5.5
red grouper	383	4.5	-0.369	10.0	0.028	0.9	0.087	7.3	0.900	5.9	40%	4.1	5.5
cero	132	1.8	0.162	7.7	-0.090	5.0	0.026	1.4	0.955	7.3	100%	9.5	5.5
yellow jack	60	0.9	0.123	7.3	0.052	2.3	0.049	2.7	0.988	10.0	100%	9.5	5.5
black grouper	29	0.5	-0.119	6.8	0.162	8.2	0.098	8.2	0.430	0.5	69%	6.4	5.1
greater amberjack	264	2.3	0.039	2.3	0.093	5.5	0.065	5.9	0.949	6.8	64%	5.5	4.7
gray triggerfish	1,072	5.5	0.045	2.7	0.095	5.9	0.066	6.4	0.748	1.8	58%	5.0	4.5
scamp	124	1.4	-0.319	9.5	-0.216	9.1	0.051	3.2	0.760	2.3	27%	1.4	4.5
spanish mackerel	7,741	10.0	0.103	5.9	0.069	2.7	0.044	2.3	0.839	3.6	34%	1.8	4.4
yellowtail snapper	2,005	6.4	-0.054	3.6	-0.129	7.7	0.064	5.0	0.825	2.7	16%	0.9	4.4
crevalle jack	2,596	6.8	-0.030	1.8	0.050	1.8	0.099	8.6	0.531	0.9	67%	5.9	4.3
vermilion snapper	1,303	5.9	0.067	4.5	0.099	6.4	0.057	4.5	0.651	1.4	38%	3.2	4.3
king mackerel	3,435	7.7	0.013	0.5	-0.032	1.4	0.034	1.8	0.987	9.5	52%	4.5	4.2
dolphin	7,454	9.5	0.026	0.9	-0.187	8.6	0.019	0.9	0.882	5.0	14%	0.5	4.2
gag	266	2.7	-0.027	1.4	0.004	0.5	0.099	9.1	0.832	3.2	38%	3.2	3.3

Table 3. Metrics and rankings for the Gulf of Mexico species prioritization based on projected impact of changes in recreational time series data on stock assessments.

Gulf of Mexico Region	MRIP AB1 (Number of Fish) Sum 2004-2011		Mean % Difference AB1 Catch		Mean % Difference B2 Catch		Relative Importance of Discards (B2 catch)		R2 Correlation Coefficient MRFSS and MRIP AB1		Avg % Recreational Landings (2004 - 2011)		Overall Priority Rank (higher values indicate greater priority)
	Value (1,000s)	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	
gray snapper	8,189	9.4	-0.088	5.0	-0.047	3.1	0.099	8.8	0.904	6.9	91%	8.8	7.0
gray triggerfish	1,824	5.6	-0.105	6.3	-0.306	7.5	0.049	3.1	0.978	9.4	96%	9.4	6.9
greater amberjack	615	3.8	-0.111	6.9	-0.212	6.9	0.089	6.3	0.905	7.5	73%	6.9	6.4
mutton snapper	238	2.5	-0.398	8.1	-0.851	10.0	0.069	4.4	0.865	5.6	78%	7.5	6.4
red grouper	1,651	5.0	-0.118	7.5	0.025	2.5	0.115	10.0	0.983	10.0	20%	1.9	6.1
gag	2,862	7.5	-0.055	3.8	0.013	1.9	0.111	9.4	0.968	8.8	69%	5.6	6.1
red snapper	6,629	8.8	-0.046	2.5	-0.100	4.4	0.090	6.9	0.957	8.1	65%	5.0	5.9
cero	211	1.3	-0.466	10.0	-0.540	8.8	0.022	1.3	0.809	3.8	100%	10.0	5.8
bluefish	1,588	4.4	0.092	5.6	0.119	5.0	0.096	8.1	0.815	4.4	63%	4.4	5.3
black grouper	93	0.6	-0.453	9.4	-0.508	8.1	0.096	7.5	0.652	1.9	60%	3.8	5.2
dolphin	2,525	6.9	-0.415	8.8	-0.646	9.4	0.033	1.9	0.562	1.3	14%	0.6	4.8
spanish mackerel	12,780	10.0	0.055	4.4	0.003	0.6	0.069	3.8	0.714	2.5	69%	5.6	4.5
cobia	298	3.1	0.047	3.1	0.062	3.8	0.081	5.6	0.763	3.1	90%	8.1	4.5
vermilion snapper	2,937	8.1	-0.004	0.6	-0.176	5.6	0.020	0.6	0.831	5.0	14%	0.6	3.4
king mackerel	2,355	6.3	0.010	1.3	-0.003	1.3	0.047	2.5	0.895	6.3	41%	3.1	3.4
scamp	229	1.9	-0.026	1.9	0.204	6.3	0.080	5.0	0.534	0.6	28%	2.5	3.0

Technical Calibration Approach

Workshop participants recognized the importance of strong, clear guidelines regarding calibration methods and how and when the methods should be used. Stock assessment scientists do not want to be in the position of developing ad hoc calibration methods on a species-by-species and region-by-region basis. While more sophisticated and time-consuming calibration approaches were discussed, workshop participants reached consensus that, prior to 2004 (or whichever year is the first year for which direct re-estimates are available, since ST is still working on re-estimation for years prior to 2004), hind-casted catch data should use a straight-forward ratio estimator (i.e., MRFSS/MRIP), either constant throughout time hind-casted time series or trended based on ancillary information. A MRFSS/MRIP ratio estimator was also suggested to approximate adjusted variances associated with the revised catch estimates.

Use of a ratio estimator approach for calibrating from MRFSS to MRIP should not preclude development of more extensive species-specific approaches as warranted. However, for many assessed species the use of a simple ratio estimator may be sufficient considering the relatively small differences found between MRFSS and MRIP numbers, and more importantly the anticipated small impact the revised recreational time series will have on assessment outcomes. The reliability and confidence in using a ratio estimator will increase considerably as more years of re-estimated MRIP numbers become available. At present, only eight years of side-by-side MRFSS-MRIP estimates (2004-2011) are available to develop ratio estimators that for some species will be applied to 23 years of data (1981-2003). ST is currently working on revised estimates for 1998-2003 and may eventually go back even further depending on the availability and quality of original data sources.

The ad-hoc working group recommends the ratio estimator be based on the “ratio of means” (across all comparison years included) rather than based on the “mean of ratios” for individual years. Based on sampling theory, the ratio of means should be less biased and more stable than the “mean of ratios” (Cochran 1977) and it also represents the least-squares estimator for a slope in a zero-intercept model when the variance of y (the MRIP estimate in this case) is proportional to x (the MRFSS estimates in this case). The estimate of the calibration factor that is a ratio of mean catches is calculated as:

Formula A

$$\hat{R}_{RM} = \frac{\bar{C}_{MRIP}}{\bar{C}_{MRFSS}} = \frac{\sum_{y=1}^n \hat{C}_{y,MRIP}}{\sum_{y=1}^n \hat{C}_{y,MRFSS}}$$

Calibrated catch estimates for 1982-2003 are then calculated as:

Formula B

$$\hat{C}_{y,\hat{R}} = \hat{R} \hat{C}_{y,MRFSS}$$

The same formulas can also be applied for calibrating variances associated with MRFSS catch estimates.

Variances of the adjusted catch estimates should include two components: 1) calibrated variance of the catch estimate, and 2) variance associated with the ratio estimator used for calibrating the catch estimate.

The variance estimator for the ratio of means derived from the formula above can be approximated as:

Formula C

$$\hat{V}(\hat{R}_{RM}) = \hat{R}_{RM}^2 \left[\frac{\hat{V}(\bar{C}_{MRIP})}{\bar{C}_{MRIP}^2} + \frac{\hat{V}(\bar{C}_{MRFSS})}{\bar{C}_{MRFSS}^2} - 2 \frac{Cov(\bar{C}_{MRFSS}, \bar{C}_{MRIP})}{\bar{C}_{MRFSS} \bar{C}_{MRIP}} \right]$$

Where

$$\hat{V}(\bar{C}) = \frac{1}{n} \frac{\sum_{y=1}^n (\hat{C}_y - \bar{C})^2}{n-1}$$

An estimate of the variance of the calibrated estimate of catch that accounts for uncertainty in the estimate of the calibration factor is calculated as:

Formula D

$$\hat{V}(\hat{C}_{y,\hat{R}}) = \hat{C}_{y,MRFSS}^2 \hat{V}(\hat{R}) + \hat{R}^2 \hat{V}(\hat{C}_{y,MRFSS}) - \hat{V}(\hat{R}) \hat{V}(\hat{C}_{y,MRFSS})$$

This assumes the estimate of the ratio is independent of the estimate of the catch that is to be calibrated. The variances of the catches in the above equation, $\hat{V}(\hat{C}_{y,MRFSS})$ are the values after being calibrated.

Ratio Estimator Approach Example – Summer Flounder

To show an example of the approach suggested above we will hind-casted summer flounder landings numbers (A+B1) estimates and variances for 2003 based on a comparison of 2004-2011 MRFSS and MRIP estimates. Table 4 shows summer flounder AB1 numbers estimates and associated variances for the eight years of MRFSS and MRIP side-by-side estimates.

Table 4. Virginia through Maine MRFSS and MRIP 2004-2011 summer flounder AB1 numbers estimates, variances, variance of means, and co-variances of means.

Year	MRFSS AB1 Numbers (in 1,000s)	MRFSS Variance (in 1,000s)	MRIP AB1 Numbers (in 1,000s)	MRIP Variance (in 1,000s)
2004	4,557	33,226	4,316	67,076
2005	4,110	42,230	4,028	58,396
2006	4,052	41,047	3,951	76,508
2007	3,393	18,420	3,109	34,795
2008	2,295	13,168	2,350	44,728
2009	1,910	9,120	1,807	16,001
2010	1,484	10,791	1,502	14,433
2011	1,782	25,722	1,830	21,439
Mean 2004-2011	2,948	24,215	2,862	41,672
Variance of the Mean	185,048	22,410,864	160,925	71,527,726
Co-variance of MRFSS and MRIP Means			150,486	28,832,853

Using the “ratio of means” approach (Formula A) the ratio estimator for landings numbers is calculated as:

$$= 2,862 / 2,948 = 0.970756$$

When this ratio is applied to the MRFSS 2003 estimate of 4,559 (X 1,000) the calibrated MRIP estimate is 4,425.7 (X 1,000).

Similarly, the ratio estimator for the landings estimate variance is calculated as:

$$= 41,672 / 24,215 = 1.7209$$

When this ratio is applied to the MRFSS 2003 variance of 33,255.2 (X 1,000) the calibrated MRIP variance is 57,228.4 (X 1,000).

The next step is to calculate the variance and PSE associated with the ratio estimator.

Using the Formula C provided above, the variance is approximated as:

$$\begin{aligned} &= 0.9708^2 * (185,048 / 2,948^2 + 160,925 / 2,862^2 - 2 * 150,486 / (2,948 / 2,862)) \\ &= 0.004964 \end{aligned}$$

The PSE is calculated as:

$$\begin{aligned} &= 100 * \text{Sqrt} (\text{Variance}) / (\text{Mean}) \\ &= 100 * \text{Sqrt} (0.004964) / (0.9708) \\ &= 7.3 \% \end{aligned}$$

Finally we calculate the variance and PSE associated with the calibrated landings estimates for each year (Formula D) as:

$$\begin{aligned} &= (4,559^2 * 0.004964) + (0.9708^2 * 57,228.4) - (0.004964 * 57,228) \\ &= 156,821.9 \end{aligned}$$

The PSE for the calibrated estimate is calculated as:

$$\begin{aligned} &= 100 * \text{Sqrt} (\text{Variance}) / (\text{Mean}) \\ &= 100 * \text{Sqrt} (156,821.9) / (4,425.7) \end{aligned}$$

= 8.95 %

Table 5. Original MRFSS AB1 landings estimates, variances and PSEs alongside hind-casted MRIP AB1 landings estimates, variances, and PSEs for summer flounder from 1982-2003.

Year	MRFSS AB1 Numbers of Fish (in 1,000s)	MRFSS Variance (in 1,000s)	MRFSS PSEs	MRFSS AB1 Numbers (in 1,000s) with Ratio Adjustment	MRFSS Variance (in 1,000s) with Ratio Adjustment	Adjusted Variance with Ratio Estimator Variance Factor	Adjusted PSE with Ratio Estimator Variance Factor
1982	15,473	16,184,368	26	15,021	27,851,679	27,296,703	34.8
1983	20,996	2,160,077	7	20,383	3,717,276	5,672,877	11.7
1984	17,475	1,954,404	8	16,965	3,363,334	4,668,685	12.7
1985	11,066	1,763,372	12	10,743	3,034,586	3,452,504	17.3
1986	11,621	661,733	7	11,282	1,138,777	1,737,870	11.7
1987	7,865	154,646	5	7,635	266,130	556,535	9.8
1988	9,960	158,723	4	9,669	273,146	748,484	8.9
1989	1,717	10,613	6	1,667	18,264	31,755	10.7
1990	3,794	23,031	4	3,683	39,634	108,607	8.9
1991	6,068	58,913	4	5,891	101,383	277,815	8.9
1992	5,002	40,032	4	4,856	68,891	188,778	8.9
1993	6,494	67,475	4	6,304	116,118	318,192	8.9
1994	6,703	71,888	4	6,507	123,713	339,002	8.9
1995	3,326	17,700	4	3,229	30,459	83,466	8.9
1996	6,997	44,062	3	6,793	75,827	314,108	8.3
1997	7,167	82,185	4	6,958	141,433	387,560	8.9
1998	6,979	77,930	4	6,775	134,110	367,494	8.9
1999	4,107	26,988	4	3,987	46,444	127,266	8.9
2000	7,801	54,770	3	7,573	94,254	390,441	8.3
2001	5,294	44,842	4	5,139	77,169	211,462	8.9
2002	3,262	17,025	4	3,167	29,298	80,285	8.9
2003	4,559	33,255	4	4,426	57,229	156,821	8.9

Guidelines for Applying Ratio Estimator Approach

The ad-hoc working group recommends the following generally guidelines for applying a ratio estimator to calibrate recreational catch and variance estimates. These guidelines may not apply, or be practical, in all cases as the impact of changes in the recreational time series data will vary by assessment or particular management need:

- Ratio estimators should be calculated using stock level aggregate data to the extent possible. Caution should be used when calculating ratio estimates at finer geographic levels or by fishing mode.
- Ratio estimators can be based on either estimated numbers of fish or weights depending on which units are used directly in the assessment model. The exception may be if ratios based on weights appear unstable due to small sample sizes of weighed fish. In such cases it may be better to calculate a ratio estimator based on numbers and apply it to the weights.
- To the extent practicable, all years for which both MRFSS and MRIP estimates are available should be used to calculate ratios. If one or two years have ratios that are different enough from the other years so as to noticeably impact the overall ratio of means, a balanced trimmed mean approach which removes both the highest and lowest ratios is preferred over simply removing just the highest or lowest year.
- Trended ratio estimators are generally not recommended at present since only eight years are available for comparison. The basic ratio estimator itself could behave poorly with very few years of paired MRFSS and MRIP observations. As additional years of side-by-side estimates are made available bias in the ratio estimator will become negligible and it may be possible to develop trended ratio estimators that better reflect different MRFSS/MRIP ratios at different parts of the time series.
- It is recommended that stock assessment scientists conduct sensitivity analyses of the hind-casted recreational catch estimates (e.g., varying them by 5, 10, 20%) and length frequencies, as available, in order to gauge the overall impact of changes in the estimates on biological reference points. If the assessment results are sensitive to changes in the recreational time series there may be justification for developing more sophisticated models for hind-casting estimates than the ratio estimator approach suggested here.
- The ad-hoc working group did not fully evaluate a ratio estimator approach for calibrating length

frequencies as data were not available at the time of this report. The group did come up with two possible options but also recognized that other options may exist: 1) Adjust the numbers at length using the same ratio as used for total numbers, or 2) Estimate length-class specific ratios and adjust by length class, then sum the adjusted length classes for an alternative adjusted total number.

References

Cochran, W. G. 1977. Sampling techniques. Third Edition. Wiley and Sons. New York.