A Report of the MRIP Sampling and Estimation Project:
Improved Estimation Methods for the Access Point Angler Intercept Survey Component of the Marine Recreational Fishery Statistics Survey

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Key Words

Access Point Angler Intercept Survey (APAIS), Marine Recreational Fishery Statistics Survey (MRFSS), stratified multi-stage cluster sampling, probability proportional to size without replacement (ppswor), weighted estimation method, unweighted estimation method, inclusion probability, alternate site sampling, time slice distribution, mean catch rate estimator, variance estimation

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Executive Summary

The Marine Recreational Fisheries Statistics Survey (MRFSS) conducted by the National Marine Fisheries Service utilizes complementary surveys: a Coastal Household Telephone Survey (CHTS) and an Access Point Angler Intercept Survey (APAIS). The CHTS is used primarily to access a target population of coastal resident marine recreational anglers, and to collect fishing activity data that can be used to estimate the total recreational effort (in number of angler fishing days) within a given two-month period. The APAIS is used to assess marine recreational angler fishing days and collect data on catch by species that estimates the mean catch per angler fishing day for the same two-month period.

The design of the APAIS is a stratified, multi-stage cluster sample. The target population consists of the set of all angler-trips within a given year, two-month wave, state, and fishing mode. The frame for this target population consists of site-days, constructed by crossing a list of available public access sites to fishing with a list of available days within the wave. The frame is stratified by month and day type (weekday and weekend). The sample within a stratum is selected in multiple stages. In the first stage, a primary sampling unit (PSU) consisting of a specific site-day combination is selected by probability proportional to size without replacement (ppswor). In the subsequent stages of selecting among a cluster of anglers or boats within a site-day or among a cluster of anglers who fished on a selected boat, the secondary (SSU) and/or tertiary (TSU) sampling units are assumed to be selected with equal probability without replacement.

In the traditional MRFSS, estimates from the APAIS rely on unweighted averages that do not reflect the complex sampling design and also contain data that are not obtained through a probability sample. These unweighted estimates are design-biased and have undergone critiques from NRC (2006) and constituents. The purpose of this report is to outline proposed changes to the estimation procedures for the APAIS. These changes will ensure that estimation methods being applied to the APAIS are statistically valid.

The most important change to the APAIS is the development of a design-based, weighted estimation method for estimating catch rate and its variance using the APAIS data. The weights used in the weighted estimation method are obtained as the inverses of the inclusion probabilities for each PSU within a stratum and for each SSU and/or TSU encountered in the multi-stage sampling design. The estimator of catch rate is, to a good approximation, design-unbiased because the method takes the weights of stratum and stages into account.

Future access point intercept surveys will need to eliminate the “alternate mode” and “alternate site” sampling allowed by the current MRFSS APAIS. In the field, samplers have been allowed to obtain samples from alternate fishing modes and alternate fishing access sites under explicit rules for the purposes of increasing productivity and minimizing the costs of the survey. However, looking back into the history of the APAIS, the pattern of alternate mode sampling was inconsistent, making it difficult to compute the inclusion probabilities for such sampled angler fishing days by any means. For this reason, alternate mode samples were excluded from this design-based, weighted estimation approach. The impact on exclusion of the alternate mode data is expected to be minimal because the size of alternate mode samples was usually small.

Although interviewers are asked to follow explicit rules when choosing alternate sites, the traditional field sampling procedures have allowed for considerable flexibility on the part of the samplers. This can make it difficult to calculate the inclusion probabilities for alternate site sampling. Since a large fraction of data (50% or more) has come from alternate sites, it would be a major loss of information if alternate site samples were not included in the estimation. For this reason, an estimated weight for alternate site sampling was developed by exploiting empirical transition rates from primary site to alternate sites in the historical database.

Lastly, a statistical adjustment is being developed to account for the fact that only a fraction of all the anglers during a sampled day are being observed at a selected site. In the traditional APAIS design, the cluster size of a specific PSU (i.e., the number of completed angler fishing days occurring within a site-day) is not observed by a sampler for the entire day because the sampler is encouraged to target only the most active part of day and is not required to stay at the site for any specified duration. An empirical time slice distribution of completed angler fishing days is obtained from the Coastal Household Telephone Survey (CHTS) and is used to expand the number of completed angler fishing days in the sampled APAIS time slice to the entire day.

The weighted estimation method can be used to estimate the mean catch rate of a given target population of angler fishing days. It can also be used to estimate the proportion of angler fishing days occurring in different water bodies and the proportion of angler fishing days covered by the sampling frame for the CHTS (i.e., anglers living in a coastal residential household that has a landline telephone). To simplify the illustration of the weighted estimation method, this report presents mean striped bass catch rates by New York private/rental boat (PR) fishing mode from 2003 to 2007 as an illustrative example. The two estimates of proportions for the target populations as mentioned above are also presented. While estimates under the new method and the historical method are quite different in many places, the direction and magnitude of differences do not exhibit any obvious patterns.

1. Introduction

Continuous monitoring of catch, effort and participation in marine recreational fisheries is needed to monitor trends of population abundance, and impacts on resources due to management regulations derived from various management scenarios. The Magnuson Fishery Conservation and Management Act of 1976 (MFCMA, P.L. 94-265) mandated collection of data for both commercial and recreational marine fisheries. In 2006, the Magnuson-Stevens Fishery Conservation and Management Reauthorization Act (MSA, P.L. 109-479) further emphasized this requirement to collect fisheries data. Following several years of testing (Human Sciences Research Inc. 1977a, 1977b, Ghosh 1981), NOAA Fisheries established the Marine Recreational Fisheries Statistics Survey (MRFSS) in 1981.

The MRFSS is a complemented surveys design that includes two independent surveys. The Access Point Angler Intercept Survey (APAIS) is an on-site approach for the collection of catch data from intercepted anglers that is used primarily as a basis for estimating a "mean catch rate" defined as the mean number of fish caught per angler day of fishing. The Coastal Household Telephone Survey (CHTS) applies a random digit dialing (RDD) sampling approach to collect data from residents of coastal county households on their marine recreational fishing activities, and it is used as a basis for estimating "fishing effort" in terms of the total number of angler fishing days. In the MRFSS, one angler day of fishing is considered to be synonymous with one "angler fishing trip". The APAIS is also used to estimate the proportion of marine recreational angler fishing trips made by the participants who could be reached via the CHTS RDD sampling frame. The inverse of this proportion is used to adjust the CHTS estimate of fishing effort to obtain an unbiased estimate of total marine recreational angler fishing effort. The APAIS and CHTS were originally implemented on all coasts but are currently only implemented to produce fishing effort and catch statistics on the Atlantic coast of the United States, on the Gulf Coast (excluding Texas), and in both Hawaii and Puerto Rico.

The current sampling approach of the APAIS is a multi-stage cluster sampling design that is stratified by month and day type (weekend or weekday) within a given year, wave, state and fishing mode (shore, private/rental boat, charter boat, or party/headboat). Sampling for each stratum utilizes a spatiotemporal frame that includes a list of public access sites to fishing and a calendar of available fishing days. The primary sampling unit (PSU) is a site-day that comprises a combination of a selected site with a selected day. A sample of site-days is selected by a probability proportional to size without replacement (ppswor) sampling scheme where the size measure for a given site-day is a prediction of the mean number of angler fishing trips that an assigned interviewer would encounter. An interviewer is assigned to each selected site-day, and the interviewer is directed to visit the "assigned site" on the "assigned day" to intercept anglers who have completed fishing for the day, observe a sample of their catch, and interview them to collect data on their place of residence, their phone ownership, the location of their fishing, and counts of any caught fish that are not available for inspection. However, the traditional procedures also allow interviewers to visit and conduct interviews at up to two additional adjacent sites (other than the assigned site) and to intercept anglers who fished in other modes (other than the assigned mode). The visits to "alternate sites" and the "alternate mode" interviews were allowed in the APAIS design as a means of maximizing the number of interviews obtained per dollar spent.

The traditional APAIS estimation method analyzes the data from different modes and sites as a simple random sample (Ghosh 1981). In other words, the angler trip data obtained for assigned sites and alternate sites and for the assigned mode and alternate modes are pooled across fishing mode, month and day-type strata into one data set, and then the pooled data set is partitioned by reported fishing mode to produce estimates of catch rate by fishing area within a given year, wave, state, and fishing mode.

In 2004, NOAA Fisheries contracted with the National Research Council (NRC) of the National Academies to conduct a review of all current marine recreational fishery survey methods funded by NOAA Fisheries. The NRC established a Committee whose mission was to review the MRFSS sampling designs and estimation methods and to make recommendations for improvement and possible alternative approaches. In 2006, the NRC published its Committee's report (NRC 2006) which expressed three major concerns regarding the traditional design of the APAIS:

(a) "…, the estimation procedure for information gathered onsite does not use the nominal or actual selection probabilities of sample design and therefore has the potential to produce biased estimates for both the parameters of interest and their variances."
(b) "The statistical properties of various sampling, data-collection, and data-analysis methods should be determined. Assumptions should be examined and verified so that biases can be properly evaluated."
(c) "The statistical properties associated with data collected through different survey techniques differ and are often unknown. The current estimators of error associated with various survey products are likely to be biased and too low. It is necessary, at a minimum, to determine how those differences affect survey results that use differing methods."

After the NRC review was completed, NOAA Fisheries began planning to re-design its marine recreational fishery survey programs and address all of the concerns raised in the 2006 NRC Report. In 2007, NOAA initiated the Marine Recreational Information Program (MRIP) as a collaborative effort involving state agencies and constituents. An MRIP project team was formed to develop and standardize the sampling design, sampling procedures, and estimation method for APAIS to address the three NRC concerns listed above.

This report presents the design-based methodologies and results of the APAIS re-design project. Section 2 describes the current sampling design of the APAIS, which is needed as the basis of the weighted estimation method. Section 3 describes the weighted estimation method, which incorporates sample weights to obtain approximately unbiased estimators of catch rates, as well as the proportions of anglers fishing in inland, near-shore, and off-shore waters, and living in coastal residential households with landline telephones. Section 4 presents the "weighted estimates" of the catch rates of striped bass (*Morone saxatilis*) from APAIS data collected in 2003-2007 New York private/rental boat fishing mode, as well as the proportions mentioned above. Section 5 discusses further changes in the sampling design, data collection procedures, and collected data elements that will help to provide a much more statistically sound on-site survey approach for estimating mean catch rates.

2. Sampling Design of APAIS

It is important to understand the current APAIS sampling design in order to apply the most appropriate weighting methods in the estimation process. In sampling theory, the weights are the inverses of the inclusion probabilities (Särndal et al. 1992). Since the sampling design of the APAIS is stratified multi-stage cluster sampling, appropriate weights must be computed for the observations for each stratum and stage.

Target population

The target population consists of the set of all angler-trips within a given year, two-month wave, state, and fishing mode. Angler-trips on the U.S. Atlantic Coast might be tied to anglers or even to boats, but it is not practical to develop a list of anglers or boats and sample from this list. Instead, the frame for this target population consists of site-days, constructed by crossing a list of available public access sites to fishing with a list of available days within the wave. It should be noted that sampling from this frame excludes fishing activities from private access points.

Fishery managers in state and federal agencies and constituents demand timely deliveries of removals by species for their in-seasonal management actions. The timely deliveries are either bi-monthly or monthly, depending on the regions. Accordingly, the frame is stratified by month and day type (weekday and weekend).). The month and day-type are used as stratification variables of the target population to account for (i) different fishing activities between weekdays and weekends and (ii) balance between sampling efforts in the first or second months of a wave.

Figure 1 shows the NMFS sub-regions for the U.S. Atlantic coast: Northeast (Sub-region 4), Mid-Atlantic (Sub-region 5), and Southeast Atlantic (Sub-region 6). The other sub-region is the Gulf of Mexico (Sub-region 7). Texas is not included. Florida is the only state that is divided into Gulf of Mexico and Southeast sub-regions (Figure 2). The current APAIS is designed to cover three different fishing modes -- shore mode (SH), private/rental boat mode (PR), and charter boat mode (CH). The sampling for Party and Charter Boat (PC) mode was officially terminated since wave 3, 2006, and replaced by CH mode sampling. The sampling of the headboat (HB) mode was originally covered as part of the PC mode, but is now being covered by a separate vessel-based survey that selects a sample of boat fishing trips that interviewers board for data collection at sea.

2.1 Sample frame

As noted above, the APAIS sampling frame is constructed from a list of public access fishing sites and the calendar of available fishing days. The sampling unit is a site-day combination, which is called the primary sampling unit (PSU). The public access fishing sites and their predicted fishing activity levels, or “fishing pressures”, are listed in the master site register (MSR) by state, fishing mode, month, and day-type

The fishing pressure is the average number of completed angler fishing trips expected to be encountered over an 8-hour period of peak activity on an average day. These fishing pressure predictions are based on the historical information collected and updated by interviewers and/or participating state agencies. The sites are categorized with respect to fishing pressure within each state/month/day type/mode as follows:

Pressure Category	Expected Range of Number of Angler-trips	Size Measure Assigned to Pressure Category
0	1~4	0.5
1	5~8	2.5
2	9~12	9
3	13~19	13
4	20~29	20
5	30~49	30
6	50~79	50
7	80+	80
8	Unable to determine	0
9	Mode not present at site or inactive sites	0

A size measure is assigned to each pressure category. The size measure determined the probability of selecting a site-day within a stratum (Särndal et al. 1992). The size measures for pressure categories 0 and 1 are reduced in order to prevent selecting an excessive number of low pressure site-days, which would significantly reduce the number of angler trip intercepts obtained per dollar spent. Pressure category 9 is used for sites that do not currently have any activity in the relevant mode. Category 8 is used as a "temporary placeholder" when a new site is identified from a variety of sources. After review, the site is either assigned to one of the active pressure categories, or transferred into category 9.

2.2. Stratification

The site-days in the sampling frame for a given target population are stratified by month and day type to help ensure that sampling is representative and balanced by month and day type throughout the two-month wave. This is especially important to prevent oversampling at the beginning or the end of the wave. Sampling of some low-pressure modes or low-pressure waves is excluded due to low sampling efficiency and cost-control.

2.3 Primary sampling unit (PSU)

For the shore (SH), private/rental boat (PR) and charter boat (CH) fishing modes, the PSUs are site-days in the list frame. Site-days are sampled via probability proportional to the expected number of angler-trips without replacement within a given month/day type stratum. Madow's method (Cochran 1977) is currently used to select PSUs. The method is a probability proportional to size without replacement (ppswor) approach that is related to systematic sampling. This method could alternatively be implemented using METHOD = PPS_SYS in the SAS PROC SURVEYSELECT.

2.4 Secondary and tertiary sampling units

The number of stages of sampling in the APAIS is dependent on the fishing mode. The CH and PR modes have three stages in which the secondary sampling unit (SSU) is boat trip within the selected site-day (PSU) and the tertiary sampling unit (TSU) is angler trip within the intercepted boat trip (SSU). The SH mode has two stages in which SSU is angler trip within the selected site-day (PSU). Both the SSU and TSU are assumed to be selected with equal probability without replacement. Note that this is an approximation to what is done in the field for selection of secondary and tertiary units. It is generally not operationally feasible to list these units and draw the sample, so the field staff typically implements a systematic design.

2.5 Variations in field sampling and estimation

Deployment of a sampler to a selected site-day is called an assignment, which is based on the selected site-day assigned to a sampler within a given year, wave, sub-region, state and mode. Variations in sampling procedures have evolved over the years due to considerations of cost-efficiency, measures of sampler productivity, and changing requirements for fisheries management.

i) *Alternate site sampling*: The traditional target of a MRFSS assignment is to obtain no less than 20 (or 30 depending on the state or sub-region) completed interviews per assignment for the assigned mode. For various reasons (such as special events occurring in the site-day, sampler missing the peak activity time interval, non-corporate anglers, etc.), the goal is not always achievable for an interviewer at the selected site-day. Thus, interviewers have traditionally been allowed to visit up to two additional “alternate sites” in that assignment. While there are explicit rules regarding the selection of an alternate site (it must be the nearest site with expected activity in the originally assigned fishing mode), evidence indicates that interviewers have not always complied with the rules. Alternate site visits that are not specified by the sampling protocol violate the rules of random sampling, making their use in survey estimation questionable at best. However, the elimination of alternate site intercepts from the estimation of the mean catch rate could result in some cases in the loss of more than 50% of the total number of interviews conducted (Wade Van Burskirk and Han-Lin Lai, personal communication, 2008), resulting in a substantial loss of data. Therefore, it is desirable to develop a method for approximating the inclusion probabilities for sites selected as alternate sites so that alternate site interviews can be included in the estimation of mean catch rates. The method proposed to obtain these “estimated inclusion probabilities” will be described further below.

ii) *Alternate mode sampling*: Alternate mode sampling is intercepting of angler trips in a fishing mode that differs from the assigned mode. Alternate mode interviews have been allowed in the past only if one of the following three conditions is met:

a) The interviewer can conduct the interview while waiting for anglers to finish fishing in the assigned mode,
b) The sampling goals in the alternate mode (i.e., total number of interviews of the alternate mode in the targeted population) are in danger of being missed for the month, wave and state,
c) Specific permission from the office of contractor or grantee has been obtained prior to sampling.

If the interviewers obey the rules, a two-phase type of probability could in principle be obtained for angler trip intercepts in an alternate mode (Jay Breidt and Jean Opsomer, personal communication, 2008). However, there is no traceable pattern in how alternate mode interviews have been collected in the historical data that would provide a reasonable basis for obtaining the appropriate two-phase probabilities. Nonetheless, alternate mode interviews are less critical in the estimation of catch rate because they represent a very small fraction of the total number of intercepts obtained for each fishing mode. (Wade Van Burskirk and Han-Lin Lai, personal communication, 2008). Starting in 2008, alternate mode interviews were no longer allowed. In the five years prior to that (2003-2007), alternate mode interviews comprised less than 13% of the total shore mode interviews, less than 11% of the total private/rental boat mode interviews, and less than 8% of the total charter boat mode interviews. Therefore, all alternate mode interviews have been excluded from the estimates of catch rate provided in this report.

iii) *Charter boat mode sampling*: Before 2002, the charter boats and partyboats (also called headboats or open boats in some regions) were combined into a party/charter boat (PC) mode. Analyses of the APAIS data performed in the late 1990's had indicated that partyboat angler trip intercepts appeared to be over-represented relative to charter boat angler trip intercepts in the traditional PC sampling. To address this issue, starting in Wave 4 of 2002, additional site-day samples have been selected for charter boat (CH) mode interviewing assignments that could not include intercepts of partyboat/headboat (HB) angler trips. The CH assignments were selected using a site-day frame and fishing pressure estimates that were specifically developed for only the charter boat fishing mode. In 2003, a headboat at-sea sampling program was introduced, but PC mode sampling was continued until the end of 2006 to allow for effective comparisons of charter boat and headboat catch rate estimates based on the traditional PC sampling with estimates based on both the HB sampling and the new supplemental CH sampling. Although the new HB at-sea sampling data are not included in the APAIS estimates presented in this report, the estimation methods developed in this report can be generally applied to analyze HB at-sea sampling data.

iv) *Catch Types*: The number of fish caught is divided into three "catch types". Type A catch is defined to include the fish brought to shore in whole form that are available to be inspected by the interviewer. The interviewers are trained to identify and count fish in the Type A catch. In some cases, the Type A catch data is collected as the catch of a group of anglers who are unable to separate out their own individual catches. At least one of the anglers who contributed to the group catch must be interviewed, and the Type A catch is counted and identified as a "mixed group catch" that is linked to that interview and any other interviews of anglers who also contributed to that group catch. Because all of the contributors to the group catch may not be interviewed, a count of the total contributors to the group catch is obtained and included with each Type A catch record. Type B1 catch is defined as the fish that were caught and killed (not released alive) but were not available to be inspected in whole form by an interviewer. Type B2 catch is defined as the fish that were caught and released alive at sea. The numbers of Type B1 and Type B2 fish are reported by individual intercepted anglers, and are never recorded as the catch of a group.

v) *Catch rates by primary area of fishing*: In the data analysis, catch rates are estimated for angler fishing trips that occurred primarily in one of three general fishing areas that distinguish between ocean and inland waters and categorize ocean location based on the distance from shore (inland waters, nearshore or state ocean waters, and offshore or federal ocean waters. The dividing line between nearshore and offshore ocean waters varies by state (3 miles in most states, 10 miles off the west coast of Florida) and is intended to correspond to the separation of state-managed and federally managed waters. The estimation of catch rates for the three fishing areas was intended in part to help meet the needs of fishery managers.

3. Estimation Method

The APAIS utilizes a stratified multi-stage cluster sampling design as described in Section 2. The alternate mode interviews are excluded from the data because there is no clear method that could be used to calculate appropriate inclusion probabilities. Using a three-stage sampling for PR and CH modes as the example, the stratified three-stage cluster sampling is summarized below:

Stratification: Stratify sampling frame by month-day type ($h = 1, \ldots, H$).

Stage I. Site-days ($i = 1, \ldots, n_h$) are sampled within stratum via ppswor. The inclusion probability of site-day *i* is π_{hi}, which is proportional to *expected* number of angler-trips for the site-day *i*.

Stage II. Sample boat-trips ($j = 1,\ldots,b_{hi}$) within each of sampled site-days via SI (simple random sampling without replacement); that is, sample b_{hi} boat-trips from a total of B_{hi} boat-trips within the *hi*-th site-day.

Stage III. Sample angler-groups ($k = 1,\ldots,m_{hij}$) within each of sampled boat-trips via simple random sampling; that is, sample m_{hij} groups from a total of M_{hij} groups at random within the *hij*-th boat-trip. (Each angler within an angler group contributes one angler-trip.)

Ideally, all site-days at stage I of sampling would have known, positive probabilities of inclusion in the sample. As noted above, the frame contains only public-access sites, so private-access sites have zero probability of selection. Further, alternate site selection complicates the computation of inclusion probability of selected PSUs.

At stage II, an ideal survey would list all boat-trips within selected site-days, and draw a simple random sample of boat-trips from the list. In practice, this list is not maintained, and the total number of boat-trips per selected site-day is not known.

Similarly, at stage III, an ideal survey would list all groups of anglers within selected boat-trips, and draw a simple random sample of angler-groups from each selected boat-trip. But in practice, the total number of groups of anglers available to be sampled is not available. This complicates estimation, as will be detailed further below.

3.1 Estimation of Catch rate and variance

The catch rate for Type A fish is estimated as a ratio-type estimator. Let

y_{hijk} = observed number of fish caught in the *k*-th group (for $k = 1,\cdots,m_{hij}$ groups sampled within the *hij*-th boat trip),

x_{hijk} = observed number of anglers in the *k*-th group,

M_{hij} = total number of groups of anglers available to be sampled in the *hij*-th boat trip,

X_{hij} = observed number of angler trips aboard the *j*-th boat trip (for $j = 1, \cdots, b_{hi}$ boat trips),

B_{hi} = total number of boat trips available to be sampled within the *hi*-th site-day (for

$i = 1, \cdots, n_h$ site-days sampled),

X_{hi} = cluster size of the *hi*-th sampled site-day, and

π_{hi} = inclusion probability of the *hi*-th sampled site-day.

The estimate of **total catch** of Type A fish is expressed by a ratio estimator:

$$\hat{t}_y = \sum_{h=1}^{H} \sum_{i=1}^{n_h} \frac{1}{\pi_{hi}} (X_{hi}) \left\{ \frac{\sum_{j}^{b_{hi}} \frac{B_{hi}}{b_{hi}}}{\sum_{j}^{b_{hi}} \frac{B_{hi}}{b_{hi}} X_{hij}} \left(X_{hij} \left(\frac{\sum_{k}^{m_{hij}} \frac{M_{hij}}{m_{hij}} y_{hijk}}{\sum_{k}^{m_{hij}} \frac{M_{hij}}{m_{hij}} x_{hijk}} \right) \right) \right\}$$

The weights or inverse inclusion probabilities of TSU and SSU within the *hi*-th site-day (M_{hij} / m_{hij} and B_{hi}/b_{hi}) are not available because M_{hij} and B_{hi} are not observed from the field. They need to be approximated by

$$\frac{\hat{M}_{hij}}{m_{hij}} = \left(\frac{1}{m_{hij}} \right) \left(\frac{X_{hij}}{m_{hij}^{-1} \sum_{k=1}^{m_{hij}} x_{hijk}} \right)$$

and

$$\frac{\hat{B}_{hi}}{b_{hi}} = \left(\frac{1}{b_{hi}} \right) \left(\frac{X_{hi}}{b_{hi}^{-1} \sum_{j=1}^{b_{hij}} X_{hij}} \right)$$

where X_{hij} is named "PARTY" or the observed number of anglers who fished on the same boat. Replacing the approximated sampling weights and assuming that X_{hi} is known, the total catch of a target population is estimated by

$$\hat{t}_y = \sum_{h=1}^{H}\sum_{i=1}^{n_h}\sum_{j=1}^{b_{hij}}\sum_{k=1}^{m_{hij}}\left(\frac{1}{\pi_{hi}}\right)\left(\frac{X_{hi}}{\sum_{j=1}^{b_{hi}} X_{hij}}\right)\left(\frac{X_{hij}}{\sum_{k=1}^{m_{hij}} x_{hijk}}\right) y_{hijk} = \sum_{h=1}^{H}\sum_{i=1}^{n_h}\frac{\hat{t}_{y,hi}}{\pi_{hi}} \tag{1}$$

Variance estimation is complicated and relies on three standard approximations: (i) Taylor series linearization to handle nonlinearity in ratio estimators (e.g., Wolter 1985); (ii) an "ultimate clusters" approximation, which uses the fact that variability of estimates between PSU's dominates the variance, rather than the SSU and TSU levels (Cochran 1977, Särndal et al. 1992), and (iii) a sampling with-replacement approximation (Särndal et al. 1992). Note that to estimate the variance of $\hat{t}_y = \sum_{h=1}^{H}\sum_{i=1}^{n_h}\frac{\hat{t}_{y,hi}}{\pi_{hi}}$, only stratum-level variance estimates $\hat{V}(\hat{t}_{y,h}) = \hat{V}\left(\sum_{i=1}^{n_h}\frac{\hat{t}_{y,hi}}{\pi_{hi}}\right)$ (h = 1,…,H) are needed for the overall variance estimate. See Appendix I for details.

Like total catch, **total effort** for a target population is estimated by

$$\hat{t}_x = \sum_{h=1}^{H}\sum_{i=1}^{n_h}\sum_{j=1}^{b_{hij}}\sum_{k=1}^{m_{hij}}\left(\frac{1}{\pi_{hi}}\right)\left(\frac{X_{hi}}{\sum_{j=1}^{b_{hi}} X_{hij}}\right)\left(\frac{X_{hij}}{\sum_{k=1}^{m_{hij}} x_{hijk}}\right) x_{hijk} = \sum_{h=1}^{H}\sum_{i=1}^{n_h}\frac{X_{hi}}{\pi_{hi}} \tag{2}$$

Note the cancellation of terms involving $\sum_{j=1}^{b_{hi}}\sum_{k=1}^{m_{hij}}$, so that the total effort estimate depends only on the expansion of cluster sizes (X_{hi}) across all sampled site-days within a given stratum.

The ratio estimator of **catch rate** for a given target population is then

$$\hat{R} = \frac{\hat{t}_y}{\hat{t}_x} = \frac{\sum_{h=1}^{H}\sum_{i=1}^{n_h}\hat{t}_{y,hi}/\pi_{hi}}{\sum_{h=1}^{H}\sum_{i=1}^{n_h}\hat{t}_{x,hi}/\pi_{hi}} \tag{3}$$

The computations in Equations (1)-(3) can be done by using SAS proc surveymeans or the R survey package (Lumley 2004, 2010). See Appendix I for examples.

Equations (1)-(3) and their associated variances can also be applied to the estimates of Type B1 and Type B2 fish by setting x_{hijk} = 1 because each individual angler is interviewed. These two types of catch are self-reported by the individual angler but unavailable to be examined by samplers. For a two-stage sampling such as in SH mode, the terms related to *j* (boat-trip) are eliminated. Equations (1)-(3) can be also applied to estimate the proportion of anglers fishing days in the three saltwater fishing areas (inland, nearshore, offshore) and the proportion of fishing days by anglers living in coastal county residential households with a landline telephone. For example, the latter proportion can be estimated by setting x_{hijk} = 1 if angler's living status agrees with the condition given above; and x_{hijk} = 0 otherwise.

Appendix II describes results of a small simulation study which illustrates properties of the weighted estimators and the corresponding variance estimators. This study assumes that cluster size and inclusion probabilities of site-days are known. In traditional MRFSS, the cluster size (X_{hi}) is not available from field data and the inclusion probability (π_{hi}) of the site-day *i* is not available due to alternate site sampling. These two design features need to be estimated, and we now turn to these estimation problems.

3.2 Cluster size of site-day (X_{hi})

The interviewers are assigned to sites in the hours of the day with the highest expected angling activity. The total number of anglers (i.e., cluster size, X_{hi}) departing the site *i* in a full day is not observed but can be estimated using the hourly distribution of angler-trips observed in telephone survey (CHTS) data. During the telephone survey, respondents are asked to enumerate fishing trips and provide departure times. Data are available for 980,000 trips by 215,000 household interviews between 1990 and 2007.

Table 1 shows number of trips by 1-hr interval by wave from 1990 to 2007 for New York PR mode based on CHTS data. The CHTS has never been conducted in New York in Wave 1, 1990-2007. Fishing activities in Waves 2 and 3 were usually low. It is necessary to "borrow strength" across target populations in order to obtain a reliable estimator for X_{hi}, a problem that we address with small area estimation techniques (Ghosh and Rao, 1994). Because departure times correspond to a 24-hour clock, the distribution at time 0 should match the distribution at time 24. The distribution is said to be "circular." We thus develop small area estimation methodology for circular data, using hierarchical Bayesian techniques.

Let T_{ijklm} denote the departure time for fishing trip m by the respondent l in state i, wave j and mode k. Given the circular nature of departure time, T_{ijklm} can be expressed as the angle of a two-dimensional random vector, suitably normalized so that 360 degrees equals 24 hours (e.g., 5:30pm is (360 degrees)(12h+5.5h)/24h=262.5 degrees). Specifically, assume that the normalized T_{ijklm} are independently distributed as projected bivariate normal random variables (denoted by PN_2); that is,

$$T_{ijklm} \overset{ind}{\sim} PN_2(\boldsymbol{\mu}_{ijkl}, \mathbf{I}_2) \tag{4}$$

The mean of the projected normal distribution can be expressed as a function of fishing trip characteristics,

$$\boldsymbol{\mu}_{ijkl} = \boldsymbol{\mu} + \mathbf{s}_i + \mathbf{w}_j + \mathbf{m}_k + \mathbf{r}_l \tag{5}$$

where each term in Equation (5) is a two-dimensional vector corresponding to grand mean ($\boldsymbol{\mu}$), state effect (**s**), wave effect (**w**), mode effect (**m**) and respondent effect (**r**), and $\mathbf{I}_2$ in Equation (4) is the 2×2 identity variance-covariance matrix. The normalization of T_{ijklm} and explicit form of PN_2 are given in Presnell et al. (1998) and Nuñez-Antonio and Gutiérrez-Pena (2005).

Hierarchical Bayesian small area estimation (Ghosh and Rao, 1994) is an effective approach to "borrow strength" across target populations to obtain reliable target population-specific estimates of the distribution of T_{ijklm}. The approach of Nuñez-Antonio and Gutiérrez-Pena (2005) is generalized to the regression case described in Equations (4) and (5). We also explore various specifications of state, wave, mode and respondent to be either random or fixed effects. In summary, we need to specify the priors for all the parameters in Equations (4) and (5). We then apply Markov Chain Monte Carlo (MCMC) techniques to obtain the posterior distributions of the parameters given the observed data. Posterior distributions of the fraction of daily departures within a given time interval and for a given state-wave-mode combination can also be obtained. Characteristics of these posterior distributions (e.g., posterior means) can then be used as the desired small area estimates.

The priors of $\boldsymbol{\mu}, \mathbf{s}_i, \mathbf{w}_j, \mathbf{m}_k, \mathbf{r}_l$ are assumed to be independent normal distributions. In the case of fixed effects, proper priors are chosen with a large variance value so that they are essentially non-informative. In the case of random effects, the variance in the prior is taken to follow an inverse gamma distribution, with parameters chosen to be non-informative (Gelman et al. 1995). The Gibbs sampler is then used to estimate the posterior distributions of all model components. The Deviance Information Criteria (DIC; Spiegelhalter, 2002) is used to choose among different model specifications of the fixed and random effects, as well as models with interactions between the factors. Appendix III contains further details.

The fraction of daily departures within time interval [t, t+Δ) for a state-wave-mode is defined as

$$P_{t,\Delta} = \int_{t}^{t+\Delta} f_T(t \mid \boldsymbol{\mu}_{ijk})dt \qquad (6)$$

where $P_{t,\Delta}$ is an explicit function of $\boldsymbol{\mu}_{ijk} = \boldsymbol{\mu} + \mathbf{s}_i + \mathbf{w}_j + \mathbf{m}_k$. Thus, its posterior distribution is obtained directly from the Gibbs sampler as well. In this report, we set Δ = 1 hr and estimate 24 fractions. The estimated fractions ($P_{t,\Delta}$) from the model are then combined with the empirical fractions from the telephone survey data, and used to expand the observed count of anglers in [t, t+Δ) to X_{hi}. Details of the composite estimator are given in Appendix III. Expansion is performed by taking the observed count of anglers at the site during the interview period (which will therefore need to be explicitly recorded; more on that below) and dividing it by the estimated fraction for that time period.

3.3 Probability with Alternate Site Sampling (π_{hi})

In prior years, a large amount of field interview data was collected at alternate sites. Between 2003 and 2007, almost 65,000 (~49%) out of 134,000 field interviews on the U.S. Atlantic coast and Gulf of Mexico were collected at alternate sites. Although alternate site sampling violates the random selection paradigm essential for valid design-based inference, there is a substantial loss of information if alternate site data are discarded. Therefore, it is desired to create "pseudo-weights" for alternate site data.

To obtain selection probabilities for alternate sites, we assume that alternate site selection follows a random process when considered across all strata, years and interviewers. The random process includes three assumptions:

(i) Alternate site-days are selected by stratified Poisson sampling (Särndal et al. 1992) among site-days not assigned as primary site-days, with the strata the same as for the primary sites.

(ii) The alternate site-day selection probabilities of alternate sites are unknown but are constant across years.

(iii) The selection probabilities do not depend on which sites were selected as primary sites and are fixed for a given site within each stratum (i.e. they do not depend on the day, only on the site).

The random process will hold if interviewers unequivocally follow the explicit rules on alternate site selection, and only approximately so if the selection is based on interviewer judgment. Under this assumed random process, it is possible to define selection probabilities and hence "pseudo-weights" for alternate sites.

We consider selection within a given stratum, defined by state, wave, mode, and day-type (weekday or weekend). For the moment, we denote stratum by the single index *h*; later, we will expand this notation. For site-day *i*, the index *i* can be rewritten as a bivariate index *kd* with site *k* and day *d*. In what follows, we maintain the subscript *d* even though, within a stratum, the inclusion probability for a given site will be modeled as constant across days. A given site-day (*k*, *d*) can be selected as: (i) primary site-day with known inclusion probability $\pi^{P}_{h,kd}$ proportional to pressure matrix of known size measures, or (ii) not selected as primary site-day but selected as alternate sites-day with unknown probability $\pi^{A}_{h,kd}$. The combined inclusion probability is

$$\pi_{h,kd} = \pi^{P}_{h,kd} + (1-\pi^{P}_{h,kd})\pi^{A}_{h,kd} \ . \qquad (7)$$

Since the alternate site-day sampling process is assumed to be stationary over time (and in particular, does not depend on the day *d*), the probability $\pi^{A}_{h,kd}$ can be directly estimated from the counts of primary and alternate site-day selections for a given site *k* across all years for each state, wave, mode and day-type stratum:

$$\hat{\pi}^{A}_{h,kd,direct} = \frac{n^{A}_{h,k}}{n^{P}_{h} - n^{P}_{h,k}} \qquad (8)$$

where n^{P}_{h} = total number of site-days selected as primary site-days,

$n^{P}_{h,k}$ = number of times (days) site *k* selected as primary site,

$n^{A}_{h,k}$ = number of times (days) site *k* selected as alternate site.

That is, the probability that site *k* is selected as an alternate site, given that it was not selected as a primary site, is estimated as (number of successes) / (number of trials). A "trial" is conducted each time a primary site other than *k* is selected, because then site *k* has an opportunity to be selected as an alternate site. A "success" occurs each time site *k* is selected as an alternate site.

A total of 134,316 site-days were visited in 2003-2007 for all state, wave, mode, and day-type strata, among which 64,692 site-days were alternate. The total number of sites is 4,391, with 3,903 of them having been used at least once as alternate site in 2003-2007. Because the sample sizes of alternate sites were very small in many strata, it was decided to investigate whether pooling estimates across strata could be used. Pooling is not possible for 688 (out of 3,903) sites, which were used as alternates only in one stratum; the direct estimate $\hat{\pi}^{A}_{h,kd,direct}$ from Equation (8) was used for these cases.

For the remaining 3,215 sites, we wish to determine whether or not the $\hat{\pi}^{A}_{h,kd,direct}$ are similar across strata, using a formal hypothesis test. Let $h \in H_k$, where H_k is the set of all strata in which site *k* appears as an alternate site. The null hypothesis of the test is

$$H_0 : \pi^{A}_{h,kd} = \pi^{A}_{h',kd} \text{ for all } h, h' \in H_k$$

The test is conducted by treating $\hat{\pi}^{A}_{h,kd,direct}$ as independent and approximately normally distributed estimators of $\pi^{A}_{h,kd}$ and applying the F-test for equality as in a one-way ANOVA. This test shows that 2,824 sites do not reject the null hypothesis. Therefore, a pooled estimator is calculated for these 2,824 sites:

$$\hat{\pi}^{A}_{h,kd,pooled} = \frac{\sum_{h \in H_k} n^{P}_{h} \hat{\pi}^{A}_{h,kd,direct}}{\sum_{h \in H_k} n^{P}_{h}} \tag{9}$$

For the 391 sites for which the null hypothesis is rejected, a logistic regression analysis was carried out to predict $\pi^{A}_{h,kd}$. Since we are going to perform a regression using the stratum characteristics as predictors, we expand the stratum index, *h*, into the four-dimensional index *(i,j,m,l)*, with state *i*, wave *j*, mode *m* and day-type *l*. We define a new binary random variable $Y_{ijml,kt}$ to represent the individual "trials" mentioned above, and for each occurrence *t* of the site *k* (= 1,…,391 sites) within stratum *(i,j,m,l)*, we let $Y_{ijml,kt}$ = 1 when site appears as alternate and $Y_{ijml,kt}$ = 0 otherwise. Under our assumptions, $\pi^{A}_{h,kd} = E[Y_{ijml,kt}]$. The linear logistic mean model is

$$\log\left(\frac{\pi^{A}_{h,kd}}{1-\pi^{A}_{h,kd}}\right) = \mu + \sum_{i=1} \alpha_i S_i + \sum_{j=1} \beta_j W_j + \sum_{m=1} \gamma_m M_m + \sum_{l=1} \lambda_l D_l \tag{10}$$

where μ is grand mean, *S, W, M* and *D* respectively represent binary variables for state, wave, mode and day-type with their coefficients α, β, γ and λ. We apply SAS proc logistic and its stepwise variable selection to estimate stratum-specific values for $\pi^{A}_{h,kd}$, denoted by $\hat{\pi}^{A}_{h,kd,\mathrm{log}reg}$. Note that the predictions from Equation (10) are the logits of probability, hence

$$\hat{\pi}^{A}_{h,kd,\mathrm{log}reg} = \frac{\exp(\mathbf{X}'\hat{\boldsymbol{\beta}})}{1+\exp(\mathbf{X}'\hat{\boldsymbol{\beta}})}$$

where $\mathbf{X}$ and $\hat{\boldsymbol{\beta}}$ are the design matrix and estimated coefficient vector from Equation (10). Appendix IV provides additional details on the statistical properties of the combined inclusion probabilities (8).

3.4 Final inclusion probability for any site

Finally, the adjusted inclusion probabilities that include both primary and alternate selections are computed for all 134,316 site-days that were visited in 2003-2007, as follows:

(i) Site-days appear as primary but never as alternate: use the original inclusion probabilities ($\pi_{h,kd}$), which are proportional to the fishing pressure size measures mentioned earlier in this report

(ii) Site-days appear as alternate but never as primary,

Case 1. if the original inclusion probability is available, then

$$\hat{\pi}_{h,kd} = \pi^{P}_{h,kd} + (1-\pi^{P}_{h,kd})\hat{\pi}^{A}_{h,kd} \tag{11}$$

Case 2. if the original inclusion probability is not available, then the site has no chance to be selected as primary. So $\pi^{P}_{h,kd} = 0$ in Equation (11), and $\hat{\pi}_{h,kd} = \hat{\pi}^{A}_{h,kd}$.

(iii) Site-days appear as both primary and alternate, use Equation (11).

4. Results

4.1 Simple simulation

A simulation study was carried out to illustrate the design properties of the weighted estimator, including its approximate design-unbiasedness. Details of the simulation are provided in Appendix II. For the estimated catch rate and its standard error, the percentages of the relative biases in estimate of catch rate and its standard error are 0.1% and -0.5% respectively (Figure 3). Note that the simulation does not evaluate the estimation of the cluster sizes (X_{hi}) nor the adjusted inclusion probabilities (π_{hi}), both of which are assumed known without error.

An example

It is worthwhile to point out that the newly developed, weighted estimation method is less susceptible to potential sources of bias. The estimation method can be applied to any species, years, waves, sub-regions, states, and modes. This report uses striped bass (*Morone saxatilis*) encountered by New York, PR mode anglers (Type A, B1 and B2 fish) in 2003-2007 as an example for the application of the weighted and unweighted methods.

The fishery statistics of fishing area (state, federal and inland waters) are important to fishery managers and anglers. Therefore, catch rates and sampling effort are listed by fishing area. Table 2 summarizes the number of selected site-days (primary and alternate site-days or PSU) and number of interviews by fishing areas, year, wave, and PR mode in New York. The major sampling effort was concentrated in waves 3, 4 and 5 in 2003-2007. The sampling effort in wave 2 was usually low, especially in state and federal waters. These phenomena were common across years and modes.

The estimated catch rates were compared between the weighted and unweighted methods by removing the alternate mode data. Note that the unweighted method used data that pooled all interviews within the target population of New York PR mode. In contrast, the traditional MRFSS method was a kind of general unweighted method, but made use of alternate mode data. Therefore, the traditional MRFSS estimates were not directly comparable with the weighted and unweighted estimates.

Table 3 lists the weighted and unweighted estimates of catch rate and its standard error for Type A, B1 and B2 catches. The differences between estimates of the two methods were substantial within wave across years. However, the differences do not show any patterns of direction and magnitude. Confidence intervals of catch rate from the two methods generally overlapped and covered the point estimates. The lower boundaries of 95% confidence intervals from both methods were negative in many cases. The unweighted method has the tendency to severely under-estimate the true variance in comparison with the weighted method (Korn and Graubard, 1995).

4.2 Proportion of anglers living in coastal county household with landline telephone

Table 4 summarizes the weighted and unweighted estimates of proportion of New York PR mode anglers living in coastal county households with landline telephones from 2003 to 2007. The differences between the two estimates were substantial, although there do not appear to be discernible patterns of direction and magnitude in differences between the two estimates.

4.3 Proportion of anglers fishing in state, federal and inland waters

Table 5 summarizes the weighted and unweighted estimates of proportion of anglers by fishing area. Although there were substantial differences between the two estimates, no pattern of direction and magnitude in differences were found. The proportion of PR mode anglers fishing in federal water is usually low as expected in the northeastern and mid-Atlantic regions.

5. Discussion

The MRFSS unweighted estimation method is described in Ghosh (1981). The method pools all interviews across all primary and alternate modes and sites, months and day-types within a given state and wave. In the analysis, the pooled data are post-stratified by angler's recorded fishing mode. A simple ratio estimator is used to calculate the estimate of catch rate, and its variance using the basic equation for simple random sampling for the "pseudo-target" population. The pooling and partitioning of the data destroy the data structure dictated by the APAIS sampling design and may cause biases in the resulting estimates of catch rate (Table 3). Also, this unweighted method leads to serious over-estimation of the precision of the catch rate because it does not account for covariance that is likely to exist due to potentially strong correlations among angler-trips that occur within the same site-day. It is clear that the unweighted estimation method is biased even though the magnitude and direction of its bias does not appear to be consistent in any predictable way from wave to wave and year to year.

The weighted estimator is design-unbiased. However, it will only provide a correct estimation method for mean catch rates when the sampling, data collection, and data processing for the APAIS are conducted in accordance with the documented sampling design. Errors may be introduced into the estimator if the data structure is not arranged in accordance with the stratified, pps multistage sampling design, or if the field sampler misinterprets the sampling and measurement protocols.

The sampling procedures for the MRFSS APAIS have incorrectly focused too much attention on the need to maximize the number of angler intercepts obtained. The total number of intercepts has been considered the “sample size” that needs to be maximized in order to maximize the statistical precision of APAIS estimates. The focus should instead be on maximizing the number of site-days sampled, because the primary sampling unit in the multistage APAIS sampling design is the site-day, not the angler trip intercept and the precision of multi-stage survey estimators depends almost exclusively on the number of primary sampling units. Future access point intercept surveys must recognize the need to increase site-day sampling as a means of increasing the statistical precision of mean catch rate estimates. In fact, a 10% increase in the average number of intercepts obtained within selected site-day assignments would have much less impact on the estimated variance of the unbiased catch rate estimator than a 10% increase in the number of site-days sampled.

There has probably not been enough emphasis placed on the need to spread out the interviews obtained within a selected site-day assignment. APAIS interviewers have often been encouraged to maximize the number of interviews obtained per hour spent on site. Because limits have been imposed on the number of interviews that an interviewer can obtain within one assigned site-day, the emphasis on maximizing interviews has often resulted in short site visits that intercept a large cluster of trips that ended near the same time. It would be more desirable to have interviewers spread out their angler trip interviews across a longer time period so that they could obtain data from more distinct time intervals and/or more distinct boat-trips (SSUs).

Future access point intercept surveys should be designed to eliminate visits to alternate sites that are not pre-determined in the probability sampling design. It is essential to understand two fundamentals in sampling design and estimation. First, sampling design is based on probability sampling and estimation is based on inverse inclusion probabilities, or weights, of individual sampling units. If clusters of sites were selected as PSUs and strict procedures were developed to determine the order and timing of the interviewer’s visits to the assigned sites within the cluster, then the inclusion probabilities of all sites within the cluster would be dictated by the sampling design. The traditional APAIS procedure to allow alternate site visits that are not predetermined at the PSU sampling stage creates unnecessary difficulty in the development of appropriate weights for the intercepts collected at the alternate sites.

Future surveys should also evaluate whether or not it makes most sense to sample different fishing modes as separate strata with their own mode-specific site frames or to just combine them into one stratum with a general site frame that covers fishing in all modes. If the choice is made to do the former, then obtaining "alternate mode" angler trip intercepts should not be a survey objective. Alternate mode interviews may be useful for assessing the different kinds of fishing activity that occur at individual sites, but the data collected from such interviews should not be used in the estimation of catch rates when sampling is stratified by mode. The difficulties of determining appropriate inclusion probabilities for alternate mode intercepts will probably always far outweigh any precision benefits that would be gained by trying to include them in the estimation of mode-specific mean catch rates.

Future access point surveys should pay more attention to getting accurate counts of the number of angler fishing trips that are completed within each site-day assignment. The total count of angler trips, including those not intercepted by the interviewer, plays a very important role in calculating the PSU cluster size. When conducting interviewing assignments for private boat and charter boat modes, it should also be an objective to get an accurate count of all of the completed boat trips so that SSU cluster sizes can be more accurately quantified. In fact, emphasis should be shifted away from maximizing the number of intercepts obtained per site-day assignment if it interferes with the ability of interviewers to obtain accurate counts of boat trips and angler trips during an assignment. For assignments at very active sites, it may also be desirable to instruct interviewers to alternate between conducting interviews and obtaining counts. Alternatively, two samplers could be assigned to a high-activity site-day so that one could obtain counts while the other is intercepting anglers and conducting interviews. Either approach could allow for more accurate accounting of cluster sizes and more accurate determination of appropriate inclusion probabilities for the SSUs and TSUs in a weighted estimation approach.

Future surveys should also consider developing an approach that would cover completed fishing trips throughout the fishing day. The traditional APAIS sampling procedure instructs interviewers to visit an assigned site during the assigned day's peak activity period for fishing. Consequently, nighttime and off-peak daytime fishing trips are generally not sampled and are assumed to be similar to trips ending during the peak period. Future surveys could circumvent this potential source of bias by establishing different time block strata so that at least some sampling would occur during all nighttime and daytime intervals when fishing occurs. The site-day sampling could be allocated among the different time-interval strata in some manner that reflects the expected distribution of fishing activity among them.

Fishery managers request to partition catch rate into fishing areas (i.e., inland, state and federal waters). However, small sample sizes (both site-days and angler-trips) in any fishing areas are major obstacle in the estimation. Obviously, the sampling design and method for model-based small-area estimation may need to be considered in the future as micro-management becomes the trend in fishery management.

Inverse-probability-weighted estimators are often quite variable due to the fluctuations of inclusion probability, especially when applied to small domains or variables with relatively rare occurrences. If the variability in the estimators is considered too high, an estimation approach that employs models to "borrow strength" across space and/or time could be investigated. Such small area estimation techniques will be a subject of future investigations.

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Tables

Table 1. Frequency distribution of departure times measured in hours of the day (0-24) by wave and day-type from the CHTS data collected in New York, PR mode, 1990-2007. (WD = weekdays and we = weekends)

	Wave									
	2		3		4		5		6	
Time	wd	we	wd	we	wd	we	wd	we	wd	we
0			10	10	14	21	7	9	5	2
1			7	7	20	8	10	4	2	4
2			4	8	9	4	2	3	2	17
3	2		2	3	3	10	2	6	1	
4					7	3	5	2		1
5	1		5	13	5	3	2	4		1
6			7	5	10	64	54	28	1	
7	4	7	26	16	14	35	10	20	9	2
8			32	12	11	10	8	35	2	3
9	1	3	4	21	16	25	8	31	7	3
10	2	10	16	13	18	33	6	27	2	4
11	1	5	15	52	29	36	11	38	8	4
12	12	3	17	43	50	60	12	43	3	2
13	2	11	24	31	75	37	21	20	6	7
14	2	8	29	26	34	82	27	30	2	4
15	18	13	29	53	88	101	29	68	12	20
16	7	7	44	58	123	84	46	55	12	10
17	12	4	28	49	101	124	46	165	12	19
18	9	5	36	50	113	95	55	90	17	12
19	19	15	87	87	92	164	46	61	3	12
20	1	7	45	43	116	114	44	61	1	8
21	9	1	44	27	79	54	14	17		2
22	6	2	49	32	132	95	13	32	1	1
23	2	7	17	5	34	21	6	2	6	
Sum	110	108	577	664	1193	1283	484	851	114	138

Table 2. Number of site-days (PSU) and number of interviews by year, wave, and fishing area. Empty cells indicate no sample available but sampling may occur.

			Year				
AREA	WAVE		2003	2004	2005	2006	2007
State Waters	2	Site-Days	7	3	3		
		Interviews	25	9	13		
	3	Site-Days	20	30	21	29	52
		Interviews	98	165	81	140	275
	4	Site-Days	42	50	35	45	45
		Interviews	188	262	137	213	245
	5	Site-Days	38	26	39	48	49
		Interviews	157	111	156	247	237
	6	Site-Days	6	8	7	12	15
		Interviews	36	47	35	74	76
Federal Waters	2	Site-Days			1		2
		Interviews			2		7
	3	Site-Days	2	5	3	3	5
		Interviews	8	11	4	5	23
	4	Site-Days	7	8	6	10	4
		Interviews	16	18	16	22	13
	5	Site-Days	7	4	2	2	6
		Interviews	13	5	5	4	12
	6	Site-Days	2		1		
		Interviews	6		4		
Inland	2	Site-Days	19	9	12	10	6
		Interviews	80	35	34	37	39
	3	Site-Days	51	40	47	50	70
		Interviews	307	237	204	228	382
	4	Site-Days	66	60	46	51	53
		Interviews	411	319	278	215	278
	5	Site-Days	50	57	34	57	70
		Interviews	221	286	170	292	389
	6	Site-Days	8	13	11	14	24
		Interviews	42	83	46	96	118

Table 3. Preliminary weighted and unweighted estimates of striped bass catch rate and standard error (StdErr) by fishing area in year, wave, New Yolk, and PR mode. Type A Catch Rate (based on counts of fish kept and observed in whole form by samplers)

			YEAR							
AREA	WAVE	ESTIMATES	2004		2005		2006		2007	
			Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted
State Waters	2	Catch Rate								
		StdErr								
	3	Catch Rate	0.0514	0.0485	0.0627	0.1235	0.0636	0.1071	0.0616	0.0582
		StdErr	0.0246	0.0188	0.0439	0.0506	0.0393	0.0492	0.0208	0.0167
	4	Catch Rate	0.0739	0.0458	0.0215	0.0730	0.0387	0.0329	0.0298	0.0286
		StdErr	0.0422	0.0214	0.0156	0.0590	0.0323	0.0122	0.0153	0.0121
	5	Catch Rate	0.1138	0.1261	0.0301	0.0385	0.0767	0.0688	0.0630	0.0506
		StdErr	0.0433	0.0385	0.0169	0.0154	0.0431	0.0268	0.0478	0.0309
	6	Catch Rate	0.1580	0.0638	0.0309	0.0857	0.0717	0.0405	0.0628	0.0789
		StdErr	0.0553	0.0468	0.0309	0.0476	0.0610	0.0299	0.0380	0.0310
Federal Waters	2	Catch Rate								
		StdErr								
	3	Catch Rate			0.6836	1.5000			0.0941	0.0870
		StdErr			0.3962	1.3013			0.0834	0.0851
	4	Catch Rate								
		StdErr								
	5	Catch Rate								
		StdErr								
	6	Catch Rate								
		StdErr								
Inland Waters	2	Catch Rate					0.1757	0.0541		
		StdErr					0.0921	0.0377		
	3	Catch Rate	0.0089	0.0127	0.1160	0.0539	0.0306	0.0395	0.0661	0.0733
		StdErr	0.0071	0.0073	0.0892	0.0186	0.0119	0.0156	0.0211	0.0199
	4	Catch Rate	0.0107	0.0063	0.0382	0.0396	0.0075	0.0093	0.0091	0.0108
		StdErr	0.0091	0.0044	0.0209	0.0147	0.0057	0.0066	0.0049	0.0062
	5	Catch Rate	0.0186	0.0210	0.0290	0.0176	0.0456	0.0205	0.0117	0.0154
		StdErr	0.0093	0.0085	0.0190	0.0101	0.0279	0.0083	0.0104	0.0081
	6	Catch Rate	0.2080	0.0964	0.1009	0.0652	0.1588	0.0313	0.0220	0.0339
		StdErr	0.1339	0.0325	0.1010	0.0366	0.0600	0.0178	0.0110	0.0167

Table 3 (continued). Preliminary weighted and unweighted estimates of striped bass catch rate and standard error (StdErr) by fishing area in year, wave, New Yolk, and PR mode. Type B1 Catch Rate (based on angler reported counts of fish released dead or kept and not observed in whole form by interviewers)

AREA	WAVE	ESTIMATES	YEAR							
			2004		2005		2006		2007	
			Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted
State Waters	2	Catch Rate								
		StdErr								
	3	Catch Rate			0.0068	0.0118	0.0303	0.0274	0.0453	0.0169
		StdErr			0.0071	0.0117	0.0244	0.0166	0.0288	0.0089
	4	Catch Rate					0.0823	0.0628	0.0093	0.0198
		StdErr					0.0423	0.0216	0.0092	0.0143
	5	Catch Rate	0.0084	0.0084	0.0469	0.0364	0.0549	0.0433	0.0888	0.0694
		StdErr	0.0081	0.0084	0.0312	0.0169	0.0226	0.0160	0.0409	0.0172
	6	Catch Rate			0.0081	0.0278	0.0364	0.0260		
		StdErr			0.0094	0.0275	0.0310	0.0182		
Federal Waters	2	Catch Rate								
		StdErr								
	3	Catch Rate			0.2874	0.1667	0.2120	0.2000	0.1222	0.0870
		StdErr			0.2596	0.1524	0.2130	0.1791	0.1098	0.0851
	4	Catch Rate								
		StdErr								
	5	Catch Rate								
		StdErr								
	6	Catch Rate								
		StdErr								
Inland Waters	2	Catch Rate					0.0035	0.0769		
		StdErr					0.0045	0.0567		
	3	Catch Rate	0.0043	0.0075	0.0296	0.0142	0.0049	0.0042	0.0067	0.0099
		StdErr	0.0044	0.0053	0.0295	0.0082	0.0048	0.0042	0.0062	0.0061
	4	Catch Rate							0.0204	0.0069
		StdErr							0.0164	0.0049
	5	Catch Rate					0.0161	0.0064	0.0010	0.0024
		StdErr					0.0142	0.0045	0.0010	0.0024
	6	Catch Rate			0.0242	0.0208	0.0076	0.0096		
		StdErr			0.0260	0.0207	0.0085	0.0096		

Table 3 (continued). Preliminary weighted and unweighted estimates of striped bass catch rate and standard error (StdErr) by fishing area in year, wave, New Yolk, and PR mode. Type B2 Catch Rate (based on angler reported counts of fish caught and released alive)

AREA	WAVE	ESTIMATES	2004 Weighted	2004 Unweighted	2005 Weighted	2005 Unweighted	2006 Weighted	2006 Unweighted	2007 Weighted	2007 Unweighted
State Waters	2	Catch Rate			0.0269	0.0667				
		StdErr			0.0333	0.0650				
	3	Catch Rate	0.2313	0.1561	0.4128	0.7412	0.3721	0.5342	0.3278	0.3243
		StdErr	0.0958	0.0436	0.3566	0.2369	0.1403	0.1754	0.1076	0.0924
	4	Catch Rate	0.1040	0.0780	0.0889	0.1133	0.2567	0.1614	0.1854	0.1349
		StdErr	0.0389	0.0208	0.0540	0.0372	0.0863	0.0531	0.0941	0.0482
	5	Catch Rate	0.4703	0.5294	0.5836	0.7636	0.4493	0.3740	0.2490	0.3551
		StdErr	0.3781	0.2710	0.1916	0.1817	0.1453	0.0907	0.0863	0.1012
	6	Catch Rate	0.6333	1.2449	0.0255	0.0833	0.4459	0.4026	0.4183	0.5455
		StdErr	0.1149	0.5891	0.0248	0.0463	0.2102	0.1306	0.1886	0.1683
Federal Waters	2	Catch Rate								
		StdErr								
	3	Catch Rate			0.2698	0.1667	1.1970	1.2000	0.5475	0.3913
		StdErr			0.1564	0.1524	0.3666	0.4387	0.1854	0.1920
	4	Catch Rate			0.7806	0.7368				
		StdErr			0.7553	0.4990				
	5	Catch Rate								
		StdErr								
	6	Catch Rate								
		StdErr								
Inland Waters	2	Catch Rate			0.0039	0.0286	0.0138	0.3077	0.0471	0.0513
		StdErr			0.0043	0.0284	0.0178	0.1521	0.0437	0.0357
	3	Catch Rate	0.1226	0.1873	0.7878	0.6209	0.4346	0.3093	0.6754	0.6650
		StdErr	0.0378	0.0805	0.3802	0.1492	0.2024	0.0803	0.1538	0.0958
	4	Catch Rate	0.2483	0.1494	0.0946	0.1058	0.0739	0.0975	0.2960	0.2257
		StdErr	0.1239	0.0421	0.0319	0.0264	0.0532	0.0510	0.1243	0.0763
	5	Catch Rate	0.2781	0.2890	0.1752	0.1878	0.2421	0.1929	0.1978	0.2120
		StdErr	0.0825	0.0634	0.0658	0.0462	0.0855	0.0389	0.0613	0.0584
	6	Catch Rate	8.4016	3.0000	0.4338	0.9792	0.9602	1.0673	0.5850	1.1496
		StdErr	3.7444	1.0152	0.3671	0.4936	0.2936	0.3833	0.3744	0.3810

Table 4. Preliminary wave-by-wave weighted and unweighted estimates of proportion of New York PR mode anglers living in coastal county households covered by the Coastal Household Telephone Survey (CHTS). The standard error (StdErr) of each estimated proportion is shown immediately below it.

WAVE	ESTIMATES	YEAR							
		2004		2005		2006		2007	
		Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted
2	Proportion	1	1	1	1	1	1	1	1
	StdErr	0	0	0	0	0	0	0	0
3	Proportion	0.9534	0.9553	0.9016	0.9437	0.9793	0.9526	0.9438	0.9220
	StdErr	0.0182	0.0098	0.0446	0.0133	0.0072	0.0109	0.0152	0.0100
4	Proportion	0.9214	0.9214	0.9615	0.9219	0.9341	0.9281	0.9051	0.9096
	StdErr	0.0150	0.0106	0.0133	0.0127	0.0173	0.0117	0.0201	0.0122
5	Proportion	0.9104	0.9222	0.9009	0.9337	0.9212	0.9336	0.9294	0.9374
	StdErr	0.0239	0.0130	0.0479	0.0137	0.0291	0.0104	0.0184	0.0094
6	Proportion	0.9688	0.9706	0.9051	0.9091	0.9459	0.9392	0.9482	0.9559
	StdErr	0.0265	0.0145	0.0360	0.0308	0.0520	0.0178	0.0285	0.0144

Table 5. Preliminary weighted and unweighted estimates of proportion of New York PR mode anglers fishing at state, federal and inland waters.

WAVE	AREA	ESTIMATES	YEAR							
			2004		2005		2006		2007	
			Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted
2	State Waters	Proportion	0.1994	0.2000	0.0475	0.2885	0	0	0	0
		StdErr	0.1528	0.0603	0.0445	0.0634	0	0	0	0
	Federal Waters	Proportion	0	0	0.0114	0.0385	0	0	0.0832	0.1522
		StdErr	0	0	0.0122	0.0269	0	0	0.0501	0.0535
	Inland Waters	Proportion	0.8006	0.8000	0.9411	0.6731	1	1.0000	0.9168	0.8478
		StdErr	0.1528	0.0603	0.0566	0.0657	0	0	0.0501	0.0535
3	State Waters	Proportion	0.4376	0.3836	0.2840	0.2815	0.3251	0.3773	0.3485	0.4100
		StdErr	0.0704	0.0229	0.0784	0.0259	0.0657	0.0247	0.0475	0.0183
	Federal Waters	Proportion	0.0206	0.0244	0.0399	0.0199	0.0430	0.0129	0.0249	0.0319
		StdErr	0.0099	0.0073	0.0240	0.0080	0.0268	0.0057	0.0116	0.0065
	Inland Waters	Proportion	0.5418	0.5920	0.6761	0.6987	0.6319	0.6098	0.6266	0.5582
		StdErr	0.0710	0.0232	0.0774	0.0264	0.0624	0.0248	0.0482	0.0185
4	State Waters	Proportion	0.3971	0.4345	0.3605	0.3247	0.4359	0.4570	0.3744	0.4557
		StdErr	0.0546	0.0195	0.0640	0.0218	0.0599	0.0226	0.0534	0.0212
	Federal Waters	Proportion	0.0409	0.0293	0.0653	0.0411	0.0638	0.0594	0.0207	0.0235
		StdErr	0.0151	0.0066	0.0389	0.0092	0.0222	0.0107	0.0119	0.0064
	Inland Waters	Proportion	0.5621	0.5362	0.5741	0.6342	0.5003	0.4836	0.6049	0.5208
		StdErr	0.0553	0.0196	0.0670	0.0224	0.0637	0.0226	0.0541	0.0213
5	State Waters	Proportion	0.2800	0.2800	0.5133	0.4701	0.3853	0.4425	0.3296	0.3640
		StdErr	0.0509	0.0218	0.0847	0.0267	0.0574	0.0207	0.0503	0.0186
	Federal Waters	Proportion	0.0194	0.0118	0.0255	0.0142	0.0169	0.0157	0.0373	0.0193
		StdErr	0.0098	0.0052	0.0240	0.0063	0.0147	0.0052	0.0200	0.0053
	Inland Waters	Proportion	0.7006	0.7082	0.4611	0.5157	0.5978	0.5418	0.6330	0.6166
		StdErr	0.0548	0.0221	0.0828	0.0267	0.0611	0.0208	0.0521	0.0188
6	State Waters	Proportion	0.3246	0.3603	0.2071	0.4091	0.2125	0.4254	0.2753	0.3775
		StdErr	0.1580	0.0413	0.0937	0.0527	0.1282	0.0369	0.0745	0.0340
	Federal Waters	Proportion	0	0	0.1971	0.0455	0	0	0	0
		StdErr	0	0	0.1189	0.0223	0	0	0	0
	Inland Waters	Proportion	0.6754	0.6397	0.5957	0.5455	0.7875	0.5746	0.7247	0.6225
		StdErr	0.1580	0.0413	0.1176	0.0534	0.1282	0.0369	0.0745	0.0340

Figures

Figure 1. Sub-regions and states in the U.S. Atlantic coast

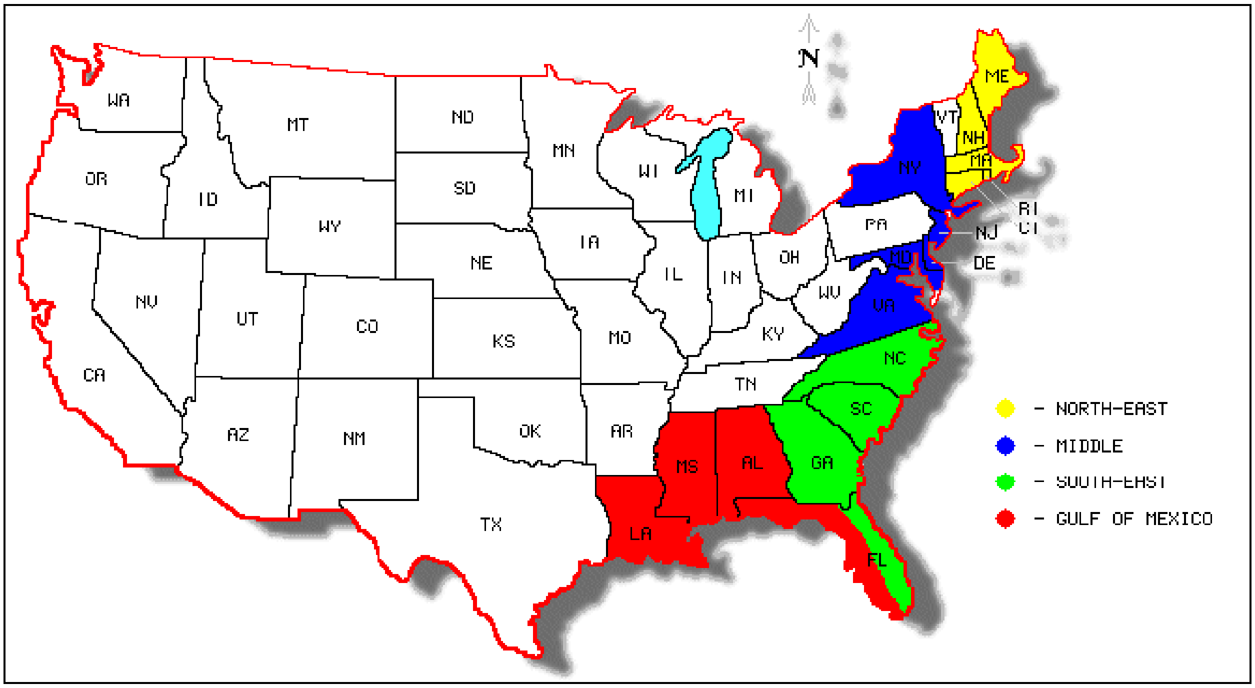


Figure 2. Counties of Florida in the Gulf of Mexico (Sub-region 7) and Southeast Atlantic (Sub-region 6).

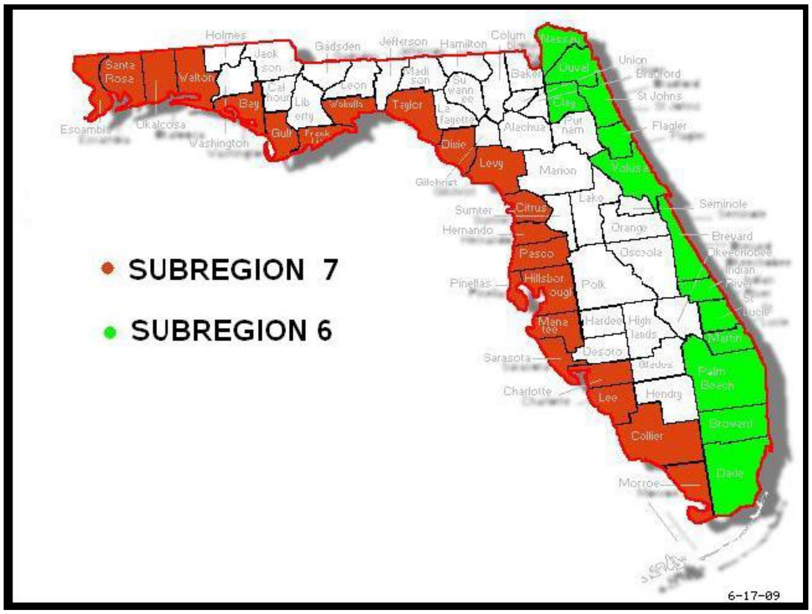
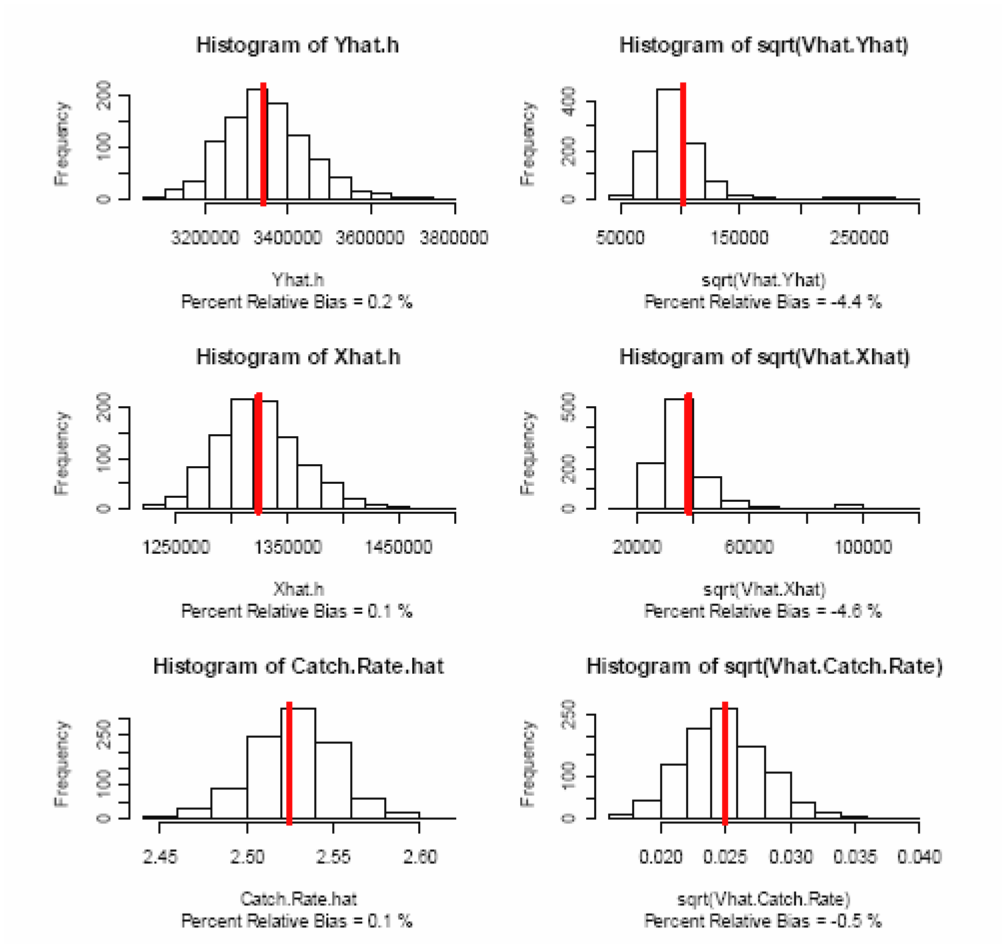


Figure 3. Results from the simulation study in Appendix II. Vertical red line indicates the true value in each experiment.



Appendices

Appendix I. Derivation of point estimate and variance of total catch for PR mode

The sampling design for PR mode is a stratified three-stage sampling. In general, each wave is stratified into $h = 1,\ldots,H$ month-KOD strata. Within a given stratum *h*, the samplings of the three stages are described below.

Stage I. Site-days ($i = 1,\ldots,n_h$) are sampled within stratum via unequal probability without replacement. The inclusion probability of site-day *i* is π_{hi}, which is proportional to *expected* number of angler-trips of the site-day.

Stage II. Sample boat-trips ($j = 1,\ldots,b_{hi}$) within each of sampled site-days via SI (simple random sampling without replacement); that is, sample b_{hi} boat-trips from a total of B_{hi} boat-trips within the *hi*-th site-day.

Stage III. Sample angler-groups ($k = 1,\ldots,m_{hij}$) within each of sampled boat-trips via simple random sampling; that is, sample m_{hij} groups from a total of M_{hij} groups at random within the *hij*-th boat-trip.

1. *Point estimate*

Let y_{hijk} = observed number of fish caught in the k-th group,

x_{hijk} = observed number of anglers in the k-th group, and

X_{hij} = observed number of angler-trips aboard the *j*-th boat-trip

The estimate of total catch within a boat-trip is

$$\hat{Y}_{hij} = X_{hij}\left(\frac{\sum_{k}^{m_{hij}} M_{hij} y_{hijk} \Big/ m_{hij}}{\sum_{k}^{m_{hij}} M_{hij} x_{hijk} \Big/ m_{hij}}\right) = X_{hij}\left(\sum_{k}^{m_{hij}} y_{hijk} \Big/ \sum_{k}^{m_{hij}} x_{hijk}\right) \qquad \text{(I.1)}$$

where M_{hij} is the total number of grouped anglers on the boat-trip *j* and m_{hij} is the sampled and interviewed groups. Although M_{hij} is not known, it does not affect the estimation. In turn, the estimate of total catch within the site-day *i* is

$$\hat{Y}_{hi} = \tilde{X}_{hi}\left(\frac{\sum_{j}^{b_{hi}} B_{hi} \hat{Y}_{hij} \Big/ b_{hi}}{\sum_{j}^{b_{hi}} B_{hi} X_{hij} \Big/ b_{hi}}\right) = \tilde{X}_{hi}\left(\sum_{j}^{b_{hi}} \hat{Y}_{hij} \Big/ \sum_{j}^{b_{hi}} X_{hij}\right) \qquad \text{(I.2)}$$

where B_{hij} and b_{hij} are the numbers of all and sampled boat-trips in the site-day *i*. Knowledge of B_{hij} and b_{hij} do not affect the estimation. The estimate of total catch within the stratum *h* is

$$\hat{Y}_{h} = \sum_{i}^{n_h} \frac{\hat{Y}_{hi}}{\pi_{hi}}\,. \qquad \text{(I.3)}$$

Replacing the estimated totals, $\hat{Y}_{hi}$ and $\hat{Y}_{hij}$ by the preceding equations yield the Equation (1). The estimate of total catch in the target population is obtained by the other ratio estimator:

$$\hat{Y} = \sum_{i}^{n_h} \frac{\hat{Y}_{hi}}{\pi_{hi}} \Big/ \sum_{i}^{n_h} \frac{\tilde{X}_{hi}}{\pi_{hi}} \qquad \text{(I.4)}$$

2. *Variance of* $\hat{Y}_h$

Reading M_{hij}/m_{hij} as the inclusion probability of the selected group of anglers, the estimate of total catch within the boat-trip *j* can be re-written in terms of π-estimators:

$$\hat{Y}_{hij} = X_{hij} \frac{\sum_{k}^{m_{hij}} y_{hijk} M_{hij}/m_{hij}}{\sum_{k}^{m_{hij}} x_{hijk} M_{hij}/m_{hij}} = X_{hij} \frac{\hat{Y}_{hij}}{\hat{X}_{hij}}$$

Note that $\hat{Y}_{hij}$ and $\hat{X}_{hij}$ are the unbiased estimators of total catch and effort within the *hij*-th boat-trip; that is, $E\left[\hat{Y}_{hij} \mid \text{Stage I}, \text{Stage II}\right] = Y_{hij}$ and $E\left[\hat{X}_{hij} \mid \text{Stage I}, \text{Stage II}\right] = X_{hij}$. However, the catch rate, $\hat{Y}_{hij} / \hat{X}_{hij}$ is not unbiased; that is, $E\left[\hat{Y}_{hij} / \hat{X}_{hij} \mid \text{Stage I}, \text{Stage II}\right] \neq Y_{hij} / X_{hij}$. Let

$$\frac{\hat{Y}_{hij}}{\hat{X}_{hij}} = \frac{Y_{hij}}{X_{hij}} + \varepsilon_{hij} \cong \frac{Y_{hij}}{X_{hij}} + \frac{1}{X_{hij}}\left(\hat{Y}_{hij} - \frac{Y_{hij}}{X_{hij}} \hat{X}_{hij}\right).$$

The above equation introduces nonlinearity into the variance estimation because estimators of total catch ($\hat{Y}_{hi}$) and total effort ($\hat{X}_{hi}$) within the *hi*-th site-day are expressed by

$$\hat{Y}_{hi} = \sum_{j}^{b_h} \frac{B_{hi} X_{hij}}{b_{hi}} \frac{\hat{Y}_{hij}}{\hat{X}_{hij}}$$

$$\cong \sum_{j}^{b_h} \frac{B_{hi} X_{hij}}{b_{hi}} \left[\frac{Y_{hij}}{X_{hij}} + \frac{1}{X_{hij}} \left(\hat{Y}_{hij} - \frac{Y_{hij}}{X_{hij}} \hat{X}_{hij} \right) \right]$$

$$= \sum_{j}^{b_h} \frac{Y_{hij} + \hat{Y}_{hij} - (Y_{hij} / X_{hij}) \hat{X}_{hij}}{b_{hi} / B_{hi}}$$

and

$$\hat{X}_{hi} \cong \sum_{j}^{b_h} \frac{B_{hi} X_{hij}}{b_{hi}}$$

Note that $\hat{Y}_{hi}$ and $\hat{X}_{hi}$ are unbiased (i.e., $E\left[\hat{Y}_{hi} \mid \text{Stage I}\right] = Y_{hi}$ and $E\left[\hat{X}_{hi} \mid \text{Stage I}\right] = X_{hi}$); however, the catch rate as the ratio of $\hat{Y}_{hi}$ and $\hat{X}_{hi}$ is biased (i.e., $E\left[\hat{Y}_{hi} / \hat{X}_{hi} \mid \text{Stage I}, \text{Stage II}\right] \neq Y_{hi} / X_{hi}$), and thus, let

$$\frac{\hat{Y}_{hi}}{\hat{X}_{hi}} = \frac{Y_{hi}}{X_{hi}} + \varepsilon_{hi} \cong \frac{Y_{hi}}{X_{hi}} + \frac{1}{X_{hi}} \left(\hat{Y}_{hi} - \frac{Y_{hi}}{X_{hi}} \hat{X}_{hi} \right).$$

Following the arguments and equations in the above, the approximate estimator of total catch within the h-th stratum ($\hat{Y}_h$) can be written by

$$\hat{Y}_h = \sum_i^{n_h} \frac{\tilde{X}_{hi}}{\pi_{hi}} \left\{ \frac{\sum_j^{b_{hi}} X_{hij} \frac{\hat{Y}_{hij}}{\hat{X}_{hij}}}{\sum_j^{b_{hi}} X_{hij}} \right\}$$

$$\cong \sum_i^{n_h} \frac{\tilde{X}_{hi}}{\pi_{hi}} \left\{ \frac{\sum_j^{b_{hi}} \frac{B_{hi} X_{hij}}{b_{hi}} \left[\frac{Y_{hij}}{X_{hij}} + \frac{1}{X_{hij}} \left(\hat{Y}_{hij} - \frac{Y_{hij}}{X_{hij}} \hat{X}_{hij} \right) \right]}{\sum_j^{b_{hi}} \frac{B_{hi} X_{hij}}{b_{hi}}} \right\}$$

$$= \sum_i^{n_h} \frac{\tilde{X}_{hi}}{\pi_{hi} \hat{X}_{hi}} \left\{ \sum_j^{b_{hi}} \frac{B_{hi} X_{hij}}{b_{hi}} \left[\frac{Y_{hij}}{X_{hij}} + \frac{1}{X_{hij}} \left(\hat{Y}_{hij} - \frac{Y_{hij}}{X_{hij}} \hat{X}_{hij} \right) \right] \right\}$$

$$= \sum_i^{n_h} \frac{\tilde{X}_{hi}}{\pi_{hi}} \frac{\hat{Y}_{hi}}{\hat{X}_{hi}}$$

$$\cong \sum_i^{n_h} \frac{\tilde{X}_{hi}}{\pi_{hi}} \left\{ \frac{Y_{hi}}{X_{hi}} + \frac{1}{X_{hi}} \left(\hat{Y}_{hi} - \frac{Y_{hi}}{X_{hi}} \hat{X}_{hi} \right) \right\}$$

The approximate variance of $\hat{Y}_h$ is

$$Var(\hat{Y}_h)$$
$$\cong Var\left(\sum_i^{n_h}\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{Y_{hi}}{X_{hi}}\right)+2Cov\left(\sum_i^{n_h}\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{Y_{hi}}{X_{hi}},\ \sum_i^{n_h}\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{1}{X_{hi}}\left(\hat{Y}_{hi}-\frac{Y_{hi}}{X_{hi}}\hat{X}_{hi}\right)\right)$$
$$+Var\left(\sum_i^{n_h}\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{1}{X_{hi}}\left(\hat{Y}_{hi}-\frac{Y_{hi}}{X_{hi}}\hat{X}_{hi}\right)\right)$$

$$=V_1+2V_2+V_3$$

Define $\Delta_{hii'}=\pi_{hii'}-\pi_{hi}\pi_{hi'}$ and write

$$V_1=\sum_i\sum_{i'}\Delta_{hii'}\left(\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{Y_{hi}}{X_{hi}}\right)\left(\frac{\tilde{X}_{hi'}}{\pi_{hi'}}\frac{Y_{hi'}}{X_{hi'}}\right);$$

$$V_2=Cov\left(\sum_i\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{Y_{hi}}{X_{hi}},\ \sum_i\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{1}{X_{hi}}\left(\hat{Y}_{hi}-\frac{Y_{hi}}{X_{hi}}\hat{X}_{hi}\right)\right)$$

$$=Cov\left(E\left[\sum_i\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{Y_{hi}}{X_{hi}}\middle|\text{Stage I}\right],\ \underbrace{E\left[\sum_i\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{1}{X_{hi}}\left(\hat{Y}_{hi}-\frac{Y_{hi}}{X_{hi}}\hat{X}_{hi}\right)\middle|\text{Stage I}\right]}_{=0}\right)$$

$$\underbrace{+Cov\left(E\left[\sum_i\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{Y_{hi}}{X_{hi}}\middle|\text{Stage I}\right],\ E\left[\sum_{i'}\frac{\tilde{X}_{hi'}}{\pi_{hi'}}\frac{Y_{hi'}}{X_{hi'}}\middle|\text{Stage I}\right]\right)}_{=0\text{ because both terms are constant given Stage I}}$$

$$=0$$

Claim that the Equation (3),

$$\hat{V}(\hat{Y}_h)=\sum_i^{n_h}\sum_{i'}^{n_h}\frac{\Delta_{hii'}}{\pi_{hii'}}\frac{\tilde{X}_{hi}}{\pi_{hi}}\left(\frac{\sum_j^{b_h}X_{hij}\left(\sum_k^{m_{hij}}y_{hijk}\Big/\sum_k^{m_{hij}}x_{hijk}\right)}{\sum_j^{b_h}X_{hij}}\right)\frac{\tilde{X}_{hi'}}{\pi_{hi'}}\left(\frac{\sum_j^{b_h}X_{hi'j}\left(\sum_k^{m_{hij}}y_{hi'jk}\Big/\sum_k^{m_{hij}}x_{hi'jk}\right)}{\sum_j^{b_h}X_{hi'j}}\right),$$

is an approximately unbiased estimator of $Var(\hat{Y}_h)=V_1+0+V_3$.

Proof. By the earlier arguments, re-write $\hat{V}$ as

$$\hat{V}=\sum_i^{n_h}\sum_{i'}^{n_h}\frac{\Delta_{hii'}}{\pi_{hii'}}\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{\hat{Y}_{hi}}{\hat{X}_{hi}}\frac{\tilde{X}_{hi'}}{\pi_{hi'}}\frac{\hat{Y}_{hi'}}{\hat{X}_{hi'}}$$

$$\cong\sum_i^{n_h}\sum_{i'}^{n_h}\frac{\Delta_{hii'}}{\pi_{hii'}}\frac{\tilde{X}_{hi}}{\pi_{hi}}\left[\frac{Y_{hi}}{X_{hi}}+\frac{1}{X_{hi}}\left(\hat{Y}_{hi}-\frac{Y_{hi}}{X_{hi}}\hat{X}_{hi}\right)\right]\frac{\tilde{X}_{hi'}}{\pi_{hi'}}\left[\frac{Y_{hi'}}{X_{hi'}}+\frac{1}{X_{hi'}}\left(\hat{Y}_{hi'}-\frac{Y_{hi'}}{X_{hi'}}\hat{X}_{hi'}\right)\right],$$

which leads to the following equation:

$$\hat{V}(\hat{Y}_h)=\sum_i^{n_h}\sum_{i'}^{n_h}\frac{\Delta_{hii'}}{\pi_{hii'}}\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{Y_{hi}}{X_{hi}}\frac{\tilde{X}_{hi'}}{\pi_{hi'}}\frac{Y_{hi'}}{X_{hi'}}\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(i)$$

$$+2\sum_i^{n_h}\sum_{i'}^{n_h}\frac{\Delta_{hii'}}{\pi_{hii'}}\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{Y_{hi}}{X_{hi}}\frac{\tilde{X}_{hi'}}{\pi_{hi'}}\frac{1}{X_{hi'}}\left(\hat{Y}_{hi'}-\frac{Y_{hi'}}{X_{hi'}}\hat{X}_{hi'}\right)\quad\cdots\cdots\cdots\cdots\cdots\cdots(ii)$$

$$+\sum_i^{n_h}\sum_{i'}^{n_h}\frac{\Delta_{hii'}}{\pi_{hii'}}\frac{\tilde{X}_{hi}}{\pi_{hi}}\frac{1}{X_{hi}}\left(\hat{Y}_{hi}-\frac{Y_{hi}}{X_{hi}}\hat{X}_{hi}\right)\frac{\tilde{X}_{hi'}}{\pi_{hi'}}\frac{1}{X_{hi'}}\left(\hat{Y}_{hi'}-\frac{Y_{hi'}}{X_{hi'}}\hat{X}_{hi'}\right)\quad\cdots\cdots\cdots(iii)$$

$$=\hat{V}_1+0+\hat{V}_3$$

Note that term (*i*) in the equation is unbiased for V_1, (*ii*) is unbiased for $0 = 2V_2$, and (*iii*) is approximately unbiased for $V_3 = Var\left(\sum_h^H \sum_i^{n_h} \frac{\tilde{X}_{hi}}{\pi_{hi}} \frac{1}{X_{hi}}\left(\hat{Y}_{hi} - \frac{Y_{hi}}{X_{hi}} \hat{X}_{hi}\right)\right)$ using the standard "ultimate cluster" arguments. #

3. ***SAS proc surveymeans***

The Equations (1) and (2) are used in SAS proc surveymeans. This SAS procedure approximates $\hat{V}$ by using the “with-replacement” approximation within strata. Define:

STRATUM = h (i.e., month-KOD)

PSU = i (i.e., site-day)

DOMAIN = area_x (1 = state waters. 2 = federal waters, 5 = Inland)

WEIGHT = $1/\pi_{hi}$

CATCH = $\tilde{X}_{hi}\left[\frac{\sum_{j}^{b_{hi}} X_{hij}\left(\sum_{k}^{m_{hij}} y_{hijk} \Big/ \sum_{k}^{m_{hij}} x_{hijk}\right)}{\sum_{j}^{b_{hi}} X_{hij}}\right]$

Note that set x_{hijk} =1 for B1- and B2-type catches.

TRIP = $\tilde{X}_{hi}$.

The SAS script of proc surveymeans corresponding to the estimation is:

```
proc surveymeans data=FISH sum varsum;
   by year wave sub_reg st mode xsp_code;
   strata stratum;
   cluster psu;
   domain area_x;
   weight weight;
   var catch trip;
   ratio 'catch Rate' catch / trip;
```

```
    ods output ratio=cpue_mrip;
  run;
```

The SH mode is a stratified 2-stage sampling without boat-trip cluster. The unbiased estimator of total catch within the *h*-th stratum is

$$\hat{Y}_h = \sum_{i}^{n_h} \frac{\tilde{X}_{hi}}{\pi_{hi}} \left\{ \sum_{k}^{m_{hij}} y_{hik} \Big/ \sum_{k}^{m_{hij}} x_{hik} \right\}.$$

and its estimated variance is

$$\hat{V}(\hat{Y}_h) = \sum_{i}^{n_h} \sum_{i'}^{n_h} \frac{\Delta_{hii'}}{\pi_{hii'}} \frac{\tilde{X}_{hi}}{\pi_{hi}} \left(\sum_{k}^{m_{hij}} y_{hik} \Big/ \sum_{k}^{m_{hij}} x_{hik} \right) \frac{\tilde{X}_{hi'}}{\pi_{hi'}} \left(\sum_{k}^{m_{hij}} y_{hi'k} \Big/ \sum_{k}^{m_{hij}} x_{hi'k} \right),$$

For the HB mode, the PSU is a HB boat-trip. The cluster size of the sampled for boat-trip *j* in stratum *h* is observed as X_{hj} (PARTY). The selection probability (π_{hj}) is calculated directly based on the sampling frame of HB mode. Unbiased estimator of total catch within the *h*-th statum is approximated by

$$\hat{Y}_h \cong \sum_{i}^{b_h} \frac{X_{hj}}{\pi_{hj}} \left\{ \sum_{k}^{m_{hij}} y_{hjk} \Big/ \sum_{k}^{m_{hij}} x_{hjk} \right\}.$$

and its estimated variance is

$$\hat{V}(\hat{Y}_h) = \sum_{j}^{b_h} \sum_{j'}^{b_h} \left(\frac{\Delta_{hjj'}}{\pi_{hjj'}} \right) \left(\frac{\tilde{X}_{hj}}{\pi_{hj}} \frac{\sum_{k}^{m_{hij}} y_{hjk}}{\sum_{k}^{m_{hij}} x_{hjk}} \right) \left(\frac{\tilde{X}_{hj'}}{\pi_{hj'}} \frac{\sum_{k}^{m_{hij}} y_{hj'k}}{\sum_{k}^{m_{hij}} x_{hj'k}} \right)$$

Appendix II. Simulation

Assume that a population that consists of $N_h = 1000$ site-days. For each of the i = 1,…, 1000-th site-day, total number of boat-trips is B_{hi} = 20. The site pressure of each site-days is simulated by binomial distribution, $P_{hi} = BN(N_h, x, p) = C(N_h, x) p^x (1-p)^{N_h - x}$, given x = 20 trials and p = 0.5 and is standardized by $P_{hi} = P_{hi} / \max(P_{hi}, i = 1, \cdots, N_h)$.

For each of the j = 1,…,20-th boat-trip within the hi-th site-day, assume that mean number of angler-trips is $\hat{\bar{x}}_{hij} = 5 + 5e^{P_{hi}} = \sum_k^{m_{hij}} x_{hijk} \Big/ m_{hij}$. The total number of angler-groups within the hij-th boat-trip is simulated by Poisson distribution, $M_{hij} = Poi(B_{hi} = 20, \lambda = \hat{\bar{x}}_{hij}) + 2$, which assures that there are minimum of two angler-groups in any boat-trip.

Within the $hijk$-th angler-group of the population, the number of angler-trips (minimum of 1) is generated from $x_{hijk} = Poi(1, \lambda = 3) + 1$. The number of angler-trips (i.e., PARTY) within the hij-th boat-trip is $X_{hij} = \sum_k^{M_{hijk}} x_{hijk}$. The number of fish caught by a angler-group is calculated by $y_{hijk} = \theta_{hij} x_{hijk} + e_{hijk}$, where $\theta_{hij} = unif(B_{hi} N_h) + 2 = \bar{y}_{hij}$ is the expected catch rate of the hij-th boat-trip and random error $e_{hijk} \sim Poi(1, \lambda = 0.1)$.

At the end of this simulation, the true population total catch (Y_h), total effort (X_h, in angler-trips), and catch rate ($\bar{Y}_h$) are obtained with their variance.

A total of 1000 replicates are generated from the population after population data are simulated. Within each replicate, $n_h = 30$ site-days are sampled with inclusion probability $\pi_{hi} = n_h P_{hi} / \sum_i^{N_h} P_{hi}$ without replacement. For each sampled site-day, $b_{hi} = 5$ boat-trips are sampled with equal probability without replacement. For each sampled boat-trip, m_{hij} = 2 angler-groups are sampled with equal probability without replacement.

The frequency distribution of estimates ($\hat{Y}_h$, $\hat{X}_h$ and $\hat{\bar{Y}}_h$) with their standard errors are shown in Figure 3. The percent Relative Bias of total catch, for example, is calculated by

$$\text{Percent Relative Bias for } \hat{Y}_h = 100\% \left\{ \left(\frac{\sum_l^{1000} \hat{Y}_{hl}}{1000 \text{ replicates}} - Y_h \right) \Big/ Y_h \right\}$$

Percent Relative Biases for $\hat{X}_h$ and $\hat{\bar{Y}}_h$ and their standard errors are calculated similarly.

Appendix III: Hierarchical Bayesian Small Area Estimation for Circular Data

1 Projected Normal Distribution

Given the circular nature of the departure times (which can be viewed as angles on $(0, 2\pi]$), we will model them as random variables having a *projected bivariate normal distribution* $PN_2(\boldsymbol{\mu}, \mathbf{I}_2)$ (Presnell et al. 1998) and build a mixed effect model based on the following factors and interactions between them: state (s), wave (w) and mode (m). The dataset contains observations for 17 states, 6 waves, and 4 modes. Some levels for the state factor were removed because of lack of data within those state levels. Because a responding household could report on multiple trips, we also investigated the addition of a household factor (h). This factor has $215,003$ levels, which will require a separate approach to incorporate into the model. Hence, we will begin by describing the model without household effect.

After normalization to the unit circle, the distribution of the departure time T_{ijkt} of respondent t in given state i, wave j and mode k is denoted as

$$T_{ijkt} \overset{ind}{\sim} PN_2\left(\boldsymbol{\mu}_{ijk}, \boldsymbol{I}_2\right),$$

where $\boldsymbol{\mu}_{ijk} = \boldsymbol{\mu} + \boldsymbol{m}_k + \boldsymbol{s}_i + \boldsymbol{w}_j$, each term being a two dimensional vector, and $\boldsymbol{I}_2$ is the 2×2 identity matrix. For now, the specification for $\boldsymbol{\mu}_{ijk}$ corresponds to a model without any interaction between the factors.

In general, the angle Θ of a 2-dimensional unit random vector $\boldsymbol{U} = \boldsymbol{X}/\|\boldsymbol{X}\|$ has a projected bivariate normal distribution $PN_2(\boldsymbol{\mu}, \boldsymbol{I}_2)$ if the random variable $\boldsymbol{X}$ has a bivariate normal distribution $N_2(\boldsymbol{\mu}, \boldsymbol{I}_2)$. The density of Θ can be written explicitly as Mardia and Jupp (2000, p.46):

$$f(\theta|\boldsymbol{\mu}) = \frac{1}{2\pi} \exp\left\{-\frac{1}{2} \parallel \boldsymbol{\mu} \parallel^2\right\} \left[1 + \frac{\mathbf{u}^T\boldsymbol{\mu}\Phi\left(\mathbf{u}^T\boldsymbol{\mu}\right)}{\phi\left(\mathbf{u}^T\boldsymbol{\mu}\right)}\right] I_{(0,2\pi]}(\theta) \qquad (1)$$

with $\mathbf{u} = (\cos\theta, \sin\theta)$, and where $\Phi(\cdot)$ and $\phi(\cdot)$ are the standard normal distribution and density functions, respectively. The distribution $PN_2(\boldsymbol{\mu}, \boldsymbol{I}_2)$ is unimodal and rotationally symmetric about the mean direction vector $\boldsymbol{\eta}$, where $\boldsymbol{\eta} = \boldsymbol{\mu}/\|\boldsymbol{\mu}\| = (\cos\omega, \sin\omega)$ and ω is called the mean direction.

In order to develop a model for the departure times, the projected normal distribution will be embedded in a hierarchical Bayesian framework that includes prior distributions for the factors. This will allow us to perform model selection, including the determination of whether fixed or random effects specifications are more appropriate for the different factors, and whether interactions between factors are need to be incorporated. The goal of the estimation is to obtain the posterior distribution of $\boldsymbol{\mu}_{ijk}$, which is the only parameter in the projected normal density (1). Once this distribution is obtained, the posterior distribution of any functional of $\boldsymbol{\mu}_{ijk}$, including the expected fraction of departures in a given time interval, can likewise be obtained. We will return to this topic below.

2 Estimation

The approach is based on the introduction of suitable latent variables to define an augmented joint distribution with $\boldsymbol{\mu}_{ijk}$ (Nuñez-Antonio and Gutiérrez-Peña, 2005). Conjugate priors will be assumed in order to ensure that we can simulate from all full conditionals required for a Gibbs sampler. Suppose first that we could observe the values of $\boldsymbol{X}_{ijkt}$ for a sample of data and that the mean vector $\boldsymbol{\mu}_{ijk}$ is

$$\boldsymbol{\mu}_{ijk} = \boldsymbol{\mu} + \boldsymbol{m}_k + \boldsymbol{s}_i + \boldsymbol{w}_j, \tag{2}$$

where the mode effect is to be modeled as a "fixed" effect and the state and wave effects are "random" effects. We are interpreting these terms in the Bayesian context, implying that a fixed effect corresponds to having a predetermined vague prior and a random effect to having a prior with a variance parameter with its own prior distribution. The conjugate priors corresponding to this model specification are

$$\boldsymbol{\mu} \sim N_2\left(\boldsymbol{\mu_0}, \sigma_0^2 \mathbf{I}_2\right)$$

$$\mathbf{m}_k \sim N_2\left(\mathbf{0}, \sigma_m^2 \mathbf{I}_2\right)$$

$$\mathbf{s}_i \mid \sigma_s^2 \sim N_2\left(\mathbf{0}, \sigma_s^2 \mathbf{I}_2\right)$$

$$\mathbf{w}_j \mid \sigma_w^2 \sim N_2\left(\mathbf{0}, \sigma_w^2 \mathbf{I}_2\right)$$

$$\sigma_s^2 \sim IG\left(\alpha_s, \beta_s\right) \propto (\sigma_s^2)^{-\alpha_s - 1} \exp\left\{-\frac{\beta_s}{\sigma_s^2}\right\}$$

$$\sigma_w^2 \sim IG\left(\alpha_w, \beta_w\right) \propto \left(\sigma_w^2\right)^{-\alpha_w - 1} \exp\left\{-\frac{\beta_w}{\sigma_w^2}\right\}.$$

It is straightforward to obtain the full conditional distributions for this model specification, after which Gibbs sampling can be used to obtain the posterior distributions of the model parameters.

In the application we are considering here, the $\boldsymbol{X}_{ijkt}$ are not observed. Let $\Theta_{ijkt} = T_{ijkt}\frac{2\pi}{24}$ represent the departure times normalized to the unit circle, and let

$$\boldsymbol{X}_{ijkt} = R_{ijkt}\boldsymbol{U}_{ijkt} = R_{ijkt}\left(\cos\Theta_{ijkt}, \sin\Theta_{ijkt}\right)^T.$$

For a given value of the random variable Θ_{ijkt}, it is possible to compute the corresponding value of $\boldsymbol{U}_{ijkt}$. However, the value of $\boldsymbol{X}_{ijkt}$ is unknown because of the unobservable component R_{ijkt}, which corresponds to the length of the vector $\boldsymbol{X}_{ijkt}$. Assuming

$$\boldsymbol{X}_{ijkt} \mid \boldsymbol{\mu}_{ijk} \overset{ind}{\sim} N_2\left(\boldsymbol{\mu}_{ijk}, \boldsymbol{I}_2\right), \tag{3}$$

it is clear that $\Theta_{ijkt} \mid \boldsymbol{\mu}_{ijk} \overset{ind}{\sim} PN_2\left(\boldsymbol{\mu}_{ijk}, \mathbf{I}_2\right)$. The structure of the model suggests that we should treat the unobserved $R_{ijkt} = \|\boldsymbol{X}_{ijkt}\|$, $l = 1, 2, \ldots, n_{ijk}$ as latent variables. This was the approach followed by Nuñez-Antonio and Gutiérrez-Peña (2005), who obtained the posterior distribution for an overall mean $\boldsymbol{\mu}$ via MCMC.

For a bivariate normal vector $\boldsymbol{X}$, consider the latent variable $R = \|\boldsymbol{X}\|$ defined on $(0, \infty)$. From equation (3), we can obtain its joint distribution with Θ by

$$f\left(\theta, r \mid \boldsymbol{\mu}_{ijk}\right) = (2\pi)^{-1} \exp\left\{-\frac{1}{2} \parallel \boldsymbol{\mu}_{ijk} \parallel^2\right\} \exp\left\{-\frac{1}{2}\left[r^2 - 2r\left(\mathbf{u}^T\boldsymbol{\mu}_{ijk}\right)\right]\right\} \mid \boldsymbol{J} \mid$$

where $\mid \boldsymbol{J} \mid$ is the jacobian of the transformation $\mathbf{x} \rightarrow (\theta, r)$. Starting from this distribution, it is in principle again possible to obtain the full conditional distributions required for the Gibbs sampler, based on the conjugate priors described above.

However, implementation of the Gibbs sampler for these data required that a number of issues be addressed. The most important problem was that, if the conjugate priors above were used, the sampling chain failed to converge even after 1000 iterations. Figure 1 shows examples of Gibbs sampler output applied to model (2) with a fixed mode effect and random state and wave effects, for the overall mean parameter and some specific mode, state and wave factors. Note that there are two traces in each plot, because each level of a factor is represented by two parameters.

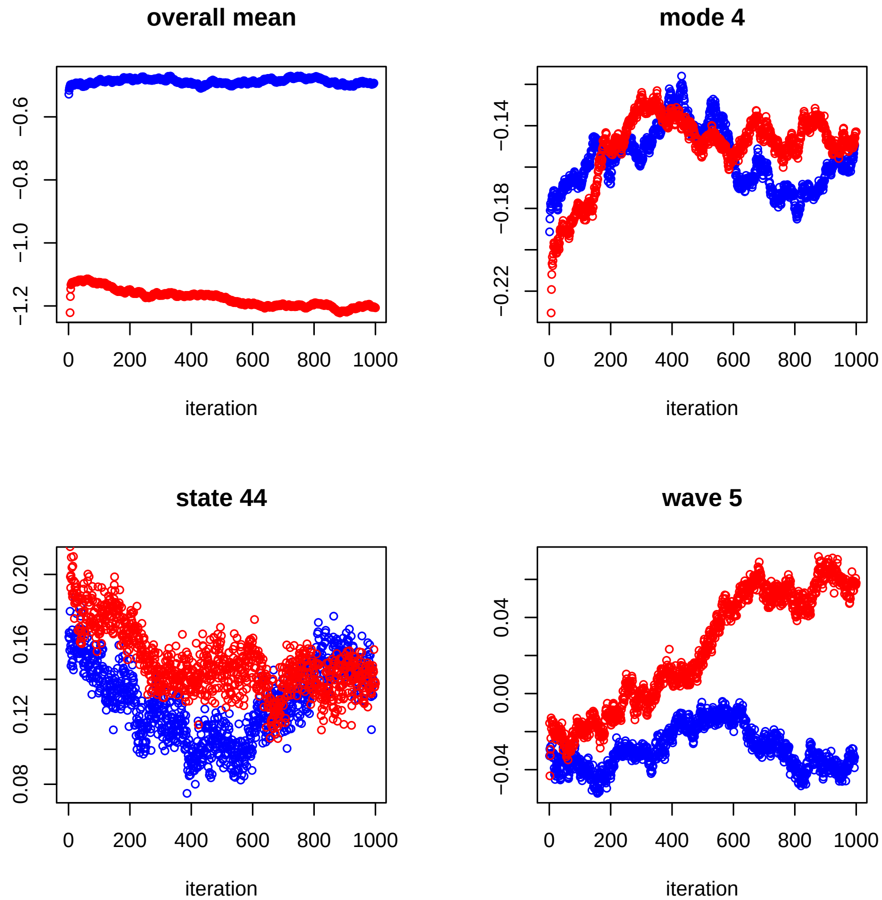


Figure 1: Output from the Gibbs sampler (before transformation) of the overall mean, the fixed effect for mode 4, and the random effects for state 44 and wave 5.

This slow mixing is most likely due to large positive or negative posterior correlations between model parameters. For an illustration of this problem, we consider the simple random-intercept model as in Gilks et al. (1998, p.94-96),

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

with $\alpha_i \sim N(0, \sigma_\alpha^2)$ and $\epsilon_{ij} \sim N(0, \sigma^2)$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. We assume σ_α^2 and σ^2 known and a flat prior on μ. Let $\boldsymbol{y}$ denote the observed data. Gelfand et al. (1995) show that posterior correlations for this model are

$$\begin{aligned} \text{Corr}\left\{(\alpha_i, \mu) \mid \boldsymbol{y}\right\} &= -\left\{1 + \frac{m\sigma^2}{n\sigma_\alpha^2}\right\}^{-1/2} \\ \text{Corr}\left\{(\alpha_i, \alpha_{i'}) \mid \boldsymbol{y}\right\} &= \left\{1 + \frac{m\sigma^2}{n\sigma_\alpha^2}\right\}^{-1}. \end{aligned}$$

Hence for this simple model, large posterior correlations and poor mixing occur when σ^2/n is small relative to σ_α^2/m, or, speaking somewhat loosely, when the number of levels of the random intercept is small relative to the number of observations. Given the number of levels for the random effects in our data, it seems likely that we are suffering from the same issue here.

Remedies to the slow mixing problem involve reparameterizations of the parameters. Vines et al. (1996) propose a parameterization called *sweeping,* since the mean of the factors are "swept" from the random and fixed effects effects and absorbed into μ. The sweeping parameterization applied to our model is:

$$\begin{aligned} \boldsymbol{\mu}' &= \boldsymbol{\mu} + \overline{\mathbf{m}} + \overline{\mathbf{s}} + \overline{\mathbf{w}} \\ \mathbf{m}'_k &= \mathbf{m}_k - \overline{\mathbf{m}} \\ \mathbf{s}'_i &= \mathbf{s}_i - \overline{\mathbf{s}} \\ \mathbf{w}'_j &= \mathbf{w}_j - \overline{\mathbf{w}}. \end{aligned}$$

Under this reparametrization, the hierarchical model for the departure times can be rewritten as

$$
\begin{aligned}
\Theta_{ijkt} &\sim PN_2\left(\boldsymbol{\mu}_{ijk}, \mathbf{I}_2\right) \\
\boldsymbol{\mu}_{ijk} &= \boldsymbol{\mu}' + \mathbf{m}'_k + \mathbf{s}'_i + \mathbf{w}'_j \\
\boldsymbol{\mu}' &\sim N_2\left(\boldsymbol{\mu}_{\mathbf{0}}, {\sigma'_0}^2 = \sigma_0^2 + \frac{\sigma_m^2}{K} + \frac{\sigma_s^2}{I} + \frac{\sigma_w^2}{J}\right) \\
\mathbf{m}'_{-K,c} &\sim N_{K-1}\left(\mathbf{0}, \sigma_m^2\left(\mathbf{I}_{K-1} - \frac{1}{K}\mathbf{J}_{K-1}\right)\right) \\
\mathbf{m}'_{K,c} &= -\sum_{k=1}^{K-1} \mathbf{m}'_{k,c} \\
\mathbf{s}'_{-I,c} \mid \sigma_s^2 &\sim N_{I-1}\left(\mathbf{0}, \sigma_s^2\left(\mathbf{I}_{I-1} - \frac{1}{I}\mathbf{J}_{I-1}\right)\right) \\
\mathbf{s}'_{I,c} &= -\sum_{i=1}^{I-1} \mathbf{s}'_{i,c} \\
\mathbf{w}'_{-J,c} \mid \sigma_w^2 &\sim N_{J-1}\left(\mathbf{0}, \sigma_w^2\left(\mathbf{I}_{J-1} - \frac{1}{J}\mathbf{J}_{J-1}\right)\right) \\
\mathbf{w}'_{J,c} &= -\sum_{j=1}^{J-1} \mathbf{w}'_{j,c} \\
\sigma_s^2 &\sim IG\left(\alpha_s, \beta_s\right) \\
\sigma_w^2 &\sim IG\left(\alpha_w, \beta_w\right)
\end{aligned}
$$

where $\mathbf{m}'_{-K,c} = \left(\mathbf{m}'_{1,c}, \ldots, \mathbf{m}'_{K-1,c}\right)^T$, $\mathbf{m}'_{k,c}$ is the c^{th} component of $\mathbf{m}'_k$ and $c = 1, 2$, with similar definitions for $\mathbf{s}'_{-I,c}$ and $\mathbf{w}'_{-J,c}$; N_p denotes a p-dimensional multivariate normal distribution; $\mathbf{I}_p$ and $\mathbf{J}_p$ are the $p \times p$ identity matrix and matrix of ones respectively. Note that the last components of each factor, $\boldsymbol{m}'_{K,c}$, $\boldsymbol{s}'_{I,c}$ and $\boldsymbol{w}'_{J,c}$, are fully determined once the other components are known.

To implement a Gibbs sampler for these new prior distributions, we need a new set of full conditional distributions. Let n represent the total sample size and

$$
\bar{\mathbf{z}} = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{t=1}^{n_{ijk}} \left(\mathbf{x}_{ijkt} - \mathbf{m}'_k - \mathbf{s}'_i - \mathbf{w}'_j\right)/n.
$$

For the mode factor $\boldsymbol{m}$, let $n_k^{(m)} = \sum_{i=1}^{I} \sum_{j=1}^{J} n_{ijk}$ denote the total number of observations for level k. Let $\mathbf{V}_1^{(m)} = \text{diag}\left(n_1^{(m)}, \ldots, n_{K-1}^{(m)}\right)$ and $\mathbf{V}_2^{(m)} = n_K^{(m)}\mathbf{J} + \frac{1}{\sigma_m^2}\left(\mathbf{I} - \frac{1}{K}\mathbf{J}\right)^{-1} +$

$\mathbf{V}_1^{(m)}$ and

$$\begin{aligned}
\mathbf{z}_{k,c}^{(m)} &= \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{n_{ijk}}\left(\mathbf{x}_{ijkt,c} - \boldsymbol{\mu}'_c - \mathbf{s}'_{i,c} - \mathbf{w}'_{j,c}\right) \quad k = 1, \ldots, K \\
\boldsymbol{\mu}_c^{(m)} &= \left(\left(\mathbf{z}_{1,c}^{(m)} - \mathbf{z}_{K,c}^{(m)}\right)/n_1^{(m)}, \ldots, \left(\mathbf{z}_{K-1,c}^{(m)} - \mathbf{z}_{K,c}^{(m)}\right)/n_{K-1}^{(m)}\right)^T.
\end{aligned}$$

We similarly define $n_i^{(s)}$, $\mathbf{V}_1^{(s)}, \mathbf{V}_2^{(s)}$, $\mathbf{z}_{i,c}^{(s)}$, $\boldsymbol{\mu}_c^{(s)}$ and $n_j^{(w)}$, $\mathbf{V}_1^{(w)}, \mathbf{V}_2^{(w)}$, $\mathbf{z}_{j,c}^{(w)}$, $\boldsymbol{\mu}_c^{(w)}$ for the remaining two factors. The conditionals for the mean function components can be written explicitly as follows:

$$p\left(\boldsymbol{\mu}' \mid \cdot\right) = N_2\left(\frac{\sigma_0'^2}{1 + n\sigma_0'^2}\left(\frac{\boldsymbol{\mu}_0}{\sigma_0'^2} + N\bar{\mathbf{z}}\right), \frac{\sigma_0'^2}{1 + N\sigma_0'^2}\mathbf{I}_2\right)$$

$$p\left(\mathbf{m}'_{-K,c} \mid \cdot\right) = N_{K-1}\left(\left(\mathbf{V}_2^{(m)}\right)^{-1}\mathbf{V}_1^{(m)}\boldsymbol{\mu}_c^{(m)}, \left(\mathbf{V}_2^{(m)}\right)^{-1}\right)$$

$$p\left(\mathbf{s}'_{-I,c} \mid \cdot\right) = N_{I-1}\left(\left(\mathbf{V}_2^{(s)}\right)^{-1}\mathbf{V}_1^{(s)}\boldsymbol{\mu}_c^{(s)}, \left(\mathbf{V}_2^{(s)}\right)^{-1}\right)$$

$$p\left(\mathbf{w}'_{-J,c} \mid \cdot\right) = N_{J-1}\left(\left(\mathbf{V}_2^{(w)}\right)^{-1}\mathbf{V}_1^{(w)}\boldsymbol{\mu}_c^{(w)}, \left(\mathbf{V}_2^{(w)}\right)^{-1}\right).$$

The conditional distributions for the random effect variances are specified by

$$\begin{aligned}
p\left(\sigma_s^2 \mid \cdot\right) &\propto IG\left(\alpha_s + I - 1, \sum_{i=1}^{I}\frac{1}{2}\mathbf{s}_i'^T\mathbf{s}_i' + \beta_s\right) * \\
&\left(\frac{1}{\sigma_0^2 + \frac{\sigma_m^2}{K} + \frac{\sigma_s^2}{I} + \frac{\sigma_w^2}{J}}\exp\left(-\frac{1}{\sigma_0^2 + \frac{\sigma_m^2}{K} + \frac{\sigma_s^2}{I} + \frac{\sigma_w^2}{J}}\left[\frac{1}{2}\left(\boldsymbol{\mu}' - \boldsymbol{\mu}_0\right)^T\left(\boldsymbol{\mu}' - \boldsymbol{\mu}_0\right)\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
p\left(\sigma_w^2 \mid \cdot\right) &\propto IG\left(\alpha_w + J - 1, \sum_{j=1}^{J}\frac{1}{2}\mathbf{w}_j'^T\mathbf{w}_j' + \beta_w\right) * \\
&\left(\frac{1}{\sigma_0^2 + \frac{\sigma_m^2}{K} + \frac{\sigma_s^2}{I} + \frac{\sigma_w^2}{J}}\exp\left(-\frac{1}{\sigma_0^2 + \frac{\sigma_m^2}{K} + \frac{\sigma_s^2}{I} + \frac{\sigma_w^2}{J}}\left[\frac{1}{2}\left(\boldsymbol{\mu}' - \boldsymbol{\mu}_0\right)^T\left(\boldsymbol{\mu}' - \boldsymbol{\mu}_0\right)\right]\right)\right).
\end{aligned}$$

Finally, the conditional distribution of the latent length of the bivariate normal vector is

$$p\left(r_{ijkt} \mid \cdot\right) \propto r\exp\left[-\frac{1}{2}r^2 + b_{ijkt}r\right]\mathrm{I}_{(0,\infty)}(r), \tag{4}$$

with $b_{ijkt} = \mathbf{u}_{ijkt}^T\boldsymbol{\mu}_{ijk}$ and $\mathbf{u}_{ijkt}^T = (\cos\theta_{ijkt}, \sin\theta_{ijkt})^T$.

The centered mean model components $\mathbf{m}'_{-K,c}$, $\mathbf{s}'_{-I,c}$ and $\mathbf{w}'_{-J,c}$ are correlated random vectors, so that they need to be generated as a block for each of the factors. Nevertheless, sampling from the corresponding multivariate normal distributions $p(\boldsymbol{\mu}' \mid \cdot)$, $p\left(\mathbf{m}'_{-K,c} \mid \cdot\right)$, $p\left(\mathbf{s}'_{-I,c} \mid \cdot\right)$, $p\left(\mathbf{w}'_{-J,c} \mid \cdot\right)$ is readily accomplished directly. Sampling from $p(\sigma_s^2 \mid \cdot)$ and $p(\sigma_w^2 \mid \cdot)$ is done via a Metropolis-Hastings algorithm with proposal distributions $IG\left(\alpha_s + I - 1, \sum_{i=1}^{I} \frac{1}{2}\mathbf{s}_i^T\mathbf{s}_i + \beta_s\right)$ and $IG\left(\alpha_w + J - 1, \sum_{j=1}^{J} \frac{1}{2}\mathbf{w}_j^T\mathbf{w}_j + \beta_w\right)$, respectively. Because the r_{ijkt} are latent, each iteration of the Gibbs sampler needs to draw n values from the conditional distribution (4). This density is concave and it can be shown that it belongs to an exponential family with canonical parameter b_{ijkt}, so that sampling from it is straightforward in principle, using algorithms such as Metropolis-Hastings or adaptive rejection sampling. However, the very large sample necessitates the use of an efficient algorithm to ensure that the Gibbs sampler can be run to convergence. In particular, we wanted to avoid having to explicitly iterate over the n random draws from the conditional distribution for each Gibbs sampler realization.

We therefore implemented a method that takes advantage of the specific form of (4). It can be shown that the constant of integration is $C(b) = \left(1 + \sqrt{2\pi} b \exp\left(\frac{b^2}{2}\right)\Phi(b)\right)^{-1}$, and that the cumulative distribution function is

$$F(r \mid \cdot) = C(b)\left[1 - \exp\left(-\frac{1}{2}r^2 + br\right) + \sqrt{2\pi} b \exp\left(\frac{b^2}{2}\right)\left[\Phi(r-b) - \Phi(-b)\right]\right].$$

In order to draw from $p(r_{ijkt} \mid \cdot)$, we transform uniform $(0,1)$ random variables by the inverse of its cumulative distribution function. We do not have an explicit form for this inverse, but it is approximated by applying the Newton-Raphson method using as initial value the mode of $p(r_{ijkt} \mid \cdot)$, which can be shown to be equal to $\frac{b_{ijkt} + \sqrt{b_{ijkt}^2 + 4}}{2}$. Sampling from $p(r_{ijkt} \mid \cdot)$ in this manner dramatically reduces the running time of the sampler.

The model specification we have used so far contains a fixed effect for mode and random effects for wave and state. We will investigate alternative model specifications including interactions and chose between them, as further described in the next section. We now return to the issue of adding a respondent household factor in the model, to account for the fact that a household can report on multiple trips. We consider here the following model

$$\boldsymbol{\mu}_{ijklt} = \boldsymbol{\mu} + \mathbf{m}_k + \mathbf{sw}_{ij} + \mathbf{h}_l \tag{5}$$

with $\boldsymbol{sw}_{ij}$ denoting a random interaction term for state and wave and $\boldsymbol{h}_l$ a bivariate

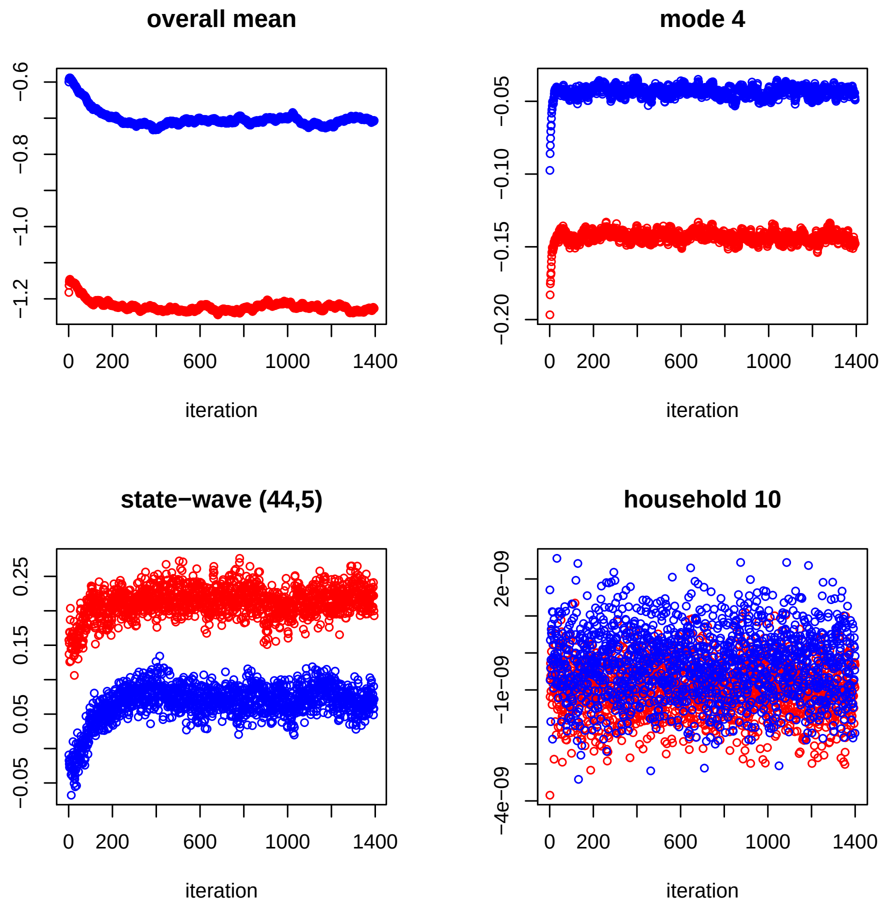


Figure 2: Output from the Gibbs sampler (after transformation) of the overall mean, the fixed effect representing mode 4, the interaction random effect representing state 44 and wave 5, and the random effect representing household 10.

random effect for household l. The sweeping adjustment discussed for the previous model specification was applied to the state-wave interaction term but not to the household term. The factor $\boldsymbol{h}$ has over 215,000 levels, so that it is simply not practical to apply the sweeping adjustment in this instance. However, based on the result by Gelfand et al. (1995) regarding the posterior correlations, we conjectured that the very large number of levels would result in negligible correlation between the levels of that factor. Figure 2 displays examples of the Gibbs sampler output applied to model (5). There do not appear to be undue convergence problems in this case.

3 Model Selection

The previous section described the set-up of a Gibbs sampler that, when run to convergence, provides an approximation to the posterior distribution of all the model parameters. In order to find a suitable model for the distribution of the departure times, we wanted to investigate and compare alternative model specifications, including considering models with some of these effects removed, different effects treated as fixed and random, and interaction between these models.

A common measure of fit in the Bayesian literature is the *deviance* (Gelman et al. 2004, p.179-184). For a general Bayesian estimation problem, the deviance is defined as $D(y,\omega) = -2\ln p(y \mid \omega)$ where y are the data, ω are the unknown parameters and $p(y \mid \omega)$ is the likelihood function. The deviance is proportional to the mean squared error if the model is normal with constant variance. The expected deviance $\mathrm{E}(D(y,\omega) \mid y)$ is a measure of how well the model fits and it can be estimated by the posterior mean deviance $\overline{D(y)} = \frac{1}{B}\sum_{b=1}^{B} D(y,\omega^b)$, where the $\omega_b, b = 1,\ldots,B$ are random draws from the posterior distribution (as obtained from an MCMC chain at convergence). Let $\bar{\omega} = \sum_{b=1}^{B} \omega_b/B$. The difference between the posterior mean deviance and the deviance at $\bar{\omega}$,

$$p_D = \overline{D(y)} - D(y,\bar{\omega}),$$

is often interpreted as a measure of the effective number of parameters of a Bayesian model. More generally, p_D can be thought of as the number of "unconstrained" parameters in the model, where a parameter counts as 1 if it is estimated without constraints or prior information, 0 if it is fully constrained or if all the information about the parameter comes from the prior distribution, or an intermediate value if both the data and prior distributions are informative. For hierarchical models, the effective number of parameters strongly depends on the variance of the group-level parameters (Gelman et al. 2004, p.182).

A common model selection criterion in the Bayesian estimation context is the *deviance information criterion* (DIC):

$$\begin{aligned} DIC &= 2\overline{D(y)} - D(y,\bar{\omega}) \\ &= \overline{D(y)} + p_D. \end{aligned}$$

The DIC can be interpreted as a measure of goodness-of-fit, i.e. the estimated expected deviance, plus a "penalty" for model complexity in the form of the total number of effective parameters. When performing model selection, models with lower values of DIC are viewed as providing a more preferable tradeoff between fit and model complexity. We therefore used DIC to compare different model specifications for the departure time data.

Table 1 shows the DIC values obtained from different models applied to the departure time data. In interpreting the number of parameters, it should be noted that a level of a factor (e.g. $\boldsymbol{m}_k$) is represented by a pair of parameters. Hence, in a sweeping-reparametrized model with only a mode effect, there are 2 parameters for the overall mean and the 4-1=3 remaining free mode levels are represented in the projected normal model by 6 parameters, for a total of 8 possible parameters. The model with only a mode effect, with results in the first row of Table 1, resulted in a value of $p_D = 7.63$. The reduction of p_D relative to the fully unconstrained value of 8 indicates that the prior distributions resulting in only a small amount of "shrinking" for this factor. It is possible to similarly interpret the values of p_D for the other model specifications.

The results in Table 1 clearly show that models containing all three factors (mode, state, wave) consistently achieve lower DIC values than models that excluded any of those factors. While not shown here, models with mode as random effect performed worse than models with mode as fixed effect. In contrast, very similar DIC values were obtained with state and wave treated as either fixed or random. When we investigated models with interactions between the three factors, those with state-wave interactions scored better than any other arrangement of two-way interactions. As shown in Table 1, treating the state-wave interactions as a random effect gave better results than treating it as a fixed effect. Finally, the addition of a household random effect gave results that were virtually identical to those without the household effect. Based on all this, the following model was selected as the best-fitting model:

$$\boldsymbol{\mu}_{ijk} = \boldsymbol{\mu}' + \mathbf{m}'_k + \mathbf{sw}'_{ij}$$

where $\boldsymbol{sw}_{ij}$ denotes a random interaction effect between state and wave, with 99 total levels.

Fixed effects	Random effects	$D(\bar{\omega})$	$\overline{D(\omega)}$	p_D	DIC
Mode		2642698	2642706	7.631472	2642713
Mode	Wave	2631890	2631908	17.85838	2631926
Mode, Wave		2631890	2631907	17.2165	2631924
Mode	State	2628456	2628496	40.25385	2628536
Mode, State		2628456	2628495	39.48233	2628535
Mode	State, Wave	2618338	2618387	49.50114	2618437
Mode, Wave	State	2618337	2618387	49.07431	2618436
Mode, State	Wave	2618337	2618387	49.17294	2618436
Mode, State, Wave		2618337	2618386	49.06591	2618435
Mode, State×Wave		2615651	2615856	205.7165	2616062
Mode	State×Wave	2615669	2615848	178.9047	2616027
Mode	State×Wave, Household	2615668	2615847	179.3620	2616026

Table 1: DIC results for different model specifications for the departure time data.

4 Prediction of Fractions of Departures

The goal of the project is to estimate the population fraction of daily departures at a fishing site between the arrival and departure times of the interviewer, for a stratum U_{ijk} determined by mode, state and wave. The population fraction between two times, t_1 and t_2, is defined as

$$F_{N,ijk}(t_1, t_2) = \frac{\sum_{U_{ijk}} I_{[t_1,t_2)}(T_{ijkt})}{N_{ijk}}, \tag{6}$$

with $I_{[t_1,t_2)}(T) = 1$ when $t_1 \leq T < t_2$ and 0 otherwise. From the survey data, a direct estimator of this quantity is given by

$$\hat{F}^D_{ijk}(t_1, t_2) = \frac{\sum_{s_{ijk}} I_{[t_1,t_2)}(T_{ijkt})}{n_{ijk}}. \tag{7}$$

This estimator is unbiased under repeated sampling from the target population, but is likely to be very variable in many strata because the sample size n_{ijk} is very small (or zero). Under the assumption of simple random sampling with replacement or with a vanishingly small sampling fraction, the variance of $\hat{F}^D_{ijk}(t_1, t_2)$ is given by

$$\mathrm{Var}(\hat{F}^D_{ijk}(t_1, t_2)) = \frac{1}{n_{ijk}} F_{N,ijk}(t_1, t_2)(1 - F_{N,ijk}(t_1, t_2)). \tag{8}$$

Under the assumption that the departure times follow a projected normal distribution, the population fraction $F_{N,ijk}(t_1,t_2)$ is expected to be very close to the probability $\Pr(t_1 \le T < t_2|\boldsymbol{\mu}_{ijk})$ under the projected normal distribution as long as N_{ijk} is sufficiently large, with $\boldsymbol{\mu}_{ijk}$ the "true" parameter value for the stratum. This is a non-random function of $\boldsymbol{\mu}_{ijk}$, so that a procedure that provides an estimate for $\boldsymbol{\mu}_{ijk}$ can be used to estimate $\Pr(t_1 \le T < t_2|\boldsymbol{\mu}_{ijk})$ and hence $F_{N,ijk}(t_1,t_2)$ as well. Hence, under the hierarchical Bayesian model described in the previous sections, we can obtain the posterior distribution of

$$F^M_{ijk}(t_1,t_2) = \Pr(t_1 \le T < t_2|\boldsymbol{\mu}_{ijk}) \tag{9}$$

given the sample data. However, obtaining this posterior distribution for each combination of state, wave, mode and time interval is far from trivial given the size of the dataset, because the integration of the projected normal density over the interval (t_1,t_2) needs to be performed at each iteration of the Gibbs sampler.

We therefore streamlined the computations by only computing the fractions for 24 1-hour intervals and by taking advantage of a number of results for the projected normal distribution. To simplify notation in what follows, let $F^M_{ijk}(t), t = 1,\ldots,24$ denote the 1-hour fractions $F^M_{ijk}(t-1,t)$. For each iteration b of the Gibbs sampler, we obtain a vector $\boldsymbol{\mu}^b_{ijk}$ for each one of the 368 combinations of indices ijk from which we compute the 24 fractions. Without simplifications, this implies that we would need to compute $368 \times 24 \times B$ integrals. To make this process more efficient, we first proved the following two identities:

$$\begin{aligned}
\int_{\theta_1}^{\theta_2} f\left(\theta \mid \boldsymbol{\mu}_{ijk}\right) d\theta &= \Phi\left(-\rho_{ijk}\sin\left(\theta_1-\omega_{ijk}\right)\right) \text{ if } \theta_2-\theta_1=\pi \\
\int_{\theta_1}^{\theta_2} f\left(\theta \mid \boldsymbol{\mu}_{ijk}\right) d\theta &= \Phi\left(-\rho_{ijk}\sin\left(\theta_1-\omega_{ijk}\right)\right)\Phi\left(-\rho_{ijk}\cos\left(\theta_1-\omega_{ijk}\right)\right) \text{ if } \theta_2-\theta_1=\frac{\pi}{2},
\end{aligned}$$

where $f\left(\theta \mid \boldsymbol{\mu}_{ijk}\right)$ denotes the density of a $PN_2\left(\boldsymbol{\mu}_{ijk}, \mathbf{I}_2\right)$ random variable normalized to the unit circle, and $\boldsymbol{\mu}^T_{ijk} = \rho_{ijk}\left(\cos\omega_{ijk}, \sin\omega_{ijk}\right)$. Then, at each iteration b the 24 fractions $F^{M,b}_{ijk}(t)$ are obtained by computing $F^{M,b}_{ijk}(1),\ldots,F^{M,b}_{ijk}(5)$ by numerical integration, followed by successive differencing using the second identity, i.e. we set $F^{M,b}_{ijk}(6) = \Phi\left(-\rho^b_{ijk}\sin\left(-\omega^b_{ijk}\right)\right)\Phi\left(-\rho^b_{ijk}\cos\left(-\omega^b_{ijk}\right)\right) - \sum_{t=1}^{5} F^{M,b}_{ijk}(t)$, $F^{M,b}_{ijk}(7) = \Phi\left(-\rho^b_{ijk}\sin\left(\frac{\pi}{12}-\omega^b_{ijk}\right)\right)\Phi\left(-\rho^b_{ijk}\cos\left(\frac{\pi}{12}-\omega^b_{ijk}\right)\right) - \sum_{t=2}^{6} F^{M,b}_{ijk}(t)$ and so on. In this manner, for each b and each ijk, we only need to compute 5 numerical integrals and 19 integrals using the formula above instead of 24 numerical integrals.

At the conclusion of this procedure, we obtain the posterior distribution of $F_{ijk}^{M}(t)$ for each 1-hour fraction in each wave, state and mode combination. Figure 3 shows boxplots corresponding to the posterior distributions of the model-estimated fractions of departures for four state-wave-mode combinations in each 1-hour period, as well as histograms of the original data in those strata, which correspond to the estimator (7). The very narrow boxplots reflect the fact that these estimated departure fractions are based on a very large sample size. It is also clear from these plots that the modeled distributions deviate substantially from the observed distributions, even when there appear to be a lot of observations in a stratum.

At this point, we have two possible estimators for the population fractions of departures in (6). The design-based estimator in (7) is unbiased but very variable in any stratum with small sample size, while the model-based estimator in (9) is very precise (low variance) but, being model-based, potentially biased if the model specification is incorrect. We now consider the model above as a component of a small area estimation procedure. The goal of small area estimation is to combine a survey estimator that only uses data from a given "small area" (defined here as a mode-state-wave combination) with a model-based estimator that is based on data from the whole sample. In this case, the survey estimator is the direct estimator (7), and the model-based estimator is the posterior mean of (9), which we will denote by $\hat{F}_{ijk}^{M}(t)$. A large literature on small area estimation is available, with a range of different parametric and, more recently, nonparametric model specifications (see Rao (2003) for an overview). However, to the best of our knowledge, no small area estimation models for circular data exist. We therefore decided to apply *composite estimation* (Ghosh and Rao, 1994), which consists of taking a convex combination of both estimators. Composite estimation is generally applicable even in non-standard situations and is easy to implement. It also provides a simple way to trade off the bias and variance of the two estimators by adjusting the linear combination weights.

The composite estimator for the fraction of departures that occur in the interval $[t-1, t), t = 1, \ldots, 24$ for state i, wave j and mode k is defined as

$$\hat{F}_{ijk}^{C}(t) = w_{ijk}\hat{F}_{ijk}^{D}(t) + (1 - w_{ijk})\,\hat{F}_{ijk}^{M}(t),$$

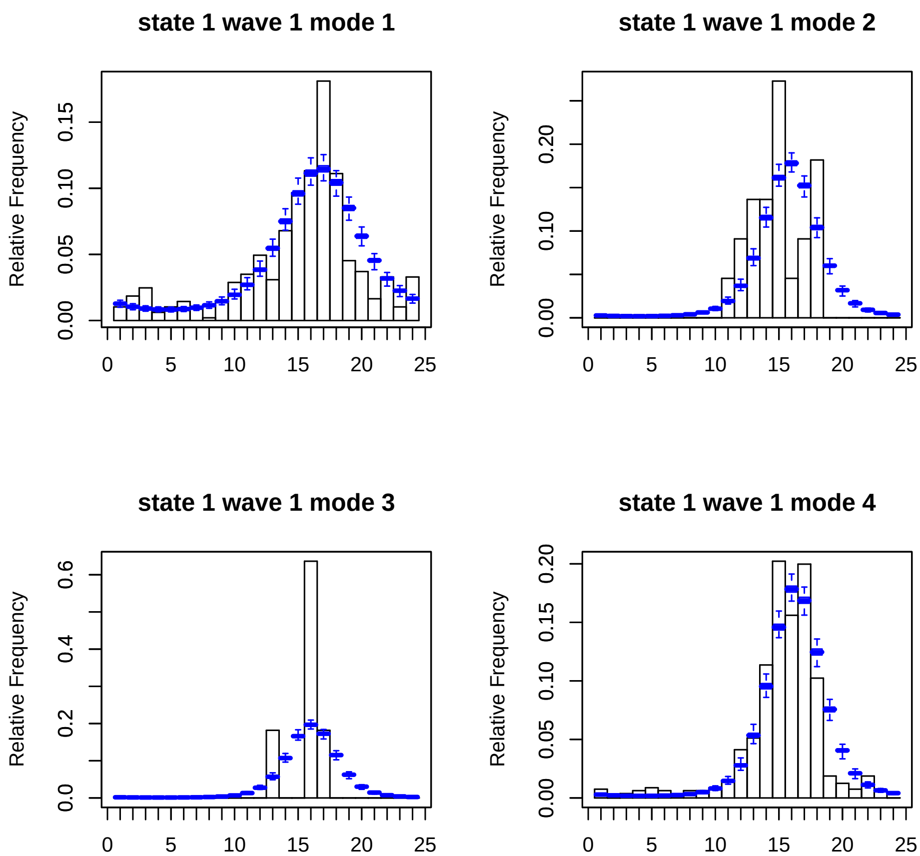


Figure 3: Histograms of departure time data and boxplots of the posterior distributions of the 1-hour fractions of departures. Four different combinations of state, wave, and mode are shown.

where $w_{ijk} \in [0,1]$ is a weight further specified below. Note that, while the weight w_{ijk} can in principle depend on t as well, we will use a single weight for all t in a small area so that the composite estimator remains a valid fraction in the sense that $\sum_{t=1}^{24} \hat{F}^C_{ijk}(t) = 1$. The optimal weight w^{opt}_{ijk} in this context minimizes $\text{MSE}(\hat{F}^C_{ijk}(t))$, averaged over t. Assuming that the direct estimator $\hat{F}^D_{ijk}(t)$ is unbiased and that the covariance $\text{Cov}\left(\hat{F}^D_{ijk}(t), \hat{F}^M_{ijk}(t)\right) = 0$,

$$w^{\text{opt}}_{ijk} = \frac{\text{MSE}\left(\hat{F}^M_{ijk}\right)}{\text{MSE}\left(\hat{F}^M_{ijk}\right) + \text{Var}\left(\hat{F}^D_{ijk}\right)}, \tag{10}$$

where we denote averaging over t by removing it from the expressions. The assumption that both estimators are uncorrelated is approximately met when the variance of the model-based estimator is negligible relative to that of the direct estimator. Given the very large sample size in this application, this is certainly reasonable here (see also Figure 3). Speaking somewhat loosely, the MSE of the direct estimator is equal to its variance, while that of the model-based estimator is equal to its bias due to model misspecification. The optimal weight w^{opt}_{ijk} therefore trades off the variance of the former and the bias of the latter. This optimal weight is unknown but will be estimated under a number of simplifying assumptions.

First, we assume that the magnitude of the bias of the model-based estimator averaged over time is approximately constant across small areas. Second, the variance of the direct estimator is assumed to be of the form C/n_{ijk}, which is reasonable given expression (8) above. Therefore, for any small area ijk, we estimate $\text{MSE}\left(\hat{F}^M_{ijk}\right)$ by

$$\widehat{MSE}(F^M) = \frac{1}{n}\sum_i \sum_j \sum_k \sum_t \left(\left(\hat{F}^D_{ijk}(t) - \hat{F}^M_{ijk}(t)\right)^2 - \frac{\hat{F}^D_{ijk}(t)\left(1 - \hat{F}^D_{ijk}(t)\right)}{n_{ijk}} \right)$$

where the latter term inside the sum is the direct estimator of the variance of $\hat{F}^D_{ijk}(t)$.

The term $\text{Var}\left(\hat{F}^D_{ijk}\right)$ in (10) is replaced by the simplified "estimator" $\widehat{V}\left(\hat{F}^D_{ijk}\right) = 0.25/n_{ijk}$, which is the largest possible value for the variance of a proportion. We are using 0.25 in the numerator instead of the average of the $\hat{F}^D_{ijk}(t)\left(1 - \hat{F}^D_{ijk}(t)\right)$ over t, because especially in small areas with small sample sizes, the $\hat{F}^D_{ijk}(t)$ were 0 for many of the time intervals, resulting in very small estimates of $\text{Var}\left(\hat{F}^D_{ijk}\right)$ and hence skewing the composite estimator towards the direct estimator despite the small sample size.

Using these estimators, the final weight for small area ijk is given by

$$w_{ijk} \;=\; \frac{\widehat{MSE}(F^M)}{\widehat{MSE}(F^M) + 0.25/n_{ijk}}.$$

In order to illustrate the effect of the above weighting procedure, Figure 4 shows the direct, model-based and composite estimators for two small areas with different sample sizes. Clearly, when n_{ijk} is small, the weighting procedure will give a relatively small weight to the direct estimator and base the composite estimator primarily on the model-based estimator. In contract, when the sample size is large, the composite estimator is very close to the direct estimator.

Currently, the direct estimator $\hat{F}^D_{ijk}(t)$ is simply taken to be the unweighted sample proportion of the departure times between $t-1$ and t in small area ijk. We plan to investigate whether replacing it by an estimator that uses sampling weights results in improved composite estimators in the future. The weighting procedure above will then be revisited, since the variance of $\hat{F}^D_{ijk}(t)$ will no longer correspond to expression (8).

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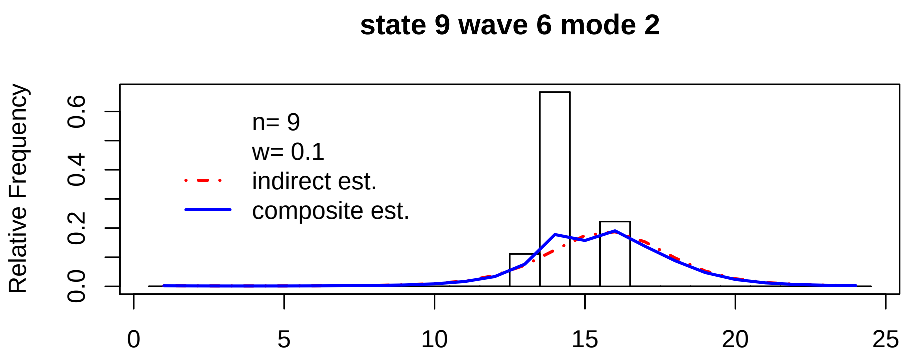

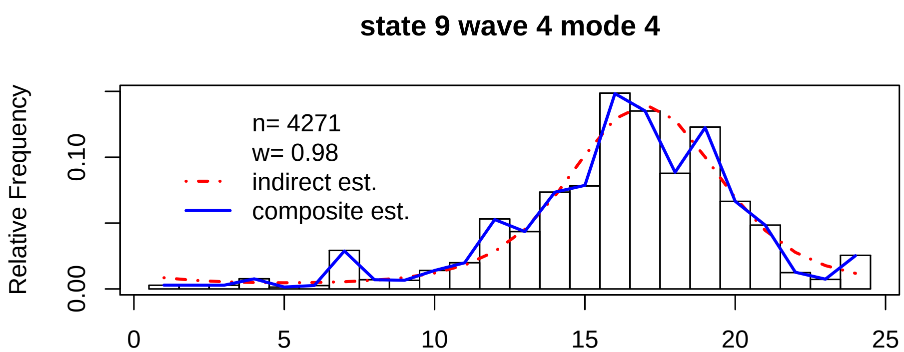

Figure 4: Examples of direct, model-based and composite estimators for the fractions of departures for two strata combinations. Top plot: stratum with small sample size; bottom plot: stratum with large sample size.

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Appendix IV: Properties of Combined Primary-Alternate Site Probabilities

We consider estimation of a total defined as

$$t_h = \sum_{(k,d)\in U_h} y_{kd},$$

where U_h denotes an arbitrary stratum of site-days within the overall target population and (k, d) is the index denoting site k and day d. The combined inclusion probabilities are defined as

$$\pi_{h,kd} = \pi^P_{h,kd} + \left(1 - \pi^P_{h,kd}\right) \pi^A_{h,kd},$$

with $\pi^P_{h,kd}$ denoting the primary site inclusion probability and $\pi^A_{h,kd}$ the alternate site inclusion probability. Note that the latter is unknown and has to be estimated. In this appendix, we ignore the uncertainty caused by the estimation of $\pi^A_{h,kd}$.

The estimator of t_h is defined as

$$\hat{t}_h = \sum_{(k,d)\in s_h} \frac{y_{kd}}{\pi_{h,kd}} = \sum_{(k,d)\in U_h} \frac{y_{kd}}{\pi_{h,kd}} \Big(I_{kd} + (1 - I_{kd}) A_{kd} \Big),$$

where s_h denotes the combined sample of primary and alternate site-days in stratum h and

$$I_{kd} = \begin{cases} 1 & \text{if the site is picked as } \textbf{primary} \\ 0 & \text{otherwise.} \end{cases}$$

$$A_{kd} = \begin{cases} 1 & \text{if the site is picked as } \textbf{alternate} \\ 0 & \text{otherwise.} \end{cases}$$

We consider the properties of $\hat{t}_h$. The unbiasedness of the estimator under the combination of the primary site and assumed alternate site selection mechanisms is immediate, since

$$\begin{aligned} \mathrm{E}(\hat{t}_h) &= \mathrm{E}\left(\mathrm{E}(\hat{t}_h | s_P)\right) \\ &= \mathrm{E}\left(\sum_{(k,d)\in U_h} \frac{y_{kd}}{\pi_{h,kd}} \Big(I_{kd} + (1 - I_{kd}) \pi^A_{h,kd} \Big) \right) \\ &= \sum_{(k,d)\in U_h} \frac{y_{kd}}{\pi_{h,kd}} \Big(\pi^P_{h,kd} + (1 - \pi^P_{h,kd}) \pi^A_{h,kd} \Big) = t_h, \end{aligned}$$

where s_P denotes the sample of primary site-days. In order to investigate the variance, we assume that the primary site-day sample is drawn under a Poisson sampling design with inclusion probabilities $\pi^P_{h,kd}$ and that, conditional on not being selected in the primary site-day sample, the alternate site-days are selected by Poisson sampling with inclusion probabilities $\pi^A_{h,kd}$. The variance can be written as

$$\mathrm{Var}(\hat{t}_h) = \mathrm{E}\left(\mathrm{Var}(\hat{t}_h|s_P)\right) + \mathrm{Var}\left(\mathrm{E}(\hat{t}_h|s_P)\right).$$

The "inner" moments in that expression can be computed as

$$\mathrm{Var}(\hat{t}_h|s_P) = \sum_{(k,d)\in U_h} \left(\frac{y_{kd}}{\pi_{h,kd}}\right)^2 (1 - I_{kd})\, \pi^A_{h,kd}\, (1 - \pi^A_{h,kd})$$

and

$$\mathrm{E}(\hat{t}_h|s_P) = \sum_{(k,d)\in U} \frac{y_{kd}}{\pi_{h,kd}} \left(\pi^A_{h,kd} + I_{kd}(1 - \pi^A_{h,kd})\right).$$

Following this, we obtain

$$\mathrm{E}\left(\mathrm{Var}(\hat{t}_h|s_P)\right) = \sum_{(k,d)\in U_h} \left(\frac{y_{kd}}{\pi_{h,kd}}\right)^2 (1 - \pi^P_{kd})\, \pi^A_{h,kd}\, (1 - \pi^A_{h,kd})$$

and

$$\mathrm{Var}\left(\mathrm{E}(\hat{t}_h|s_P)\right) = \sum_{(k,d)\in U} \left(\frac{y_{kd}}{\pi_{h,kd}}\right)^2 (1 - \pi^A_{h,kd})^2\, \pi^P_{kd}\, (1 - \pi^P_{kd}).$$

Combining these results and after some simplification, we conclude that

$$\mathrm{Var}(\hat{t}_h) = \sum_{(k,d)\in U_h} y^2_{kd} \frac{1 - \pi_{h,kd}}{\pi_{h,kd}},$$

corresponding the variance of $\hat{t}_h$ under a single-phase Poisson sampling design with probabilities $\pi_{h,kd}$.

Appendix V. Glossary of Terms Used in the Main Document

TERM	DESCRIPTION
Alternate mode interview	An interview that is obtained with an angler who has completed fishing for the day in a mode other than the mode assigned for interviewing. For example, an opportunistic interview with a shore angler or charter boat angler would be an "alternate mode interview" if the interviewer was specifically directed to obtain interviews with private/rental boat anglers.
Alternate site	An alternate site is a site adjacent to the assigned, or primary, site for the interviewing assignment that has fishing pressure estimated in the assigned mode for interviewing. The current methods allow an interviewer to visit up to two alternate sites in addition to the primary site during an interviewing assignment.
Angler fishing trip (or angler trip)	An angler day of fishing in a specific fishing mode. An angler trip is not complete until the angler has finished his/her day of fishing.
Angler group	An angler group is a "group" of one or more anglers who fished together, combined their catch, and are unable to separate that catch so that an interviewer can observe and identify the specific fish caught by each angler.
APAIS	The Access Point Angler Intercept Survey is the on-site survey component of the MRFSS that has been used to collect catch data from angler fishing trips and estimate the mean numbers of fish caught per trip for different finfish species.
Catch type	The catch for an intercepted angler fishing trip is assigned to a specific catch type based on whether or not it can be observed directly by an interviewer.
Catch type A	Landed catch that can be directly observed in whole form and identified by an interviewer. This type may also be called "observed catch".
Catch type B	Catch that was reported by an intercepted angler as either kept or released at sea that cannot be directly observed in whole form by an interviewer. This type may also be called "unobserved catch".
Catch type B1	Unobserved catch that was reported by an intercepted angler as either kept or released dead at sea.

TERM	DESCRIPTION
Catch type B2	Unobserved catch that was reported by the angler as released alive at sea.
CHTS	The Coastal Household Telephone Survey is the off-site component of the MRFSS that has been used to collect fishing trip data from residents of coastal county households and estimate the mean number of angler fishing trips per household.
Cluster sampling	Cluster sampling refers to sampling from a survey frame that identifies subsets, or clusters, of elements in the target population. For example, each site-day unit in the APAIS frame that is selected for a private/rental boat interviewing assignment represents a cluster of vessel fishing trips that could be intercepted. Each vessel trip that is intercepted represents a cluster of angler fishing trips that could be intercepted.
Cluster size	The number of elements (or clusters of elements) from which a sample is drawn at each stage in a multi-stage cluster sampling design. For boat modes, this would be the number of boat trips (each has a cluster of anglers) that could potentially be sampled within a site-day assignment, or it would be the number of angler trips that could potentially be sampled within each intercepted boat trip. For the shore mode, this would be the number of angler trips that could potentially be sampled within each site-day assignment.
Day type	Days are stratified into "weekday" and "weekend/holiday" day types. Federal government holidays are combined with Saturdays and Sundays in the latter day type. All other days are considered to be "weekdays".
Departure time	The time that an angler departs from a day of fishing. This is the time at which an angler reports having completed a day of fishing in a given fishing mode.
Domain	A domain is a subpopulation of the target population for which separate survey estimates are desired. Domain estimates can be obtained by partitioning the data collected from a survey sample. Domains are not synonymous with "strata", because they are typically subpopulations that cannot be easily separated for the purpose of independent sampling. In the APAIS, separate domain estimates of catch are produced for different species and fishing areas.

TERM	DESCRIPTION
Fishing pressure	In the MRFSS, “fishing pressure” for a given fishing access site is defined as the estimated number of angler fishing trips completed within an 8-hour period that comprises the peak activity period for the site. Fishing pressure estimates are made for each site in each fishing mode and for each month and day type within a given mode.
Frame	A frame (or sampling frame) is a list or device that provides access to elements in a target population for the purpose of drawing a representative sample . The selected frame for a given survey may not provide access to all elements in the target population for the study and it may also include access to elements not in the target population.
Inclusion probability	The probability that a given primary, secondary, or tertiary sampling unit gets selected for observation at a given stage of sampling.
Interviewing Assignment	An interviewing assignment is specific to a given sampling stratum defined by the fishing mode, month, and day type, as well as to a specific site-day combination that is selected in the sampling conducted for that stratum.
Master site register (MSR)	The master site register is a complete list of fishing access sites in each coastal state that includes site-specific estimates of fishing pressure for each possible combination of fishing mode, month, and day-type. This register comprises a frame that can be used for stratified sampling of sites in which strata are defined by fishing mode, month, and day type. The MSR also includes information on the location of each site, driving directions to the site, and specific types of fishing present at the site.
Mixed group catch	A mixed group catch (or group catch) is a collection of observed fish (Type A catch) that were caught by more than one angler and mixed together so that they cannot be easily separated by angler. The group catch is recorded with the count of the anglers who contributed to the catch, and all contributing anglers comprise an “angler group” (see above).
Mode of fishing	Angler fishing trips are differentiated into different fishing mode categories as follows:
Shore fishing mode (SH)	Shore fishing trips are those made by anglers who are saltwater fishing from beaches, banks, piers, docks, jetties, breakwaters, bridges, causeways, and other man-made structures.

TERM	**DESCRIPTION**
Private/Rental boat mode (PR)	Private/rental boat trips are those made by anglers who are saltwater fishing from privately owned boats or rented boats.
Charter boat mode (CH)	Charter boat trips are those made by anglers who are fishing from a charter boat. A charter boat is one that usually takes anglers in a pre-formed group who paid in advance for the services of the captain and/or crew on a specific scheduled date.
Headboat mode (HB)	Headboat trips are those made by anglers who are fishing on a headboat, partyboat, or open boat. A headboat is one on which the anglers typically pay as individuals (on a "per head" basis) to fish.
Party/Charter boat mode (PC)	This for-hire boat mode of fishing (both charter boat and headboat fishing) was used to define a sampling stratum before separate sampling of the charter boat and headboat modes was initiated.
MRFSS	The Marine Recreational Fishery Statistics Survey is comprised of two complemented surveys – a Coastal Household Telephone Survey (CHTS) and an Access Point Angler Intercept Survey (APAIS).
NMFS	The National Marine Fisheries Service is a branch agency of NOAA and is synonymous with the NOAA Fisheries Service
NOAA	This is the abbreviation for the National Oceanic and Atmospheric Administration
Pressure category	A pressure category corresponds to a specific range of estimated fishing pressure. Each site is assigned to a specific pressure category in each mode/month/day-type stratum based on its estimated fishing pressure.
Primary area of fishing	The primary area, or water body, in which fishing occurred on a given angler fishing trip. If more than one area was visited, the angler is asked to report the area in which most of the fishing took place.
Inland area	The inland area includes the brackish or saltwater portions of sounds, rivers, bays, or inlets, and does not include any part of the open ocean. The water bodies included in this area category are combined with the nearshore ocean area to comprise State waters.
Nearshore ocean area	The nearshore area is the area of the open ocean that extends up to 3 miles from the shoreline (up to 10 miles off the Gulf coast of Florida) and comprises the ocean portion of the State territorial seas.

TERM	DESCRIPTION
Offshore ocean area	The offshore area is the area of open ocean that extends beyond 3 miles from shore (beyond 10 miles from the Gulf coast of Florida) and comprises Federal waters.
Primary sampling unit (PSU)	The PSU is the sampling unit selected in the first stage of a multi-stage sampling design. For the APAIS, the PSU is a site-day.
Probability proportional to size (PPS) sampling	PPS sampling is a special type of unequal probability sampling where the inclusion probability of a particular frame element is proportional to its value for a specific size measure. In the APAIS, sites are selected in proportion to their fishing pressure, and this is an example of PPS sampling.
Sampling without replacement	This refers to the type of sampling that does not allow any individual frame unit to be selected more than once.
Site-day	A site-day is the combination of a selected fishing access site with a selected day.
Secondary sampling unit (SSU)	The SSU is the sampling unit selected in the second stage of a multi-stage sampling design. For the APAIS, the SSU is a boat trip for boat mode sampling and an angler trip for shore mode sampling.
Small area estimation	"Small area estimation" refers to any of several statistical techniques involving the estimation of parameters for small sub-populations, generally used when the sub-population of interest is included in a larger survey.
Stratified sampling	Sampling is stratified if the frame population is divided into subpopulations called "strata", and each stratum is sampled independently. If strata are defined such that the elements of each stratum are relatively homogeneous with respect to the parameter of study and most of the frame population variability is due to differences among strata, then stratified sampling can lead to substantial gains in the precision of point estimators of the study parameters.
Target population	The population about which information is desired. The population that is actually surveyed is the study population.
Tertiary sampling unit (TSU)	The TSU is the sampling unit selected in the third stage of a multi-stage sampling design. For the APAIS, the TSU is an angler trip for boat mode sampling.

TERM	DESCRIPTION
Unweighted estimation method	An estimation method that does not properly weight survey observations to account for the probability sampling design that was used.
Wave of sampling	The term "wave" is used in this document to describe the particular time frame for periodic telephone surveys of fishing effort. If telephone surveys are conducted bimonthly, then the length of the wave is two months. If conducted monthly, then the length of the wave is one month. The term is also used to describe the "temporal stratification" of sampling and estimates for such periodic surveys.
Weighted estimation method	An estimation method that properly weights survey observations to account for the probability sampling design that was used. Individual observations must be weighted to reflect their known (or approximated) probabilities of inclusion in the survey sample.

Appendix VI. Table of Notation

Estimation of catch rate and variance	
H	The number of sampling strata in a target population
	The number of site-days sampled within stratum *h* ( )
	The number of boat-trips sampled within the *hi*-th site-day ( )
	The number of angler-groups sampled within the *hij*-th boat ( )
	The observed number of fish caught in *hijk*-th angler-group ( )
	The observed number of anglers in the *hijk*-th angler-group
	The total number of groups of anglers available to be sampled in the *hij*-th boat trip
	The observed number of angler trips aboard the *hij*-th boat trip
	The total number of boat trips available to be sampled within the *hi*-th site-day
	Cluster size of the *hi*-th sampled site-day
	Inclusion probability of the *hi*-th sampled site-day
	Total catch in a target population
	Total effort in a target population
	Catch rate for a target population

Cluster size of *hi*-th site-day ( )	
	Departure time for fishing trip *m* by respondent *l* in state *i*, wave *j* and mode *k*
	Fraction of daily departure within time interval $[t, t+\Delta)$

Inclusion probability of the *hi*-th sampled site-day with alternate site sampling ( )	
	Probability that site-day with site *k* and day *d* in stratum *h* is selected as primary site-day
	Probability that site-day with site *k* and day *d* in stratum *h* is selected as alternate site-day, given that it is not selected as a primary site
	The total number of site-days selected as primary site-days in stratum *h*
	The number of times (days) site *k* selected as primary site in stratum *h*
	Number of times (days) site *k* selected as alternate site in stratum *h*
	The set of all strata in which site *k* appears as an alternate site