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Atlantic Blacktip Shark Benchmark Stock Assessment

Life history – Vital rates

SEDAR 65 (Review Workshop)

October 29, 2020



Multiple methods used to estimate r_{max}

1. Classical Euler-Lotka equation (life table) and Leslie matrix
2. Smith et al. (1998) rebound potentials tweaked by Au et al. (2015) (and multiplied by 2 to represent r_{max})
3. Skalski et al. (2008) (=Myers et al. (1997)="extinction risk")
4. Eberhardt et al. (1982)
5. Demographic Invariant Method (Niel & Lebreton 2005, Dillingham 2010)

Data requirements/assumptions of different methods

Method	Age at maturity/ first breeding	Maximum age	Fecundity	M	Survival to age at maturity/ first breeding
Euler-Lotka/Leslie matrix	Yes	Yes	Age-dependent	Age-dependent	Yes
Rebound potential (Au et al. 2009)	Yes	Yes	Constant	Constant	Yes
Eberhardt et al. (1992)	Yes	Yes	Constant	Constant	Yes
Skalski et al. (2008)	Yes	No	Constant	Constant	Yes
DIM	Yes	No	No	Constant	No

Methods used to estimate M (natural mortality)

Natural mortality (M) estimated through multiple methods:

1, 2) Jensen (1996) estimators: $M = \frac{1.65}{t_{mat}}$ and $M = 1.5K$

3) Hoenig new estimator (Then et al. 2015):

$$M = 4.899t_{\max}^{-0.916}$$

4) Pauly new estimator (Then et al. 2015):

$$M = 4.118K^{0.73}L_{\inf}^{-0.33}$$

Methods used to estimate M (natural mortality)

5) Chen and Yuan's (2006) estimator:

$$\ln(M) = 1.46 - 1.01 \ln \left(t_0 - \frac{\ln(0.05)}{K} \right)$$

6) Peterson and Wroblewski (1984) estimator:

$$M = 1.92W^{-0.25}$$

Population dynamics parameters of interest

r_{max} : intrinsic rate of increase (maximum population growth rate in ideal conditions after exploitation has ceased)

Generation time : multiple definitions. Will report here \bar{A}
(mean age of parents of offspring in a stable age distribution)

Net reproductive rate (R_0) or unexploited spawners per recruit (ϕ_0) : average number of females produced by each female over her lifetime. Is also used to compute the maximum lifetime reproductive rate:

$$\hat{\alpha} = S_0 \Phi_0$$

Steepness : ranges from 0.2 to 1, uses alpha hat in its computation:

$$h = \frac{\hat{\alpha}}{4 + \hat{\alpha}}$$

Population dynamics parameters of interest

SPR_{MER} : analogous to steepness. Spawning Potential Ratio at Maximum Excess Recruitment (the closer to 100% the less exploitation permitted):

$$SPR_{MER} = \frac{1}{\sqrt{\hat{\alpha}}}$$

R : analogous to SPR_{MER} . It's the position of the inflection point of population growth curves (assumed to be 0.5 in traditional production models): the closer to 1, the less exploitation permitted

Fertility, juvenile survival, and adult survival elasticity: the proportional contribution of these different stages to population growth rate.

Distributions used to incorporate uncertainty in vital rates for blacktip shark in the Monte Carlo simulation using the Leslie matrix

Lifespan: $U \square (t \max(obs), t \max(theor))$

Growth parameters: $MVN \square \left(\mu = \begin{pmatrix} L_{\infty} \\ K \\ t_0 \end{pmatrix}, \Sigma \right)$

Maturity ogive: $MVN \square \left(\mu = \begin{pmatrix} a \\ b \end{pmatrix}, \Sigma \right)$

Natural mortality: $Min \square (method1, method2, method3, method4, method5, method6)$

Deterministic estimates of r_{max} for Atlantic blacktip sharks obtained through six methods

	Lifespan=18 yr	Lifespan=31 yr
Euler-Lotka	0.039 (0.25)	0.060 (0.34)
Leslie matrix	0.039 (0.25)	0.060 (0.34)
Au et al. (2016)	0.092	0.092
Eberhardt et al. (1982)	0.058	0.058
Skalski et al. (2008)	0.040	0.057
Neil and Lebreton's (2005) DIM*	0.084	0.084
Mean	0.059	0.069
Values in parentheses are steepness		
*DIM=Demographically Invariant Method		

Estimates of population dynamics parameters for Atlantic blacktip sharks obtained from Monte Carlo simulation of vital rates with a Leslie matrix approach

	Median	LCL	UCL
r_{\max}	0.042	-0.027	0.085
Generation time	12.5	11.2	20.1
Net reproductive rate (R_0)	1.815	0.781	3.307
Age-0 survivorship (S_0)	0.84	0.79	0.88
Steepness (h)	0.28	0.20	0.42
SPR_{MER}	0.81	0.59	1.00
R	0.72	0.61	1.00

Steepness inferred from the other methods

Method	r_{max}	steepness
Euler-Lotka	0.060	0.34
Leslie matrix		
Au et al. (2016)	0.092	0.52
Eberhardt et al. (1982)	0.058	0.33
Skalski et al. (2008)	0.057	0.32
Neil and Lebreton's (2005) DIM*	0.084	0.47
Mean		0.40
Mean steepness from 33 assessments		0.46



Conclusions/Recommendations

1. Combination of relatively high M_0 and low ϕ_0 produces low steepness
2. Productivity and other parameter estimates found here are likely underestimates because life history information used is likely not representative of ideal conditions
3. Recommend using a steepness value on the order of 0.40 (mean for 33 stock assessments from Cortes and Brooks (2018) was 0.46)
4. Potentially use lower and upper values of the range obtained with deterministic methods (0.32, 0.52) for low and high productivity scenarios

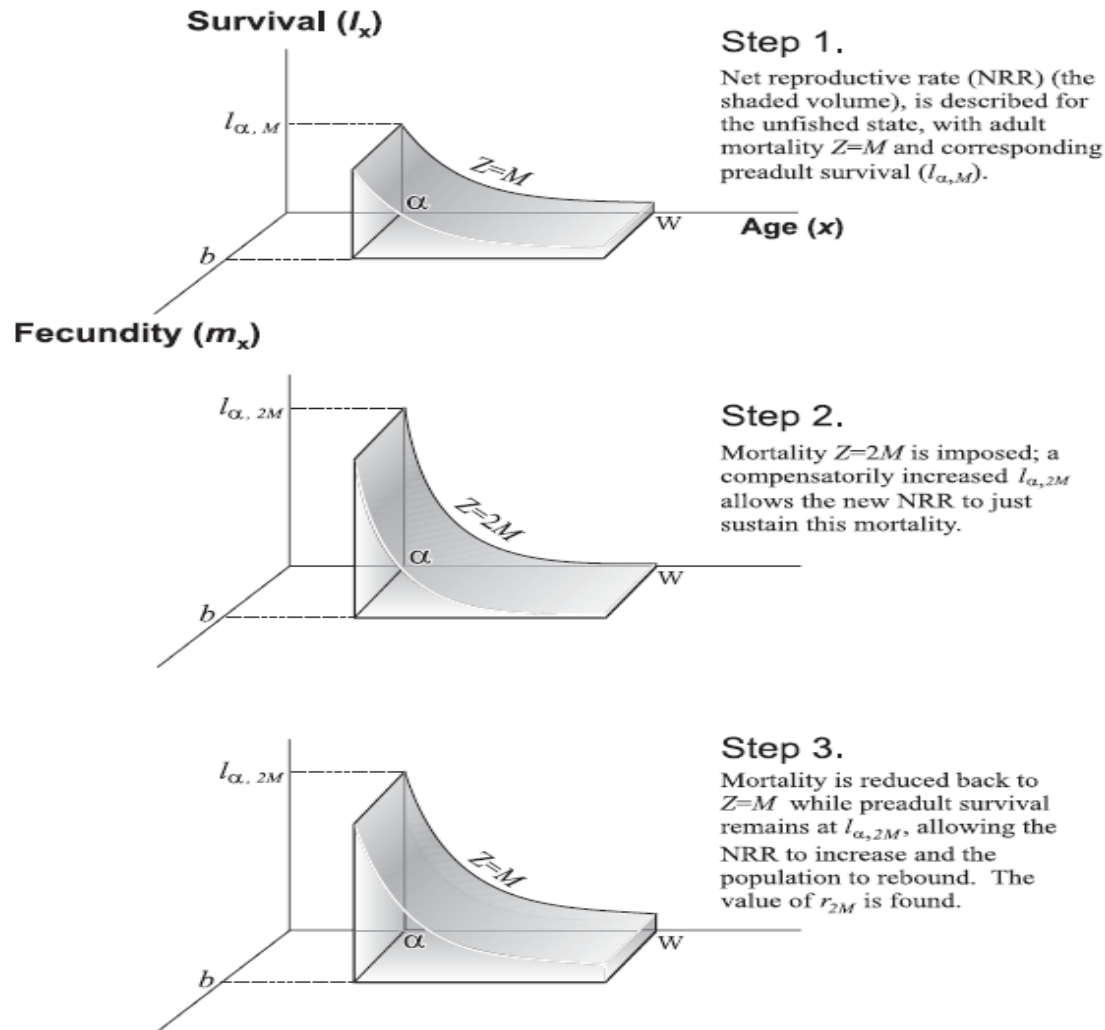
Additional slides



1. Euler-Lotka equation

$$\sum_{x=\alpha}^{x=w} l_x m_x e^{-rx} = 1$$

Smith et al.'s density-dependent model



Step 1: Describe the Net Reproductive Rate for an unfished population in terms of survival to the average age at maturity

Step 2: Determine pre-adult survival when adults suffer total mortality $Z=F+M=2M$ at all ages above the age at maturity

Step 3: The rebound level of population increase is determined by removing F from adults

Fig. 1. Three steps for determining population productivity or rebound rate (r_{2M}) based on mortality-induced changes to the net reproductive rate ($\sum I_x m_x$), a quantity representing the expectation of reproductive output in the lifetime of a female. b , average fecundity/2 (female pups/adult female).

3. Extinction risk

- Originally developed by Myers et al. (1997)
- Subsequently modified by Myers and Mertz (1998) to explicitly incorporate level of fishing that could be imposed on a stock before it went extinct

$$e^{r_m a} - e^{-M} (e^{r_m})^{a-1} - \tilde{\alpha} = 0$$

$$\tilde{\alpha} = ml_a$$

3. Skalski et al.'s (2008)

- Is identical to the correct Myers et al. (1997) formula
- Does not require an estimate of longevity
- a is age at first reproduction

$$e^{ra} - e^{-M} (e^r)^{a-1} - ml_a = 0$$

4. Eberhardt et al.'s (1982)

- Also assumes constant adult survival and fecundity rates, but includes lifespan (w)

$$e^{ra} - e^{-M} (e^r)^{a-1} - ml_a \left(1 - \left(\frac{e^{-M}}{e^r} \right)^{w-a+1} \right) = 0$$

5. Demographic Invariant Method

- Combines an age-based matrix model with an allometric model
- The allometric model is based on relationships between r_{max} and T and body mass (M), such that $r_{max} = a r M^{-0.25}$ and $T = a^T M^{-0.25}$, which when multiplied yield the dimensionless maximum rate of increase per generation or $r_{max} T = a^r a^T = a^{rT}$.
- Niel & Lebreton (2005) found that $a^{rT} \approx 1$ for birds and Dillingham et al. (2016) recently found that $a^{rT} \approx 1$ for several vertebrate taxa (birds, mammals, and elasmobranchs), thus r_{max} can be obtained from knowledge of a and S only.

$$e^r = e^{\left(a_{rT} \frac{1}{\left(a + \frac{e^{-M}}{e^r - e^{-M}} \right)} \right)}$$

6. Leslie Matrix

The fertility term can be expressed in different ways:

$f_x = p_0 m_x$ (reproduction first, then survival: pre-breeding census)

$f_x = p_x m_{x+1}$ (survival first, then reproduction: post-breeding census)

The Leslie Matrix: pre-breeding census ($fx=p_0mx$), ages 1 to $n=\max$

$$\mathbf{A} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 & \dots & \mathbf{F}_{n-1} & \mathbf{F}_n \\ \mathbf{P}_1 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_2 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P}_3 & \dots & \mathbf{0} & \mathbf{0} \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{P}_{n-1} & \mathbf{0} \end{pmatrix}$$

Biological input values (females and males) used in the stock assessment

Parameter(s)	Value(s)	Reference(s)
Growth relationships	Female / Male / Sexes combined	
L_{∞} (cm)	166.23 (2.47) / 145.03 (1.82) / 159.30 (1.87)	SEDAR65-DW-02
K	0.16 (0.01) / 0.23 (0.02) / 0.17 (0.01)	SEDAR65-DW-02
t_0 (years)	-2.59 (0.16) / -1.97 (0.16) / -2.51 (0.13)	SEDAR65-DW-02
Maximum observed age (years)	17.5 / 13.5	SEDAR65-DW-02
Sample size	269 / 278 / 547	SEDAR65-DW-02
Length-weight relationships		
PCL (cm)	$PCL = 1.92990 + 0.885043*FL$	SEDAR65-DW-15
NTL (cm)	$NTL = 4.89349 + 1.15734*FL$	SEDAR65-DW-15
STL (cm)	$STL = 9.00754 + 1.16776*FL$	SEDAR65-DW-15
Wt (kg)	$Wt = (4.63 \times 10^{-6})FL^{3.21575}$	SEDAR65-DW-15
Age at 50% maturity		
Female	$t_{mat} = 6.69$ years $a = -12.07 (2.52)$ $b = 1.80 (0.35)$	SEDAR65-DW-01
Male	$t_{mat} = 5.34$ years $a = -9.09 (1.72)$ $b = 1.70 (0.29)$	SEDAR65-DW-01
Size at 50% maturity		
Female	$FL_{mat} = 123.05$ cm FL $a = -30.09 (4.66)$ $b = 0.24 (0.04)$	SEDAR65-DW-01
Male	$FL_{mat} = 115.15$ cm FL $a = -31.41 (5.34)$ $b = 0.27 (0.04)$	SEDAR65-DW-01
Reproductive cycle	Biennial	Castro 1996, Gelsleichter pers. comm.
Fecundity	4.09 (SD = 0.13) pups per brood	SEDAR65-DW-01
Maternal age/fecundity relationship	Brood size = $-0.04078 + 0.38445*Age$	SEDAR65-DW-01
Maternal size/fecundity relationship	Brood size = $-5.82556 + 0.06857*FL$	SEDAR65-DW-01
Gestation	11 months	Castro 1996, Ulrich et al. 2007
Pupping month	late May / June	Castro 1996, Ulrich et al. 2007, Frazier pers. comm.



M (instantaneous natural mortality rate) used in the reference case (minimum)

Females		Males	
Age	M	Age	M
0	0.198	0	0.273
1	0.198	1	0.237
2	0.198	2	0.203
3	0.185	3	0.183
4	0.171	4	0.170
5	0.161	5	0.161
6	0.153	6	0.155
7	0.147	7	0.150
8	0.143	8	0.147
9	0.139	9	0.144
10	0.136	10	0.142
11	0.133	11	0.140
12	0.131	12	0.139
13	0.130	13	0.138
14	0.128	14	0.137
15	0.127	15	0.137
16	0.126	16	0.136
17	0.125	17	0.136
18	0.125	18	0.136
19	0.124	19	0.136
20	0.123	20	0.135
21	0.123	21	0.135
22	0.123		
23	0.122		
24	0.122		
25	0.122		
26	0.122		
27	0.122		
28	0.121		
29	0.121		
30	0.121		
31	0.121		
Average	0.139	Average	0.158



M (instantaneous natural mortality rate) used in sensitivity analyses (mean)

Females		Males	
Age	M	Age	M
0	0.261	0	0.340
1	0.252	1	0.328
2	0.247	2	0.322
3	0.244	3	0.318
4	0.241	4	0.315
5	0.239	5	0.314
6	0.238	6	0.312
7	0.237	7	0.311
8	0.236	8	0.311
9	0.235	9	0.310
10	0.235	10	0.310
11	0.234	11	0.309
12	0.234	12	0.309
13	0.234	13	0.309
14	0.233	14	0.309
15	0.233		
16	0.233		
17	0.233		
18	0.233		
Average	0.239	Average	0.315

