#### SEDAR 77 Assessment webinar II: Dynamic Factor Analysis (DFA) of Accepted Age-0 Scalloped Hammerhead Indices

PW08-RD11

Received: 10/24/22



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#### Dynamic Factor Analysis (DFA) of Accepted Age-0 Scalloped Hammerhead Indices

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SEDAR 77 (Assessment Webinar II)

July 15, 2022

# **Dynamic Factor Analysis (DFA) SEDAR 77**

 https://sedarweb.org/sedar-77-hms-hammerhead-sharksassessment-process-0

> Reconciling age-0 indices of relative abundance of the U.S. Atlantic and Gulf of Mexico scalloped hammerhead (*Sphyrna lewini*)

> > Dean Courtney, Robert J. Latour, and Cassidy D. Peterson

#### SEDAR77-AW05

20 June 2022





# **DFA Methods Followed SEDAR 65**

https://sedarweb.org/sedar-65-assessment-process

Reconciling indices of relative abundance of the Atlantic blacktip shark (Carcharhinus limbatus)

Robert J. Latour and Cassidy D. Peterson

SEDAR65-AW03

6 March 2020





# **DFA Approach Simulation Tested**

• Application of DFA to time-series of relative abundance simulation tested in Cassidy et al. 2021



ICES Journal of Marine Science (2021), 78(5), 1711-1729. doi:10.1093/icesjms/fsab051

#### Dynamic factor analysis to reconcile conflicting survey indices of abundance

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Peterson, C. D., Wilberg, M. J., Cortés, E., and Latour, R. J. Dynamic factor analysis to reconcile conflicting survey indices of abundance. – ICES Journal of Marine Science, 78: 1711–1729.

Received 31 August 2020; revised 2 March 2021; accepted 3 March 2021; advance access publication 2 May 2021.



### SEDAR Procedural Workshop 8 November 2-4, 2022 in Tampa, Florida



SEDAR Procedural Workshop 8

http://sedarweb.org/pw-08-sedar-fishery-independent-indices-terms-reference

"2. Review past assessments and document types of issues that arose when indices of abundance were combined or due to changing survey design. Possible issues may include:

Combining multiple surveys:



#### Accepted age-0 scalloped hammerhead indices

April 2022

#### HMS Hammerhead Sharks

Table 9. Recommended base indices of abundance for the Age 0 scalloped hammerhead including index name, the value of catch per unit effort, the area sampled and SEDAR document number (See Table 3 for the regions recommended for base model and sensitivity analysis with each index). CV is the coefficient of variation for the annual index value. Missing values in a given year correspond to zero catches (index value of 0 and no CV), where no sampling occurred (ns), or when the model did not converge (nc).

	TXPWD-Gills	net	GULFSPAN		COASTSPA	N-LL	SCCOASTG	N-LONG		SCCOASTGN - SHORT	
	SEDAR77 DV	V-16	SEDAR77 D	W-17	SEDAR77-0	DW-30	SEDAR77-0	W-31		SEDAR77 DW-32	
	Stockwide/0	Sulf of Mexico	Stockwide/	Gulf of Mexico	Stockwide	/Atlantic	Stockwide,	/Atlantic		Stockwide/Atlantic	1
	sharks per n	et per hour	our sharks per net per hour sharks per 100 hook hours sharks per net hour			sharks per net hour					
year	index	CV	Index	CV	index	CV		index	CV	index	CV
1982	0.00033					12000	-1				
1983	0.00042	0.912									
1984	0.00000						1				
1985	0.00015						1				
1986	0.00035	0.732		17							
1987	0.00000			- U							
1988	0.00050	0.618			No.	11					
1989	0.00012			_	15 3						
1990	0.00090	0.603			1 3 2						
1991	0.00053	0.749				bi l					
1992	0.00000					10.					
1993	0.00032	0.819				-					
1994	0.00027	0.848			11		0.1				
1995	0.00010	1.165				1 10					
1996	0.00093	0.536	0.009	0.294		100	10	_	1		
1997	0.00172	0.666	0.016	0.461		11 3	1 10				
1998	0.00031	0.842	0.002	0.548		0				1	
1999	0.00021	0.781	0.091	0.312							
2000	0.00048	0.589	0156	0.253		-	1000		-		
2001	0.00150	0.603	0148	0 302		1 1 1 1	15	1 250	0.479		
2002	0.00033	0.822	0.15	0.166		1.04	11	0.788	0.518		
2003	0.00183	0.577	0.102	0.181			1 7	2.742	0.450		
2004	0.00075	0.689	0.07	0.227		12	-	0.541	1 432		
2005	0.00254	0.517	0.048	0.373	5:454	0.529	-	0.625	0 538		
2005	0.00069	0.630	0.079	0.22	8 119	0.416		0.981	1.018		
2007	0.00079	0.778	0.168	0.171	1.976	1 128		1952	0 533	0 171	0.423
2008	0.00075	0.703	0.172	0.189	1,720	1.165		1 384	0.707	0.286	0.581
2009	0.00095	0.560	0 163	0.2	3 482	0.654	_	7 298	1 383	0.000	
2010	0.00213	0 598	0.208	0.211	9 376	0.327		2 297	0.854	0.114	0.581
2011	0.00091	0 563	0.159	0.201	3,976	0.372	-	1.487	0.540	0.112	0 307
2012	0.00124	0.540	0.093	0.217	1 907	0.469		8 180	0.527	0.115	0 307
2013	0.00484	0.428	0129	0.215	2.507	0427	-	4.058	0.451	0.090	0.423
2014	0.00198	0.420	0.141	0.207	2.052	0.548		2 204	0.695	0.090	5.463
2015	0.00283	0.565	0.069	0.252	1100	0.554		0.969	0.616	0.000	0.591
2015	0.001243	0.500	0.134	0.225	1.158	0.419	-	1.675	0.010	0.020	0.351
2017	0.00051	0.330	0.124	0.235	1,899	0.519	-	6.909	0.358	0.098	0.331
2018	0.00482	0.499	0.21	0.225	0.729	0.565	-	3.208	0 547	0.000	
2010	0.00482	0.435	0.176	0.225	1.030	1.175		3.725	0.347	0.000	0.6.01
2019	0.00248	0.514	0.176	0.265	1,029	1-112		3.305	0.423	0.021	0.561



- Accepted age-0 scalloped hammerhead indices
  - Decision
  - Removed years with "zero" relative abundance

```
Scalloped HH Age 0
TXPWD-Gillnet
SEDAR77 DW-16
Gulf of Mexico
1984, 1987, 1992
```

Scalloped HH Age 0 SCCOASTGN - SHORT SEDAR77 DW-32 Atlantic sharks per net hour

2009, 2014, 2017, 2018



#### • DFA methods SEDAR65\_DW03:

#### Methods

*Dynamic Factor Analysis* The general form of a DFA model can be written as follows (Zuur et al. 2003a):

> $y_t = \Gamma \alpha_t + \varepsilon_t, \text{ where } \varepsilon_t \sim MVN(0, R)$  $\alpha_t = \alpha_{t-1} + \eta_t, \text{ where } \eta_t \sim MVN(0, Q)$

where  $y_t$  is the vector  $(n \ge 1)$  of estimated z-scored index values from all time-series of relative abundance in year t,  $\alpha_t$  is the vector  $(m \ge 1)$  of common trends (m < n),  $\Gamma$  is the matrix  $(n \ge m)$ of loadings on the trends which indicates the strength of each time-series in determining the resulting trend, and R and Q denote the variance-covariance matrices associated with the observation error vector  $\varepsilon_t$   $(n \ge 1)$  and process error vector  $\eta_t$   $(m \ge 1)$ , respectively. Both observation and process error terms assume a multivariate normal distribution. To ensure that the model is identifiable, Q is set to equal to the identity matrix while R is free to take on various forms. All factor loadings, common trends, and fitted values are unitless.



• DFA methods SEDAR65\_DW03:

"Application of DFA to time-series of relative abundance requires some care to preserve the underlying error structure and the relative scale of the survey indices. Accordingly, the following analytical approach was adopted: (1) all time-series of relative abundance were logtransformed, thereby normalizing the time-series error, (2) each time-series was centered and demeaned by subtracting and dividing each by its mean, (3) the global standard deviation (GSD) was calculated for all relative abundance time-series after being log-transformed and demeaned, (4) each time-series was then divided by the GSD, (5) the DFA model was fitted, (6) the resulting DFA-predicted common trend was then multiplied by the GSD and back-transformed. Since the stock assessment model relies heavily on trend rather than magnitude of relative abundance indices, bias correcting will have little impact. However, standard errors estimated by the DFA model for the annual indices were multiplied by the GSD to preserve scale of uncertainty relative to the trend."



• DFA methods SEDAR65\_DW03:

"The above approach does not work well in situations where the log-transformed relative abundance mean was close to zero or negative, because the second step would essentially involve dividing by zero or a negative value, respectively. This was the case for a few of the Atlantic blacktip shark time-series of relative abundance. Simulation analyses have also shown that DFA model fitting is fairly robust when the standard deviation of each time-series resulting from step four are approximately one (Peterson et al. 2021). Accordingly, the Atlantic blacktip time-series of relative abundance were first multiplied by a survey-specific constant, c, to ensure that the resulting time-series approximately achieved these two general criteria. Multiplying indices by a constant is comparable to redefining effort such that the scale of the index changes. Best practices suggest that time-series be z-scored prior to DFA model fitting (Holmes et al. 2020), so in effect, the above analytical approach was developed in the spirit of maintaining consistency with that recommendation."



• DFA Implemented in R consistent with SEDAR Procedural Workshop 8 DFA TUTORIAL FROM ICES MANUSCRIPT: DOI: 10.1093/icesjms/fsab051

# REQUIREMENTS FOR C -- change c vector

y.bar # means must be >0 (otherwise demeaning will change sign/direction of index); means should be >1; ideally >~2 (which isn't always possible)

gsd # gsd should be small! <~0.1

- sd = apply(dat.z, 1, sd, na.rm=T) # MOST IMPORTANT DIAGNOSTIC: sd should be 1.
- # this rescaling approach is to approximate a z-score.

# if sd for an index is <1, then increase corresponding element in c matrix; vice versa || index specific c values

- # helpful to tune 1 at a time
- min(datL, na.rm=T) # ensure that minimum datL >0



#### Decisions

1) DFA input CV obtained from average CV from each index (after removing zeros)

sharks per net per hour	sharks per net per hour	sharks per 100 hook hours	sharks per net hour	sharks per net hour
Gulf of Mexico	Gulf of Mexico	Atlantic	Atlantic	Atlantic
SEDAR77 DW-16	SEDAR77 DW-17	SEDAR77-DW-30	SEDAR77-DW-31	SEDAR77 DW-32
TXPWD-Gillnet	GULFSPAN	COASTSPAN - LL	SCCOASTGN - LONG	SCCOASTGN - SHO

Average CV	0.665	0.255	0.618	0.663	0.460
Count	32.000	24.00	15.000	19.00	9.000

2) DFA input "z-score" scaler "c" obtained iteratively for each index following methods in SEDAR65\_DW03 (after removing zeros)

	dat = dat.a * c #	multiply da	t.a by vector of co	onstants (c)	
с	1042950.536	18596.21	60.94239372	108.564844	4126.117767
	TXPWD-Gillnet	GULFSPAN	COASTSPAN - LL	SCCOASTGN - LONG	SCCOASTGN - SHORT



- DFA obtained separately by region using the CVs and scalers "c" obtained above
  - DFA #1: Combined Gulf and Atlantic (5)

TXPWD-Gillnet	GULFSPAN	COASTSPAN - LL	SCCOASTGN - LONG	SCCOASTGN - SHORT
SEDAR77 DW-16	SEDAR77 DW-17	SEDAR77-DW-30	SEDAR77-DW-31	SEDAR77 DW-32
Gulf of Mexico	Gulf of Mexico	Atlantic	Atlantic	Atlantic
sharks per net per hour	sharks per net per hour	sharks per 100 hook hours	sharks per net hour	sharks per net hour

#### • DFA #2: Gulf (2)

TXPWD-Gillnet	GULFSPAN
SEDAR77 DW-16	SEDAR77 DW-17
Gulf of Mexico	Gulf of Mexico
sharks per net per hour	sharks per net per hour

#### • DFA#3: Atlantic (3)

Atlantic	Atlantic	Atlantic	
SEDAR77-DW-30	SEDAR77-DW-31	SEDAR77 DW-32	
COASTSPAN - LL	SCCOASTGN - LONG	SCCOASTGN - SHOR	



• Diagnostics followed SEDAR65\_DW03:

"The underlying assumptions of a DFA model are equivalent to those of a linear regression, which include normality, independence, and homogeneity of residuals (Zuur et al. 2003b). Model validation was therefore based on standard diagnostic tools (QQ plots, analysis of residuals). Additionally, 'fit ratio' statistics were calculated as  $\Sigma_t y_{it}^2 / \Sigma_t \varepsilon_{it}^2$ , where i denotes an individual time-series. High fit ratios (i.e.,  $\geq$  0.6) suggest that the DFA model poorly fits the time series, or a few years in the time series (Zuur et al. 2003b)."



#### Results

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#### • Results

TXPWD-Gillnet	GULFSPAN	COASTSPAN - LL	SCCOASTGN - LONG	SCCOASTGN - SHORT
SEDAR77 DW-16	SEDAR77 DW-17	SEDAR77-DW-30	SEDAR77-DW-31	SEDAR77 DW-32
Gulf of Mexico	Gulf of Mexico	Atlantic	Atlantic	Atlantic
sharks per net per hour	sharks per net per hour	sharks per 100 hook hours	sharks per net hour	sharks per net hour

V1

V2







#### Results

TXPWD-Gillnet	GULFSPAN	COASTSPAN - LL	SCCOASTGN - LONG	SCCOASTGN - SHORT
SEDAR77 DW-16	SEDAR77 DW-17	SEDAR77-DW-30	SEDAR77-DW-31	SEDAR77 DW-32
Gulf of Mexico	Gulf of Mexico	Atlantic	Atlantic	Atlantic
sharks per net per hour	sharks per net per hour	sharks per 100 hook hours	sharks per net hour	sharks per net hour
<ul> <li>Fit ratios</li> </ul>	s (Fit ratio >=0.6 suggest poo	r fit)		
• \/1	FitBatio			
VI /	Survey1 Survey2 Surv	AVA SURVAVA	SURVAV5	
0	5007528 0 2845202 0 5460	640 0 6501031 0	501 VEy5	
0.	090/538 0.2845292 0.5460	0.0591031 0	. 003//22	
>	mean(FitRatio)			
[1	.] 0.5689646			
Surv	ey 2, Good fit; but Survey 1, 4, 5 poor fit			
• \/2	FitPatio	, ,		
× -	FICKACIO	Lov 2 Europa	Supvov5	
	Surveyi Surveyz Sur	veys Survey4	Surveys	
0.	/185246 0.4142/06 0.41/	/663 0.63603/1	0.4331884	
>	mean(FitRatio)			
[1]	0.5239574			
	Tond A cimilar poor fit co in \	14		
<ul> <li>Survey</li> </ul>	i and 4 similar poor fit as in v			
Survey 2	2 reduced fit compared to V1			

- Survey 3 and 4 improved fit compared to V1
  - Overall fit ratio improved



- Manual QQ plots "raw residuals" from fit (dat.z-fit.b)
- Survey 2 skewed in both V1 and V2
- Survey 5 has low sample size

V1



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V2

- QQ plots "standardized residuals"
- Survey 2 still has some skew in V1 but reduced in V2



- QQ plots State Vector "standardized residuals"
- Some skew, but reduced in V2



Cholesky standardized state smoothation (xT) residuals. The residuals should be Gaussian. Note if the data have many missing values, the state residuals will not be Gaussian. In that case, manually remove the states residuals associated with missing data and redo the qq plot. Cholesky standardized state smoothation (xtT) residuals. The residuals should be Gaussian. Note if the data have many missing values, the state residuals will not be Gaussian. In that case, manually remove the states residuals associated with missing data and redo the qq plot.



- Analysis of survey residuals to fit
- Survey 2 residual pattern reduced in V2



V1





Innovations (one-step ahead) residuals. These residuals should not have a temporal trend and 95% of residuals should fall within the Cls. A violation of this indicates that the model cannot fit the data.

Innovations (one-step shead) residuals. These residuals should not have a temporal trend and 95% of residuals should fall within the CIs. A violation of this indicates that the model cannot fit the data.



- Analysis of survey standardized residuals
- Scale (influence) of Survey 2 outliers reduced in V2



V1





rend. Residuals outside the Cholesky standardized model smoothation (ytT) residuals. These residuals should not have a temporal trend. Residuals outside the +/- 2 limits are potential outliers.

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Cholesky standardized model smoothation (ytT) residuals. These residuals should not have a temporal trend. Residuals outside the +/- 2 limits are potential outliers.



- Analysis of state vector standardized residuals
- Scale (influence) of outliers reduced but trend remains V2

V1





x(t+1)

 $\label{eq:cholesky standardized state smoothation (xtT) residuals. Residuals outside the +/- 2 limits are potential outliers of x(t) to x(t+1).$ 

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- Analysis of standardized residuals autocorrelation (acf)
- Survey 2 has autocorrelation (significant at lag 1 in V2)





Figure 1. Raw time-series of relative abundance indices for age-0 individuals for combined Gulf of Mexico and Atlantic indices (Table 1) along with back-transformed common trend resulting from the DFA model fitted to the age-0 time-series of relative abundance (Table 3). All indices are divided by their mean for plotting.





Figure 4. Back-transformed common trend resulting from the DFA model fitted to the age-0 time-series of relative abundance for combined Gulf of Mexico and Atlantic (Table 3). The shaded interval denotes the approximate 95% confidence interval.



- Decision
- Recommend V2 for use in Sensitivity Analysis
- Remove outlier survey 2 (1998)

TXPWD-Gillnet	GULFSPAN		COASTSPAN - LL	S	CCOASTGN	- LONG	SCCOAST	GN - SHORT
SEDAR77 DW-16	SEDAR77 DV	V-17	SEDAR77-DW-30	SI	EDAR77-DV	N-31	SEDAR77	DW-32
Gulf of Mexico	Gulf of Mexi	co	Atlantic	At	tlantic		Atlantic	
sharks per net per hour	sharks per n	et per hour	sharks per 100 hook h	iours sh	harks per n	et hour	sharks per	net hour
	index	CV						
	0.009	0.294						
	0.016	0.461						





TXPWD-Gillnet	GULFSPAN
SEDAR77 DW-16	SEDAR77 DW-17
Gulf of Mexico	Gulf of Mexico
sharks per net per hour	sharks per net per hour





#### • Results

TXPWD-Gillnet	GULFSPAN
SEDAR77 DW-16	SEDAR77 DW-17
Gulf of Mexico	Gulf of Mexico
sharks per net per hour	sharks per net per hour

V1





V2



#### Results

TXPWD-Gillnet	GULFSPAN	
SEDAR77 DW-16	SEDAR77 DW-17	
Gulf of Mexico	Gulf of Mexico	
sharks per net per hour	sharks per net per hour	

- Fit ratios (Fit ratio >=0.6 suggest poor fit)
- V1 > FitRatio Survey1 Survey2 0.6521836 0.1869621 > mean(FitRatio) [1] 0.4195729
  - Survey 2, Good fit; but Survey 1, 4, 5 poor fit;

```
    V2 > FitRatio
Survey1 Survey2
0.6227688 0.3028268
> mean(FitRatio)
[1] 0.4627978
```

- Survey 1 similar poor fit as in V1
- Survey 2 reduced fit compared to V1
  - Overall fit ratio slightly worse than V1



- Manual QQ plots "raw residuals" from fit (dat.z-fit.b)
- Survey 2 skewed in both V1 and V2

V1





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V2

- QQ plots "standardized residuals"
- Survey 2 skew in V1 reduced in V2



- QQ plots state vector "standardized residuals"
- Some skew, but reduced in V2



Cholesky standardized state smoothation (xtT) residuals. The residuals should be Gaussian. Note if the data have many missing values, the state residuals will not be Gaussian. In that case, manually remove the states residuals associated with missing data and redo the qq plot.

data and redo the qq plot.



- Analysis of survey residuals to fit
- Survey 2 outlier pattern reduced in V2



V1

#### V2



Innovations (one-step ahead) residuals. These residuals should not have a temporal trend and 95% of residuals should fall within the Cls. A violation of this indicates that the model cannot fit the data.

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- Analysis of survey standardized residuals
- Scale (influence) of Survey 2 outliers reduced in V2



V1

V2



Cholesky standardized model smoothation (ytT) residuals. These residuals should not have a temporal trend. Residuals outside the +/- 2 limits are potential outliers.



- Analysis of state vector standardized residuals
- Scale (influence) of outliers reduced but trend remains V2

V1





x(t+1)



- Analysis of standardized residuals autocorrelation (acf)
- Survey 2 has autocorrelation (significant at lag 1 in V2)



# **DFA #2: Gulf (V2)**



Figure 5. Raw time-series of relative abundance indices for age-0 individuals for Gulf of Mexico indices (Table 1) along with back-transformed common trend resulting from the DFA model fitted to the age-0 time-series of relative abundance (Table 3). All indices are divided by their mean for plotting.



# **DFA #2: Gulf (V2)**



Figure 8. Back-transformed common trend resulting from the DFA model fitted to the age-0 time-series of relative abundance for Gulf of Mexico (Table 3). The shaded interval denotes the approximate 95% confidence interval.



# **DFA #2: Gulf (V2)**

- Decision
- Recommend V2 for use in Sensitivity Analysis
- Remove outlier survey 2 (1998)

TXPWD-Gillnet	GULFSPAN	
SEDAR77 DW-16	SEDAR77 DW-17	
Gulf of Mexico	Gulf of Mexico	
sharks per net per hour	sharks per net per hour	
	index	CV
	0.009	0.294
	0.016	0.461
	0.002	0.548



#### • Results





• Results





#### Results

COASTSPAN - LL	SCCOASTGN - LONG	SCCOASTGN - SHORT
SEDAR77-DW-30	SEDAR77-DW-31	SEDAR77 DW-32
Atlantic	Atlantic	Atlantic
sharks per 100 hook hours	sharks per net hour	sharks per net hour

• Fit ratios (Fit ratio >=0.6 suggest poor fit)

```
> FitRatio
Survey3 Survey4 Survey5
0.3782546 0.7334609 0.3071547
> mean(FitRatio)
[1] 0.4729567
```

• Fit to Survey 4 is poor (Fit ratio >=0.6)



- Manual QQ plots "raw residuals" from fit (dat.z-fit.b)
- Survey 3 and 4 reasonable
- Survey 5 has low sample size





- QQ plots "standardized residuals"
- Survey 3 and 4 reasonable
- Survey 5 has low sample size





- QQ plots state vector "standardized residuals"
- Some skew but reasonable



Cholesky standardized state smoothation (xtT) residuals. The residuals should be Gaussian. Note if the data have many missing values, the state residuals will not be Gaussian. In that case, manually remove the states residuals associated with missing data and redo the qq plot.



- Analysis of survey residuals to fit
- Some outlier pattern similar to DFA #1 and #2 V2 above



Innovations (one-step ahead) residuals. These residuals should not have a temporal trend and 95% of residuals should fall within the CIs. A violation of this indicates that the model cannot fit the data.



- Analysis of survey standardized residuals
- Scale (influence) of outliers similar to DFA #1 and #2 V2



Cholesky standardized model smoothation residuals

Cholesky standardized model smoothation (ytT) residuals. These residuals should not have a temporal trend. Residuals outside the +/- 2 limits are potential outliers.



- Analysis of state vector standardized residuals
- Scale (influence) of outliers reduced relative to survey,
  - but there is some pattern in residuals



 $\label{eq:cholesky standardized state smoothation (xtT) residuals. Residuals outside the +/- 2 limits are potential outliers of x(t) to x(t+1).$ 



- Analysis of standardized residuals autocorrelation (acf)
- Survey 3 has some autocorrelation (significant at lag 2)







Figure 9. Raw time-series of relative abundance indices for age-0 individuals for Atlantic indices (Table 1) along with back-transformed common trend resulting from the DFA model fitted to the age-0 time-series of relative abundance (Table 3). All indices are divided by their mean for plotting.



Figure 12. Back-transformed common trend resulting from the DFA model fitted to the age-0 time-series of relative abundance for Atlantic (Table 3). The shaded interval denotes the approximate 95% confidence interval.



- Decision
- Recommend for use in Sensitivity Analysis

